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MORE-FOR-LESS PARADOX IN A SOLID TRANSPORTATION PROBLEM

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Abstract. In this paper, we discuss more-for-less paradox in a solid transportation problem. Thereby, we demonstrate a theorem which gives a comfortable condition for the existence of paradox in this type of problem. Next we present an algorithm to find out all the paradoxical pairs as well as paradoxical range of flow and paradoxical pair for a specified flow if paradox exists. Also we illustrate a numerical example in support of the given algorithm.

Keywords: Solid Transportation Problem, Paradox in a Solid Transportation Problem, Paradoxical Range of Flow

1. Introduction

The solid transportation problem(STP) is a generalization of the classical transportationproblem(TP). Haley [13] was the pioneer in this field. The STP includes three types of constraints viz., source constraint, destination constraint and capacity (e.g., conveyance, various types of products, etc.) constraint instead of two constraints viz., source constraint and destination constraint in classical TP. The STP has an immense application in real life problems. Matvenco[14]considered combinatorial approach to the problem of solvability of the solid (three-index) transportation problem. Gen et al. [11] considered the STP in fuzzy environment. Basuet al.[8] gave an algorithm for obtaining the optimum time-cost trade-off in STP. They[9]also developed an algorithm for finding the optimum solution of solid fixed charge transportation problem. Basu and Acharya[5] developed an algorithm for the optimum time-cost trade-off in generalized STP. They [6] also considered on quadratic fractional generalized bi-criterion STP. Yang and Feng[20] studied on bi-criteria STP with fixed charge under stochastic environment. Ojhaet al.[15] considered bi-criterion an entropy based STP under fuzzy environment. They [16] also developed STP for an item with fixed charge, vechicle cost and price discounted varying charge using genetic algorithm. Tao and Xu[18] considered a class of rough multiple objective programming and its application to STP.

In some cases of the classical TP, an increase in the supplies and demands or in other words, increase in the flow results a decrease in the optimum transportation cost. This type of behavior which means paradoxical, is called transportation paradox. Basically, the papers of Charnes and Klingman[10] and Szwarc[17] are treated as the sources of transportation paradox for the researchers. In the paper of Charnes and Klingman, they name it “more-for-less" paradox and wrote “The paradox was first observed in the early days of linear programming history (by whom no one knows) and has been a part of the folklore known to some (e.g. A.Charnes and W.W.Cooper), but unknown to the great majority of workers in the field of linear programming”. Subsequently, in the paper of Appa[4], he mentioned that this paradox is known as “Doig Paradox” at the London School of Economics, named after Alison Doig. Gupta et al.[12] established a sufficient condition for a paradox in a linear fractional transportation problem with mixed constraints. Adlakha and Kowalski [3] derived a sufficient condition to identify the cases where the paradoxical situation exists. Deinekoet al.[19] developed a necessary and sufficient condition for a cost matrix which is immuned against the transportation paradox. Basuet. al.[7] provided an algorithm for obtaining paradoxical range of flow and paradoxical flow for a specified flow.Acharyaet al. [1] developed an algorithm for obtaining paradoxical range of flow and paradoxical flow for a specified flow in a fixed charge transportation problem. They [2] also considered paradox in a fuzzy transportation problem with linear constraints.

In this paper, we present a method for solving solid transportation problem with linear constraints. Thereby, we state a sufficient condition for existence of paradox. Then we give an algorithm for obtaining all paradoxical pairs, paradoxical range of flow and paradoxical pair for a specified flow in such type of problem. We also justify the theory by illustrating a numerical example.

2. Problem Formulation

In this paper, our goal is to obtain a transportation plan which satisfies all required demands and minimizes the overall transportation cost of the problem:

$$P_1: \ Min\, Z = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} p_{ijk}x_{ijk}$$

subject to the constraints,

$$\sum_{j=1}^{n}\sum_{k=1}^{l} x_{ijk} = a_i \ \forall \ i \in I = (1,2,3,\ldots..m)$$
$$\sum_{i=1}^{m}\sum_{k=1}^{l} x_{ijk} = b_j \ \forall \ j \in J = (1,2,3,\ldots..n)$$
$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = c_k \ \forall \ k \in K = (1,2,3,\ldots..l)$$

and $x_{ijk} \geq 0 \quad \forall \quad (i,j,k) \in I \times J \times K$

where,

x_{ijk}= the amount of k-th type of product transported from the i-thorigin to the j-thdestination,

p_{ijk}= the cost involved in transporting per unit of the k-th type of product from the i-thorigin to the j-thdestination,

a_i= the number of units available at the i-thorigin,

b_j= the number of units required at the j-thdestination,

c_k= requirement of the number of units of the k-th product.

