ISSN 1746-7659, England, UK
Journal of Information and Computing Science
Vol. 10, No. 4, 2015, *pp*. 243-248

Finite-time synchronization of time-delay Hindmarsh- Rose system with external disturbance

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(*Received November 04, 2014, accepted April 23, 2015)*

Abstract. In this paper, the finite-time synchronization of time-delay Hindmarsh-Rose with external disturbance is investigated. Based on Lyapunov stability theory, a scheme is proposed and controllers are constructed to realize it. Finally, numerical simulations are given to verify the theoretical results.

Keywords: Finite-time synchronization, Hindmarsh-Rose system, time-delay, external disturbance

1. Introduction

Recently, chaos synchronization has received much attention [1-5]. Many kinds of methods have been proposed to study the synchronization of chaotic system, such as drive-response synchronization method [6], adaptive control method [7], backstepping method [8], and so on. Various synchronization have been observed, such as complete synchronization [9], Lag synchronization [10], phase synchronization [11], etc. With further investigation, the time to realize synchronization received attention. Therefore finite-time synchronization is studied in many fields [12-17], especially in neuron system.

Experimental studies [18] suggested that synchronization has significant meaning in the information transferring of neurons. Meanwhile, in information transformation among neurons, not only the time-delay always exists, but also the external disturbance is inevitable. Therefore, it is necessary to investigate the synchronization of time-delay neural system with disturbance.

In this paper, finite-time synchronization of time-delay Hindmarsh-Rose system with external disturbance is to be explored. Other parts are arranged as follows. Section 2 gives some preliminaries. In section 3, a scheme is descripted to realize finite-time synchronization of time- delay Hindmarsh-Rose system with disturbance. Section 4 gives some numerical simulations. Result is given in Section 5.

2. Preliminaries

$$\dot{x} = f(x) + F(x)\alpha + d(x,t), \tag{1}$$
$$\dot{y} = g(y) + G(y)\beta + u(x,t), \tag{2}$$

where x , $y \in R^n$ are state vectors. $f(x), g(y) \in R^{n+1}$ are linear matrix functions. $F(x), G(y) \in R^{n+1}$ are nonlinear matrix functions. α, β are parameter vectors. $d(x,t) \in R^n$ is external disturbance. $u(x,t)$ is controller vector.

Hypothesis 1(H1): The nonlinear matrix function $g(x)$ satisfies Lipschitz condition, that is

$$\|g(x) - g(y)\| \le L_g \|x - y\|,$$

where L_g is an appropriate positive constant. $\|\cdot\|$ denotes the norm of matrix or vector, defined as $\|A\| = (\sum_{j=1}^{m}\sum_{i=1}^{n} a_{ij}{}^2)^{\frac{1}{2}}$ for matrix $A = (a_{ij})_{m\times n}$ or $\|x\| = (\sum_{i=1}^{n} x_i^2)^{\frac{1}{2}}$ for vector $x = (x_1, \cdots, x_n)$.

Hypothesis 2(H2): The uncertain parameters α, β and disturbance $d(x,t)$ are all bounded in terms of norm, namely, there exist positive constants $\theta_\alpha, \theta_\beta, \theta_d$ such that

Let $e = y - x$, subtracting (1) from (2) yields

$$\dot{e} = \dot{y} - \dot{x} = g(y) - f(x) + G(y)\beta - F(x)\alpha + u(x, t) - d(x, t). \tag{3}$$

Therefore, to realize the finite-time synchronization of systems (1) and (2) means to obtain finite-time stability of error system (3). For this end, following definition of finite-time synchronization and some necessary lemmas are introduced as follows.

Definition 1[19] Consider two chaotic systems

$$\dot{x}_m = f(x_m),$$

$$\dot{x}_s = h(x_s), \tag{4}$$

where x_m, x_s are two n-dimensional vectors. The subscripts 'm' and 'n' stand for the master and slave systems, respectively. $f: R^n \to R^n$ and $h: R^n \to R^n$ are vector-valued functions. If there is a positive constant T such that

$$\lim_{t\to T}\|x_m - x_s\| = 0,$$

and $\|x_m - x_s\| \equiv 0$ if $t \geq T$, then it is said that the finite-time synchronization between two systems of (4) can be achieved.

