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Pinning synchronization of Hyperchaotic network with time delay via one controller

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Abstract. In this paper, pinning synchronization of unilateral coupled Hyperchaotic network with time delay is investigated. According to Lyapunov stability theory, an appropriate controller and particular Lyapunov function are designed, then the pinning synchronization of a network is realized in which every node is a hyperchaotic Lorenz system. Numerical simulations are given to verify the effectiveness of the proposed method.

Keywords: Pinning synchronization; Time delay; Unilateral coupling

1. Introduction

Synchronization has been studied in many fields [1, 2] and many kinds of it are observed, such as, mutual synchronization [3], chaotic synchronization [4], complete synchronization [5], phase synchronization [6], generalized synchronization [7], etc.. Some approaches have been proposed to reach synchronization of chaotic systems [8-12]. At the same time, with the development of nonlinear dynamics, complex network attracted more and more attention of researchers, for example, the WWW, the Internet, social network and cited network. To know complex network better, many people start to study the dynamics of complex network [13-17]. As an important behavior of complex network, pinning synchronization received much attention in the past few years.

Based on above, pinning synchronization of unilateral coupled hyperchaotic network is to be investigated in this paper. Other parts of this paper are arranged as follows. Hyperchaotic network is presented in Section 2. Scheme to realize the pinning synchronization is introduced in Section 3. Section 4 gives some numerical simulations. Conclusion is shown in Section 5.

2. System description

In this paper, Lorenz system with time-delay is considered as follows:

$$\begin{aligned} \dot{x} &= a(y-x) + rw(t-\tau), \\ \dot{y} &= cx - y - xz, \\ \dot{z} &= xy - bz, \\ \dot{w} &= -yz - dw, \end{aligned} \tag{1}$$

where $\tau > 0$ is the time delay. When $\tau = 0$, system (1) is the hyperchaotic system [18]. If parameters are chosen as $a = 10$, $b = 8/3$, $c = 28$, $d = 1$, $\tau = 1$, system (1) has two positive Lyapunov exponents[19], $\lambda_1 = 0.6513$, $\lambda_2 = 0.1394$. It means that system (1) has hyperchaotic behavior. The hyperchaotic attractors of system (1) are depicted in Fig. 1 (3D overview).