Hence this problem consists of m origins and n destinations along with l types of products. We assume that $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \sum_{k=1}^{l} c_k$and $a_i, b_j, c_k > 0$ for all i, j, k which is known as balanced STP. If the STP P_1 is unbalanced then we convert it to a balanced STP by using dummy variables with zero cost.

Let $X^\alpha = \{x_{ijk}^\alpha | (i,j,k) \in I \times J \times K\}$ be a basic feasible solution corresponding to the basis B of theproblemP_1, the corresponding value of the objective function $Z^\alpha = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} p_{ijk}x_{ijk}^\alpha$ and the flow $F^\alpha = \sum_{i\in I} a_i = \sum_{j\in J} b_j = \sum_{k\in K} c_k$

Definition 2.1. The pair (Z^α, F^α) is called the **cost-flow pair** corresponding to the feasible solution X^α.

Condition of optimality: The condition of optimality of the problem P_1 is $p'_{ijk} = (u_i + v_j + w_k) - p_{ijk} \leq 0$ for all $(i,j,k) \notin B$, where u_i, v_j, w_k are the dual variables corresponding to the basis B such that $p_{ijk} = (u_i + v_j + w_k)$[11].

Let $Z^0 = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} p_{ijk}x_{ijk}^0$ and $F^0 = \sum_{i\in I} a_i = \sum_{j\in J} b_j = \sum_{k\in K} c_k$ be the optimum cost and flow respectively corresponding to the optimum solution$X^0 = \{x_{ijk}^0 | (i,j,k) \in I \times J \times K\}$.

Definition 2.2 In a STP, if we can obtain flow $F^1 > F^0$ with cost $Z^1 < Z^0$then we say **paradox** occurs.

Definition 2.3 The pair (Z^1, F^1) is called the **paradoxical cost-flow pair** if paradox exists.

Definition 2.4A cost-flow pair (Z^2, F^2) is called an **improved paradoxical cost-flow pair** if $Z^2 < Z^1$ and $F^2 > F^1$.

Definition 2.5 The paradoxical cost-flow pair (Z^*, F^*) where (Z^i, F^i), $1 < i < g$, such that $Z^i < Z^{i-1}$and $F^i > F^{i-1}\ \forall\, i$ be all paradoxical cost-flow pairs and $Z^* = Z^g$ and $F^* = F^g$, is called the **best paradoxical cost-flow pair**.

Definition 2.5 The pair $[F^0, F^*]$ is defined as **paradoxical range of flow**.

Theorem 2.1. The sufficient condition for the existence of paradoxical solution of the problem $P_1$is thatif ∃at least one cell(r,s,t) ∉B in the optimum table of $P_1$where $a_r, b_s$and $c_t$are replaced by $a_r + q$, $b_s + q$and$c_t + q$respectively $(q > 0)$then $(u_r + v_s + w_t) < 0$.

Proof: Let (Z^0, F^0)be the optimum cost-flow pair corresponding to the optimum solution $X^0 = \{x_{ijk}^0 | (i,j,k) \in I \times J \times K\}$ of the problemP_1. The dual variables u_i, v_j and w_k satisfy the equation$p_{ijk} = (u_i + v_j + w_k)\ \forall\ (i,j,k) \in B$.

Then

$$Z^0 = \sum_i \sum_j \sum_k c_{ijk}\, x_{ijk}^0$$
$$= \sum_i \sum_j \sum_k (u_i + v_j + w_k)\, x_{ijk}^0$$
$$= \sum_i \left(\sum_j \sum_k x_{ijk}^0\right) u_i + \sum_j \left(\sum_k \sum_i x_{ijk}^0\right) v_j + \sum_k \left(\sum_i \sum_j x_{ijk}^0\right) w_k$$
$$= \sum_i a_i u_i + \sum_j b_j v_j + \sum_k c_k w_k,$$

and$F_c^0 = \sum_{i \in I} a_i = \sum_{j \in J} b_j = \sum_{k \in K} c_k$

Now, let ∃at least one cell(r,s,t) ∉B, where $a_r, b_s$and $c_t$are replaced by $a_r + q, b_s + q$and$c_t + q$respectively $(q > 0)$ in such a way that the optimum basis remains same, then the value of the objective function $\hat{Z}$is given by

$$\hat{Z} = \sum_{i, i \neq r} a_i u_i + \sum_{j, j \neq s} b_j v_j + \sum_{k, k \neq t} c_k w_k + u_r(a_r + q) + v_s(b_s + q) + w_t(c_t + q)$$
$$= [Z^0 + q(u_r + v_s + w_t)],$$

The new flow $\hat{F}$is given by $\hat{F} = \sum_{i \in I} a_i + q = \sum_{j \in J} b_j + q = \sum_{k \in K} c_k + q$. Therefore $\hat{F} - F^0 = q > 0$.