Lemma 1[20] Assume that a continuous, positive-definite function $V(t)$ satisfies the following differential inequality:

$$\dot{V}(t) \leq -cV^{\eta}(t),\ \forall t \geq t_0, V(t_0) \geq 0, \tag{5}$$

where $c > 0$, $0 < \eta < 1$ are all constants. Then for any given t_0, $V(t)$satisfies following inequality:

$$V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - c(1-\eta)(t-t_0),\ \ t_0 \leq t \leq t_1, \tag{6}$$

and

$$V(t) \equiv 0, \forall t \geq t_1 \tag{7}$$

with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{c(1-\eta)}. \tag{8}$$

Proof. Consider differential equation:

$$\dot{X}(t) = -cX^{\eta}(t), X(t_0) = V(t_0). \tag{9}$$

Although equation (9) doesn't satisfy the global Lipschitz condition, the unique solution of it can be found as

$$X^{1-\eta}(t) = X^{1-\eta}(t_0) - c(1-\eta)(t-t_0). \tag{10}$$

Therefore, from the comparison Lemma [20], it can be gotten that

$$V^{1-\eta} \leq V^{1-\eta}(t_0) - c(1-\eta)(t-t_0),\ \ t_0 \leq t \leq t_1, \tag{11}$$

and

$$V(t) \equiv 0, \forall t \geq t_1 \tag{12}$$

with t_1 given in (8).

Lemma 2[21] Suppose $0 < r \leq 1$, a, b are positive constants, then the following inequality is quite straightforward:

$$(|a| + |b|)^r \leq |a|^r + |b|^r.$$

3. Finite-time synchronization of time-delay Hindmarsh-Rose system with disturbance

In this paper, Hindmarsh-Rose (HR) system with time-delay is considered as following:

$$\begin{aligned} \dot{x} &= ax^2 - bx^3 + y - z(t-\tau) + I_{ext}, \\ \dot{y} &= c - dx^2 - y, \\ \dot{z} &= r(S(x+k) - z), \end{aligned} \tag{13}$$

where $\tau > 0$ is the time delay. When$\tau = 0$, model (13) is a mathematical representation of the firing behavior of neuron proposed by Hindmarsh and Rose [22]. In system (13), the variables x, y and z represent the membrane potential of the neuron, the recovery variable, and the adaptation current, respectively. The current I_{ext} represents an external influence on the system. a, b, c, d, r, S, k are real constants.

Model (13) may describe regular bursting or chaotic bursting for certain domains of the parameters. When $\tau = 1$, other parameters are chosen as $a = 3.0$, $b = 1.0$, $c = 1.0$, $d = 5.0$, $r = 0.006$, $S = 4.0$, $k = 1.6$, system (13) can show various complex dynamical behaviors with the changing of I_{ext}. For example, when $I_{ext} = 2.6$ and $I_{ext} = 3.1$, system (13) is regular bursting (Fig.1) and chaotic bursting (Fig.2), respectively.

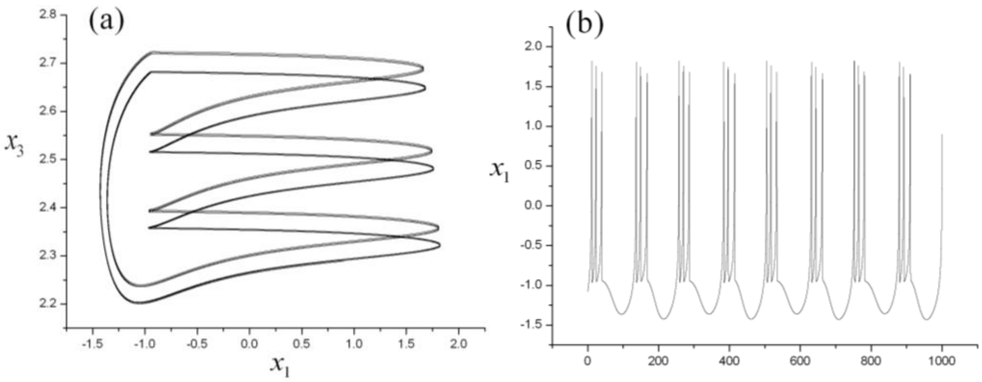

Fig.1. Regular bursting of system (13) for $I_{ext} = 2.6$. (a) Phase portrait, (b)Time series

