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Finite-time hybrid synchronization of time-delay hyperchaotic Lorenz system

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Abstract. In this paper, based on Lyapunov stability theory, a scheme to realize finite-time hybrid synchronization of time-delay hyperchaotic Lorenz system is proposed. By theoretical analysis, the controllers are constructed. Finally, numerical simulations are given to verify the theoretical results.

Keywords: Finite-time hybrid synchronization, hyperchaotic Lorenz system, time-delay

1. Introduction

In the past few years, synchronization of chaotic system drew much attention of researchers because of its potential applications in many fields. Many kinds of synchronization have been investigated [1-5]. Accordingly, many effective schemes have been proposed [6-11]. With further research on synchronization, more and more people have realized the importance of the time in achieving synchronization. For this end, some methods have been proposed to investigate finite-time synchronization because of its showing the robustness and disturbance rejection properties of system [12]. Therefore, finite-time synchronization has been widely studied [13-18].

Based on the existing results, finite-time hybrid synchronization of time-delay hyperchaotic Lorenz system is investigated in this paper. Other parts of this paper are arranged as follows. Section 2 gives some preliminaries. In Section 3, the scheme to realize the finite-time hybrid synchronization of time-delay hyperchaotic Lorenz system is proposed. Simulation results are given in Section 4. Conclusion is drawn in Section 5.

2. Preliminaries

Definition 1. Suppose that x, y are n-dimensional state vectors and $f: R^n \to R^n$, $g: R^n \to R^n$ are vector-valued functions. Considering two chaotic systems as follows:

$$\dot{x} = f(x),$$

$$\dot{y} = g(y). \tag{1}$$

If there exists a positive constant T such that

$$\lim_{t \to T} \|x - y\| = 0,$$

and $\|x - y\| \equiv 0$ when $t \geq T$, then it is said that the two systems of (1) can achieve finite-time synchronization.

Lemma 1[19]. Assume that a continuous, positive-definite function $V(t)$ satisfies following differential inequality:

$$\dot{V}(t) \leq -cV^{\eta}(t), \forall t \geq t_0, V(t_0) \geq 0, \tag{2}$$

where $c > 0$, $0 < \eta < 1$ are all constants. Then for any given t_0, $V(t)$ satisfies following inequality:

$$V^{1-\eta} \leq V^{1-\eta}(t_0) - c(1-\eta)(t - t_0), t_0 \leq t \leq t_1, \tag{3}$$

and

$$V(t) \equiv 0, \forall t \geq t_1, \tag{4}$$

with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{c(1-\eta)}. \tag{5}$$

Proof. Consider differential equation:

$$\dot{X}(t) = -cX^{\eta}(t), X(t_0) = V(t_0). \tag{6}$$

Although equation (6) doesn't satisfy the global Lipschitz condition, the unique solution of it can be found as

$$X^{1-\eta}(t) = X^{1-\eta}(t_0) - c(1-\eta)(t - t_0). \tag{7}$$

Therefore, from the comparison Lemma [20], it can be gotten that

$$V^{1-\eta} \leq V^{1-\eta}(t_0) - c(1-\eta)(t - t_0), t_0 \leq t \leq t_1,$$

and

$$V(t) \equiv 0, \forall t \geq t_1$$

with t_1 given in (5).

Lemma 2[21]. Suppose $0 < r \leq 1$, a, b are all positive numbers, then the following inequality is quite straightforward:

$$(|a| + |b|)^r \leq |a|^r + |b|^r. \tag{8}$$

3. Finite-time hybrid synchronization of time-delay hyperchaotic Lorenz system

In this section, hyperchaotic Lorenz system with time-delay is considered as following:

$$\begin{aligned} \dot{x} &= a(y - x) + rw(t - \tau), \\ \dot{y} &= cx - y - xz, \\ \dot{z} &= xy - bz, \\ \dot{w} &= -yz - dw, \end{aligned} \tag{9}$$

where $\tau > 0$ is time delay. When $\tau = 0$, system (9) is hyperchaotic Lorenz system [22]. If $a = 10$, $b = 8/3$, $c = 28$, $d = 1$, $\tau = 1$, system (9) has hyperchaotic behavior with two positive Lyapunov exponents[23] $\lambda_1 = 0.6513$, $\lambda_2 = 0.1394$. The hyperchaotic attractors are shown in Fig. 1 (3D overview).