Hence for the existence of paradox we must have $\hat{Z} - Z^0 < 0$.So the sufficient condition for the existence of paradox is that∃at least one cell(r,s,t) ∉B in the optimum table of $P_1$where $a_r, b_s$and $c_t$are replaced by $a_r + q, b_s + q$and $c_t + q$respectively (q> 0) then $q(u_r + v_s + w_t) < 0$, i.e. $(u_r + v_s + w_t) < 0$.

Now we state two algorithms.

Algorithm 2.1 To obtain all the paradoxical pairs.

Step 1: Find the optimum cost-flow pair (Z^0, F^0) for the optimum solutionX^0.
Step 2: $i = 1$.
Step 3: Find all cells (r,s,t) ∉B such that $(u_r + v_s + w_t) < 0$ if it exists, otherwise go to step 8.
Step 4: $F^i = F^{i-1} + 1$
Step 5: Obtain X^i and Z^i corresponding toF^i. Write(Z^i, F^i).
Step 6: $i = i + 1$
Step 7: go to Step 3.
Step 8: Write the best paradoxical pair $(Z^*, F^*) = (Z^i, F^i)$for the optimum solution $X^* = X^i$.
Step 9: End.

Algorithm 2.2 To obtain the paradoxical pair for a specified flow $\overline{F}$.

Step 1: Find the optimum cost-flow pair (Z^0, F^0) for the optimum solution X^0.
Step 2: $i = 1$.
Step 3: Find all cells (r,s,t) ∉B such that $(u_r + v_s + w_t) < 0$.
Step 4: $F^i = F^{i-1} + 1$.
Step 5: Obtain X^i and Z^i corresponding to F^i. Write (Z^i, F^i).

Step 6: If $\overline{\boldsymbol{F}} = F^i$ go to step 9.
Step 7: $i = i + 1$.
Step 8: go to Step 3.
Step 9: We write the paradoxical solution for a specified flow $(\overline{\boldsymbol{Z}}, \overline{\boldsymbol{F}}) = (Z^i, F^i)$ corresponding to the optimum solution $\overline{\boldsymbol{X}} = X^i$.
Step 10: End.

3. Numerical Example

We consider the following problem (Figure 1)

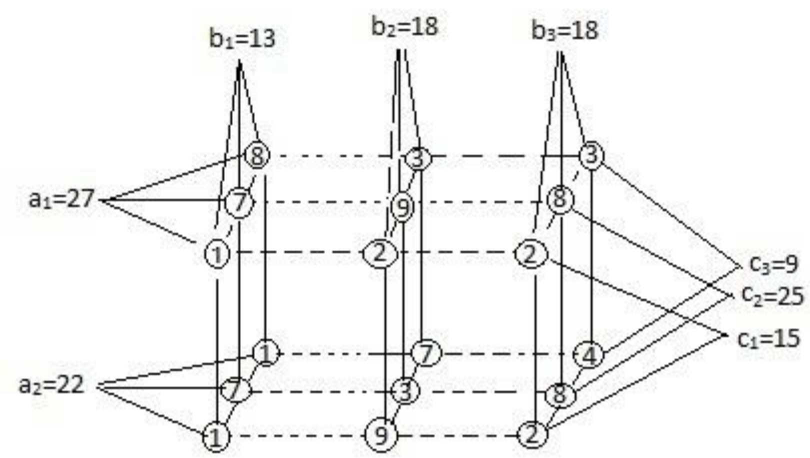

Figure 1: numerical example

Applying algorithm 2.1, the optimum cost-flow pair $(Z^0, F^0) = (150,49)$ corresponding to the optimumsolution$X^0 = \{x_{111} = 2, x_{112} = 7, x_{213} = 4, x_{222} = 18, x_{131} = 13, x_{133} = 5\}$(Figure 2).

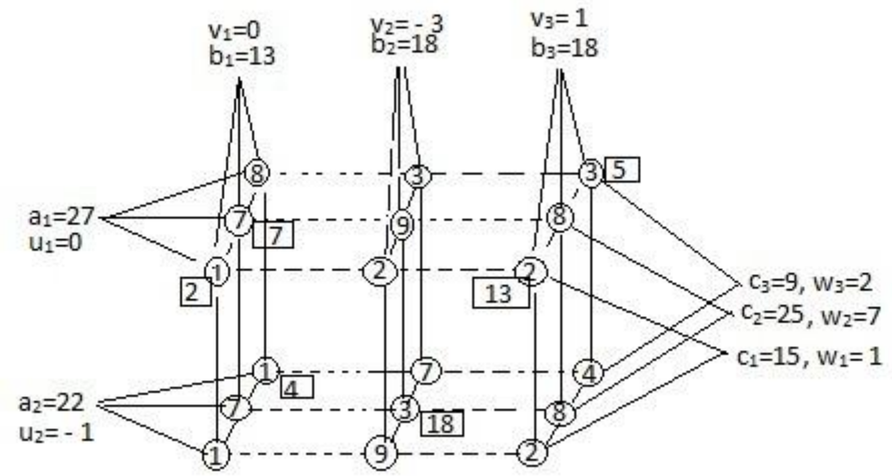

Figure 2 : optimum solution

We get the cells (1,2,1), (1,2,3), (2,2,1) and (2,2,3) with respective costs 148, 149, 147and 148. The paradoxical cost-flow pair $(Z^1, F^1) = (147,50)$ (Figure 3).