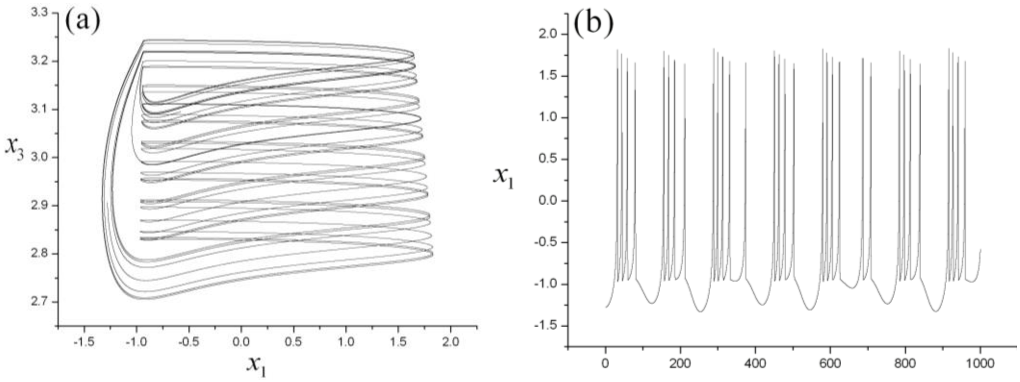

Fig.2. Chaotic bursting of system (13) for $I_{ext} = 3.1$. (a) Phase portrait, (b)Time series

For simplicity, let $rS = p$, $rSk = q$, then system (13) can be written as following:

$$\begin{aligned} \dot{x} &= ax^2 - bx^3 + y - z(t-\tau) + I_{ext}, \\ \dot{y} &= c - dx^2 - y, \\ \dot{z} &= px + q - rz. \end{aligned} \tag{14}$$

The drive system is written as

$$\begin{aligned} \dot{x}_1 &= ax_1^2 - bx_1^3 + x_2 - x_3(t-\tau) + I_{ext} + d_1(x,t), \\ \dot{x}_2 &= c - dx_1^2 - x_2 + d_2(x,t), \\ \dot{x}_3 &= px_1 + q - rx_3 + d_3(x,t). \end{aligned} \tag{15}$$

and the controlled HR system can be considered as

$$\begin{aligned} \dot{y}_1 &= ay_1^2 - by_1^3 + y_2 - y_3(t-\tau) + I_{ext} + u_1(x,t), \\ \dot{y}_2 &= c - dy_1^2 - y_2 + u_2(x,t), \\ \dot{y}_3 &= py_1 + q - ry_3 + u_3(x,t). \end{aligned} \tag{16}$$

Let $x = (x_1, x_2, x_3)^T$, $y = (y_1, y_2, y_3)^T$, then systems (15) , (16) and the error system can be rewritten as follows, respectively:

$$\dot{x} = f(x) + F(x)\alpha + d(x,t), \tag{17}$$

$$\dot{y} = g(y) + G(y)\beta + u(x,t), \tag{18}$$

$$\dot{e} = \dot{y} - \dot{x} = g(y) - f(x) + G(y)\beta - F(x)\alpha + u(x,t) - d(x,t). \tag{19}$$

where $f(x) = \begin{pmatrix} x_2 + I_{ext} \\ c - x_2 \\ q \end{pmatrix}$, $F(x) = \begin{bmatrix} x_1^2 & -x_1^3 & 0 & 0 & 0 & -x_3(t-\tau) \\ 0 & 0 & -x_1^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & x_1 & -x_3 & 0 \end{bmatrix}$,