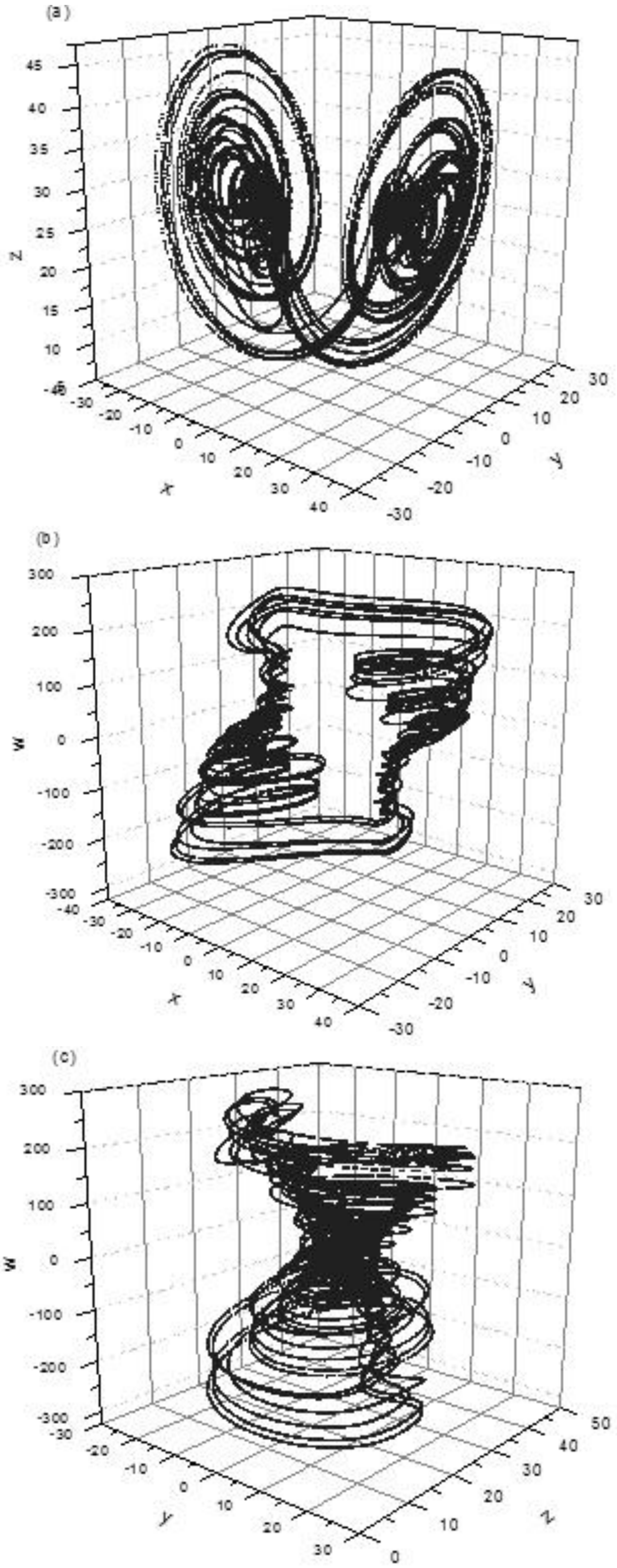

Fig. 1. 3D overview hyperchaotic attractor of system (1) . (a) (x, y, z), (b) (x, y, w), (c)(y, z, w).

The unilateral coupled hyperchaotic system with time delay is expressed as follows.

$$\dot{x}_1 = a(y_1 - x_1) + rw_1(t - \tau),$$

$$\dot{y}_1 = cx_1 - y_1 - x_1 z_1,$$

$$\dot{z}_1 = x_1 y_1 - b z_1,$$

$$\dot{w}_1 = -y_1 z_1 - d w_1,$$
$$\dot{x}_i = a(y_i - x_i) + r w_i(t-\tau) + H_i(x_{i-1} - x_i),$$

$$\dot{y}_i = c x_i - y_i - x_i z_i,$$

$$\dot{z}_i = x_i y_i - b z_i,$$

$$\dot{w}_i = -y_i z_i - d w_i, \quad (2)$$

where H_i $(i = 2, \cdots, N)$ are coupling intensity between nodes.

3. Control scheme

In this section, appropriate controller is designed to realize the pinning synchronization between system (2) and any given reference signal $s(t)$. For this, the controlled unilateral coupling network is given as follows:

$$\begin{aligned} \dot{x}_1 &= a(y_1 - x_1) + r w_1(t-\tau), \\ \dot{y}_1 &= c x_1 - y_1 - x_1 z_1 + u, \\ \dot{z}_1 &= x_1 y_1 - b z_1, \\ \dot{w}_1 &= -y_1 z_1 - d w_1, \\ \dot{x}_i &= a(y_i - x_i) + r w_i(t-\tau) + H_i(x_{i-1} - x_i), \\ \dot{y}_i &= c x_i - y_i - x_i z_i, \\ \dot{z}_i &= x_i y_i - b z_i, \\ \dot{w}_i &= -y_i z_i - d w_i, \end{aligned} \quad (3)$$

where u is the controller to be designed. To get the synchronization between the output signal x_1 of system (3) and signal $s(t)$, the controller u should be given appropriately, where $s(t)$ may be any given signal, even as equilibrium point, periodic orbit, or chaotic orbit.

Let $e = x_1 - s(t)$, corresponding result is given as following **Theorem 1**.

Theorem 1 For any given reference signal $s(t)$, the output signal x_1 in system (3) can synchronize it if the controller u is taken as following (4)

$$u = (3\beta - c - {}^{\alpha}/_{a})x_1 + (1 - 3\beta)y_1 + ({}^{r}/_{a})(d - 3\beta)w_1(t-\tau) + x_1 z_1 + \dot{x}_1 + ({}^{r}/_{a})\, y_1(t-\tau) z_1(t-\tau) + ({}^{1}/_{a})(\ddot{s} - 3\beta\dot{s} - \alpha s)\,. \quad (4)$$

That is

$$\lim_{t\to\infty} e = \lim_{t\to\infty}[x_1 - s(t)] = 0, \quad (5)$$

where α and β are positive gain coefficients.