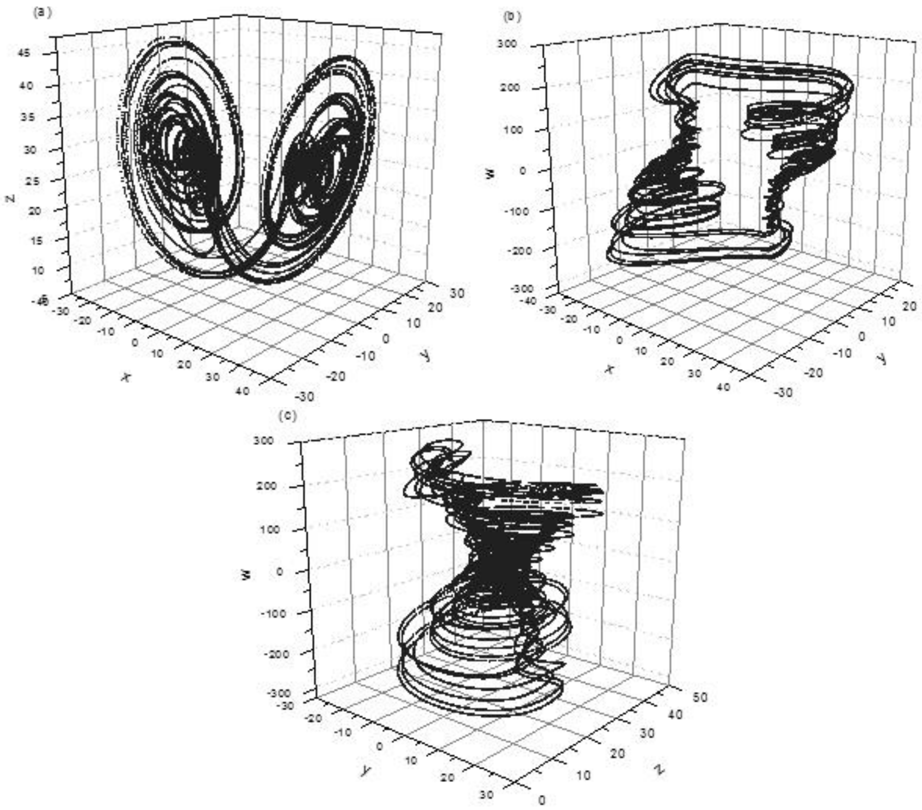

Fig. 1. Hyperchaotic attractors of system (9) (a) (x, y, z), (b)(x, y, w), (c)(y, z, w).

The drive system is given as (10):

$$\begin{aligned} \dot{x}_1 &= a(y_1 - x_1) + rw_1(t - \tau), \\ \dot{y}_1 &= cx_1 - y_1 - x_1 z_1, \\ \dot{z}_1 &= x_1 y_1 - bz_1, \\ \dot{w}_1 &= -y_1 z_1 - dw_1, \end{aligned} \tag{10}$$

and the response system is written as (11):

$$\dot{x}_2 = a(y_2 - x_2) + rw_2(t - \tau) + u_1,$$

$$\dot{y}_2 = cx_2 - y_2 - x_2 z_2 + u_2,$$

$$\dot{z}_2 = x_2 y_2 - bz_2 + u_3,$$

$$\dot{w}_2 = -y_2 z_2 - dw_2 + u_4, \tag{11}$$

where u_1, u_2, u_3, u_4 are controllers to be constructed.

To realize the finite-time hybrid synchronization between systems (11) and (10), that is, some variables of the two systems gain finite-time anti-synchronization while other variables reach finite-time complete synchronization. For this end, let

$$e_1 = x_2 + x_1,\ e_2 = y_2 + y_1,\ e_3 = z_2 - z_1,\ e_4 = w_2 + w_1, \tag{12}$$

and suppose following **Assumption 1** is satisfied.

Assumption 1(A1): Due to the bounded trajectories of hyperchaotic system, there exists a positive constant M meeting $|x_i| < M, |y_i| < M$, $|z_i| < M, |w_i| < M$ $(i = 1,2)$.