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$d(x,t) = \begin{pmatrix} d_1(x,t) \\ d_2(x,t) \\ d_3(x,t) \end{pmatrix}$, $g(y) = \begin{pmatrix} y_2 + I_{ext} \\ c - y_2 \\ q \end{pmatrix}$, $G(y) = \begin{bmatrix} y_1^2 & -y_1^3 & 0 & 0 & 0 & -y_3(t-\tau) \\ 0 & 0 & -y_1^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & y_1 & -y_3 & 0 \end{bmatrix}$,

$$u(x,t)=\begin{pmatrix}u_1(x,t)\\u_2(x,t)\\u_3(x,t)\end{pmatrix},\ \alpha=\beta=(a,b,d,p,r,1)^T.$$

It is easy to see that H1 and H2 hold for systems (17) and (18). There exist constants $M>0$ and $L_g=\sqrt{2}$ such that

$$\|f(x)-g(x)\|=0<M,$$
$$\|g(x)-g(y)\|\le L_g\|x-y\|.$$

The following Theorems can be obtained.

Theorem 1 When the parameters of systems (15) and (16) are known, the finite-time synchronization of them can be achieved if H1, H2 hold and the controller is chosen as

$$u(x,t)=-L_g e-(\theta_d+1)\frac{e}{\|e\|}-(G(y)-F(x))\alpha,$$

namely,

$$u_1=-L_g e_1-(\theta_d+1)\frac{e_1}{\|e\|}-\left(ae_1(y_1+x_1)-be_1(y_1^2+y_1x_1+x_1^2)-e_3(t-\tau)\right),$$
$$u_2=-L_g e_2-(\theta_d+1)\frac{e_2}{\|e\|}-de_1(y_1+x_1),$$
$$u_3=-L_g e_3-(\theta_d+1)\frac{e_3}{\|e\|}-(pe_1-re_3).$$

Proof. The Lyapunov function is chosen as

$$V=\frac{1}{2}e^Te,$$

then

$$\dot V=e^T\dot e=e^T(g(y)-f(x)+(G(y)-F(x))\alpha+u(x,t)-\ d(x,t))$$
$$\le L_g\|e\|^2+e^T\left((G(y)-F(x))\alpha+u(x,t)\right)+\theta_d\|e\|$$
$$=L_g\|e\|^2+e^T\left((G(y)-F(x))\alpha-L_g e-(\theta_d+1)\frac{e}{\|e\|}-(G(y)-F(x))\alpha\right)+\theta_d\|e\|$$
$$=-\|e\|\le-(\|e\|^2)^{\frac{1}{2}}=-2^{\frac{1}{2}}V^{\frac{1}{2}}.$$

According to Lemma 1, there exists a constant $T>0$, such that $\|e\|\equiv 0$ when $t\ge T$.

According to Definition1, the finite-time synchronization of time-delay Hindmarsh-Rose system with disturbance can be realized when parameters are known.

Theorem 2 When the parameters of systems (15) and (16) are unknown, the finite-time synchronization of them can be achieved if H1, H2 hold and the controller is chosen as

$$u(x,t)=-L_g e-(\theta_d+1)\frac{e}{\|e\|}-(G(y)-F(x))\alpha$$

and parameter update law is selected as

$$\dot{\hat\alpha}=\frac{\alpha-\hat\alpha}{\|\alpha-\hat\alpha\|},$$

namely,

$$u_1=-L_g e_1-(\theta_d+1)\frac{e_1}{\|e\|}-\left(ae_1(y_1+x_1)-be_1(y_1^2+y_1x_1+x_1^2)-e_3(t-\tau)\right),$$
$$u_2=-L_g e_2-(\theta_d+1)\frac{e_2}{\|e\|}-de_1(y_1+x_1),$$
$$u_3=-L_g e_3-(\theta_d+1)\frac{e_3}{\|e\|}-(pe_1-re_3).$$