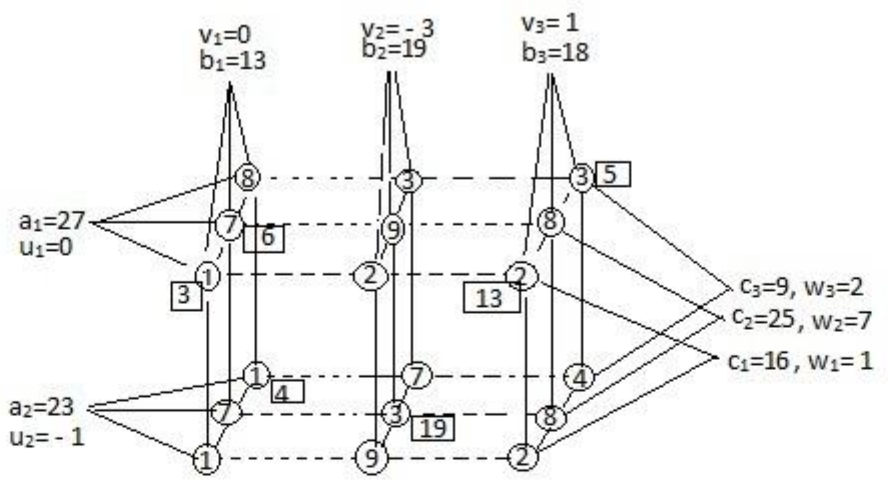

Figure 3 : paradoxical solution (Z^1, F^1)

All the paradoxical pairs are (150,49), (147,50), (144,51), (141,52), (138,53), (135, 54), (132, 55), (129,56).

Hence the best paradoxical pair is $(Z^*, F^*) = (129, 56)$ corresponding to the optimumsolution $X^* = \{x_{111} = 9, x_{213} = 4, x_{222} = 25, x_{131} = 13, x_{133} = 5\}$ (Figure 4).

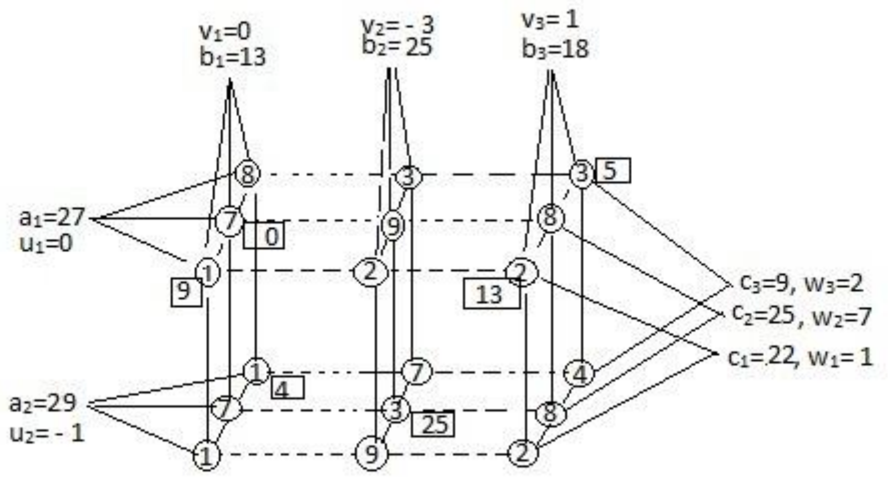

Figure 4 : best paradoxical solution

4. Conclusion

In real life, we face many problems which belong to three dimensional (solid) rather than two dimensional classical transportation problem. Practically, paradox in a solid transportation problem may occur quite frequently. But till date, researchers do not give any attention in this area.

In this paper, we discuss a paradox (so called ``more-for-less'' paradox) in a solidtransportation problem. Thereby, we develop a new efficient algorithm for solving paradox in a solid transportation problem if paradox exists. In this procedure, we not only obtain the best paradoxical pair but also all the paradoxical pairs as well as the paradoxical pair for a specified flow. Today, calculation is very simple and not time taking if one solves this type of problem using mathematical software. The managers in decisions such as increasing warehouse/plant capacity and/or advertising efforts to increase demand at some destinations and/or for some types of products may use this paradoxical analysis to increase his business under the same environment. Hence, in practically it is an important part of solid transportation problem.

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