According to (10) and (11), the error system (12) is governed by the following dynamical system

$$\dot{e}_1 = a(e_2 - e_1) + re_4(t-\tau) + u_1,$$

$$\dot{e}_2 = ce_1 - e_2 - z_2 e_1 + x_1 e_3 + u_2,$$

$$\dot{e}_3 = -x_2 e_2 - y_1 e_1 - be_3 + u_3,$$

$$\dot{e}_4 = y_1 e_3 - z_2 e_2 - de_4 + u_4. \tag{13}$$

Based on above, following **Theorem** can be gotten.

Theorem Let $u_1 = -ae_2 - re_4(t-\tau) - e_1^{\beta}$, $u_2 = -Me_3 - e_2^{\beta}$, $u_3 = -Me_2 - e_3^{\beta}$, $u_4 = -e_4^{\beta}$. where M is the constant in A1, $\beta = \frac{n}{m}$ is a proper rational number, m, n are positive odd integers satisfying $m > n$, then the finite-time hybrid synchronization of the drive system (10) and response system (11) can be achieved.

Firstly, let $u_1 = -ae_2 - re_4(t-\tau) - e_1^{\beta}$, then $\dot{e}_1 = -ae_1 - e_1^{\beta}$, choose the first Lyapunov function as

$$V_1 = \frac{1}{2} e_1^2 ,$$

and we have

$$\dot{V}_1 = e_1 \dot{e}_1 = -ae_1^2 - e_1^{\beta+1} \le e_1^{\beta+1} = -2^{\frac{\beta+1}{2}} \left(\frac{1}{2} e_1^2\right)^{\frac{\beta+1}{2}} = -2^{\frac{\beta+1}{2}} V_1^{\frac{\beta+1}{2}},$$

According to Lemma 1, there exists a constant $T_1 > 0$, such that $e_1 \equiv 0$ if $t \ge T_1$. Thus the last three equations of (13) becomes

$$\dot{e}_2 = -e_2 + x_1 e_3 + u_2,$$

$$\dot{e}_3 = -x_2 e_2 - be_3 + u_3,$$

$$\dot{e}_4 = y_1 e_3 - z_2 e_2 - de_4 + u_4.$$

Secondly, let $u_2 = -Me_3 - e_2^{\beta}$, $u_3 = -Me_2 - e_3^{\beta}$, and the second Lyapunov function is taken as

$$V_2 = \frac{1}{2}(e_2^2 + e_3^2) V_2 = \frac{1}{2}(e_2^2 + e_3^2) ,$$

Using Lemma 2, we have

$$\begin{aligned}
\dot{V}_2 &= e_2 \dot{e}_2 + e_3 \dot{e}_3 \\
&= e_2\left(-e_2 + x_1 e_3 - Me_3 - e_2^{\beta}\right) + e_3\left(-x_1 e_2 - be_3 + Me_2 - e_3^{\beta}\right) \\
&= -e_2^2 - be_3^2 - e_2^{\beta+1} - e_3^{\beta+1} \\
&\le -(e_2^{\beta+1} + e_3^{\beta+1}) \\
&= -2^{\frac{\beta+1}{2}}((\tfrac{1}{2} e_2^2)^{\frac{\beta+1}{2}} + ((\tfrac{1}{2} e_3^2)^{\frac{\beta+1}{2}} \le -2^{\frac{\beta+1}{2}}[\tfrac{1}{2}(e_2^2 + e_3^3)]^{\frac{\beta+1}{2}} \\
&= -2^{\frac{\beta+1}{2}} V_2^{\frac{\beta+1}{2}}.
\end{aligned}$$

According to Lemma 1, there exists a constant T_2 $(T_2 > T_1 > 0)$, such that $e_2 \equiv 0$， $e_3 \equiv 0$ if $t \ge T_2$. Thus the last equation of (13) becomes $\dot{e}_4 = -de_4 + u_4$.