Proof. The Lypunov function is chosen as

$$V=\frac{1}{2}(e^Te+(\alpha-\hat\alpha)^T(\alpha-\hat\alpha)),$$

Using Lemma 2, and we have

$$\dot V=e^T\dot e-(\alpha-\hat\alpha)\dot{\hat\alpha}$$
$$=e^T\left(g(y)-f(x)+(G(y)-F(x))\alpha+u(x,t)-\ d(x,t)\right)-(\alpha-\hat\alpha)\frac{\alpha-\hat\alpha}{\|\alpha-\hat\alpha\|}$$
$$\le e^T\left(L_g e+(G(y)-F(x))\alpha-L_g e-(\theta_d+1)\frac{e}{\|e\|}-(G(y)-F(x))\alpha\right)-(\alpha-\hat\alpha)\frac{\alpha-\hat\alpha}{\|\alpha-\hat\alpha\|}$$
$$\le-\|e\|-\|\alpha-\hat\alpha\|$$
$$\le-\left((\|e\|^2)^{\frac{1}{2}}+(\|\alpha-\hat\alpha\|^2)^{\frac{1}{2}}\right)\le-(\|e\|^2+\|\alpha-\hat\alpha\|^2)^{\frac{1}{2}}=-2^{\frac{1}{2}}V^{\frac{1}{2}}.$$

According to Definition1, the finite-time synchronization of time-delay Hindmarsh-Rose system with disturbance can be realized when parameters are unknown.

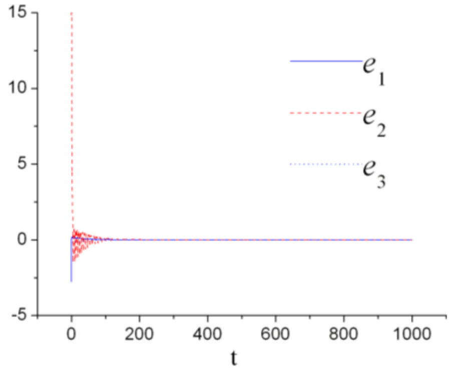
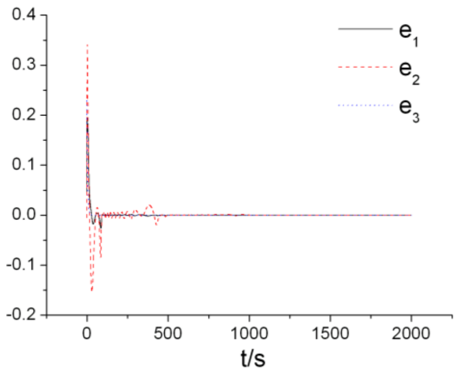


Fig. 3. Evolution of error of $e = (e_1, e_2, e_3)^T$ when parameters are known.

4. Numerical simulations

In this section, numerical simulations are given to verify the effectiveness of the proposed scheme. In the simulations, $d(x,t) = (0.02x_2 sint, 0.01x_3 cos2t, 0.1x_1 sin2t)^T$. The parameters of HR system is taken as $a = 3.0$, $b = 1.0$, $c = 1.0$, $d = 5.0$, $r = 0.006$, $S = 4.0$, $k = 1.6$ and $I_{ext} = 3.1$. The initial conditions of the master system and slave system are set as $x_1(0) = 0.1$, $x_2(0) = 0.9$, $x_3(0) = 0.7$ and $y_1(0) = 0.3$, $y_2(0) = 0.5$, $y_3(0) = 0.6$, respectively. Fig.3 gives the evolution of error $e = (e_1, e_2, e_3)^T$ when parameters are known. Fi.4 gives the curve of synchronization error $e = (e_1, e_2, e_3)^T$ when parameters are unknown. From Fig.3 and Fig.4, it's obvious to see that the finite-time synchronization of time-delay HR system with disturbance can be achieved whether the parameters of the system are known or unknown.



Fi.4 Curve of synchronization error $e = (e_1, e_2, e_3)^T$ when parameters are unknown.

5. Conclusion

In this paper, according to Lyapunov stability theory, a scheme is proposed to realize the finite-time synchronization of time-delay Hindmars-Rose with disturbance in two cases. In one case, the parameters are known. In another case, the parameters are unknown. Numerical simulations show that the proposed method is effective and practicable.

6. References

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