Thirdly, let $u_4 = -e_4^{\beta}$, choose the third Lyapunov function as

$$V_3 = \frac{1}{2}e_4^2,$$

and we can get

$$\dot{V}_3 = e_4\dot{e}_4 = -de_4^2 - e_4^{\beta+1} \le e_4^{\beta+1} = -2^{\frac{\beta+1}{2}}\left(\frac{1}{2}e_4^2\right)^{\frac{\beta+1}{2}} = -2^{\frac{\beta+1}{2}}V_3^{\frac{\beta+1}{2}},$$

According to Lemma 1, there exists a constant T_3 $(T_3 > T_2 > T_1 > 0)$ such that $e_4 \equiv 0$, when $t \ge T_3$.
Then it is said that the finite-time hybrid synchronization of hyperchaotic Lorenz system can be realized.

4. Numerical simulation

In this part, numerical simulations are given to verify the effectiveness of the proposed scheme. In simulations, the parameters are chosen as a=10, b=8⁄3, c=28, d=1 , the time delay is τ=1, with which the Lorenz system (9) is hyperchaotic. The initial values of the drive system (10) and response system (11) are set to be 〖(x〗_1 (0),〖 y〗_1 (0),z_1 (0),w_1 (0))=(0.8,0.28,0.45 ,0.1) and 〖(x〗_2 (0),〖 y〗_2 (0),z_2 (0),w_2 (0))=(0.76,0.25,0.35,0.2), respectively. β is chosen as 0.76. Time evolution of the variables of systems (10) and (11) are depicted in Fig.2. The dynamical behaviors of error system (13) is shown in Fig.3. From Fig.2 and Fig.3, it is obvious to see that the finite-time hybrid synchronization can be realized via the proposed scheme.

5. Results

In this paper, according to Lyapunov stability theory, a scheme to obtain the finite-time hybrid synchronization of hyperchaotic system is proposed from theoretical analysis. At the same time, the theoretical result is verified via numerical simulations.

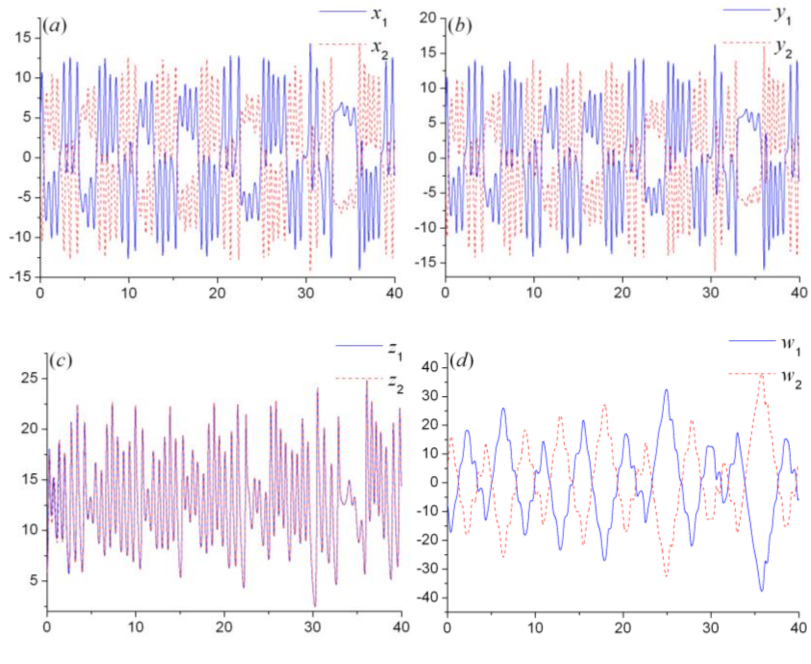

Fig.2 Time evolution of the variables of system (10) and (11) (a)(x_1, x_2), (b) (y_1, y_2), (c) (z_1, z_2), (d) (w_1, w_2).

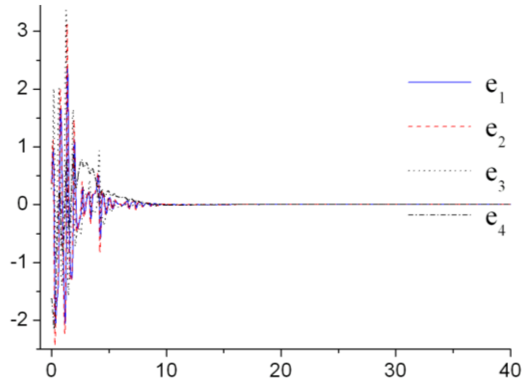

Fig.3 The dynamical behaviors of error system (13).

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