Proof. The positive Lyapunov function is given as following changeable

$$V_1 = \alpha e^2 + (\dot{e} + \beta e)^2, \quad (6)$$

where $\dot{e}$ denotes the differential variable e to time. The differential of V_1 to time t is

$$\begin{aligned} \dot{V}_1 &= 2\alpha e\dot{e} + 2(\dot{e} + \beta e)(\ddot{e} + \beta\dot{e}) \\ &= -2\beta V_1 + 2\alpha e(\dot{e} + \beta e) + 2\beta(\dot{e} + \beta e)^2 + 2(\dot{e} + \beta e)(\ddot{e} + \beta\dot{e}) = -2\beta V_1 + 2(\ddot{e} + 2\beta\dot{e} + \alpha e + \beta^2 e)(\dot{e} + \beta e\,. \end{aligned} \quad (7)$$

Substitute (3) and (4) into (7), and we can get

$$\dot{V}_1 = -2\beta V_1 - 2\beta(\dot{e} + \beta e)^2 < 0\,. \quad (8)$$

According to Lyapunov stability theory and (8), the synchronization of reference signal $s(t)$ and the output signal x_1 in system (3) can be reached, that is

$$\lim_{t\to\infty} e = \lim_{t\to\infty}[x_1 - s(t)] = 0.$$

The proof is complete.

If x_i $(i = 2,3, \cdots, N)$ can pinning synchronize x_1, while x_1 of system (3) can synchronize $s(t)$, then the pinning synchronization between system (3) and $s(t)$ can be realized ultimately.

For this end, let

$$e_i = x_i - x_1 \ (i = 2,3, \cdots, N), \quad (9)$$

and the following **Theorem 2** can be obtained.

Theorem 2 x_i $(i = 2,3,\cdots,N)$ with time delay can pinning synchronize x_1 in system (3) under the controller u in (4).

Proof. According to (9) and system (3), it can be gotten that

$$\dot{e}_i = (-a - H_i)e_i + H_i e_{i-1} + a(y_i - y_1) + r(w_i(t-\tau) - w_1(t-\tau)), (i = 2,3,\cdots,N). \quad (10)$$

For simplicity, let $e = (e_2, e_3, \cdots, e_N)^T$, $\phi = (y_2 - y_1, y_3 - y_1, \cdots, y_N - y_1)^T$,

$\Psi = (w_2(t-\tau) - w_1(t-\tau), w_3(t-\tau) - w_1(t-\tau), \cdots, w_N(t-\tau) - w_1(t-\tau))^T$,

$$A = \begin{bmatrix} -a - H_2 & 0 & \cdots & 0 & 0 \\ H_3 & -a - H_3 & \cdots & 0 & 0 \\ 0 & H_4 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & H_N & -a - H_N \end{bmatrix},$$

and (10) can be written as

$$\dot{e} = Ae + a\phi - r\Psi \,. \quad (11)$$

Lyapunov function is considered as following

$$V = e^T e. \quad (12)$$

It's obvious to see that Lyapunov function V in (12) is positive definite. Using (11), time derivative of V in (12) can be expressed as

$$\begin{aligned} \dot{V} &= \dot{e}^T e + e^T \dot{e} \\ &= (Ae + a\phi - r\Psi)^T e + e^T (Ae + a\phi - r\Psi) \\ &= e^T (A^T + A)e + (a\phi - r\Psi)^T e + e^T (a\phi - r\Psi) \\ &= e^T (A^T + A)e + 2e^T (a\phi - r\Psi). \end{aligned} \quad (13)$$

Since a chaotic system has bounded trajectories, there exists a positive constant M, such that $|x_i| < M, |y_i| < M, |z_i| < M, |w_i(t-\tau)| < M, (i = 1, 2, \cdots, N)$, therefore ϕ, Ψ are bounded. For suitable H_i $(i = 2, \cdots, N)$, $A^T + A$ is negative definite, so we have $\dot{V} < 0$. Based on Lyapunov stability theory, it's easy to know that error system (10) or (11) is asymptotically stable. That is to say, unilateral coupled x_i $(i = 2, \cdots, N)$ with time delay can pinning synchronize x_1 in system (3) under the controller u in (4).

According to Theorems 1 and Theorem 2, for any given signal, unilateral coupling hyperchatic network can pinning synchronize it under appropriate controller.

4. Simulations

In this section, simulations are given to illustrate the effectiveness of the proposed scheme. For simplicity, in simulations, coupling intensity H_i $(i = 2, \cdots, N)$ is supposed to be the same value. The original values are chosen as constants. Each node is a hyperchaotic Lorenz system. $s(t)$ is taken as the behavior of an isolated node. The parameters are chosen as $a = 10$, $b = 8/3$, $c = 28$, $d = 1$, and the time delay is chosen as $\tau = 1$, with which the Lorenz system is hyperchaotic. The number of nodes is taken $N = 20$ and coupling intensity is supposed as $H_i = 20$. Evolution of errors is illustrated in Fig.2, from which it is obvious to see that the hyperchaotic network can pinning synchronize the given signal $s(t)$.

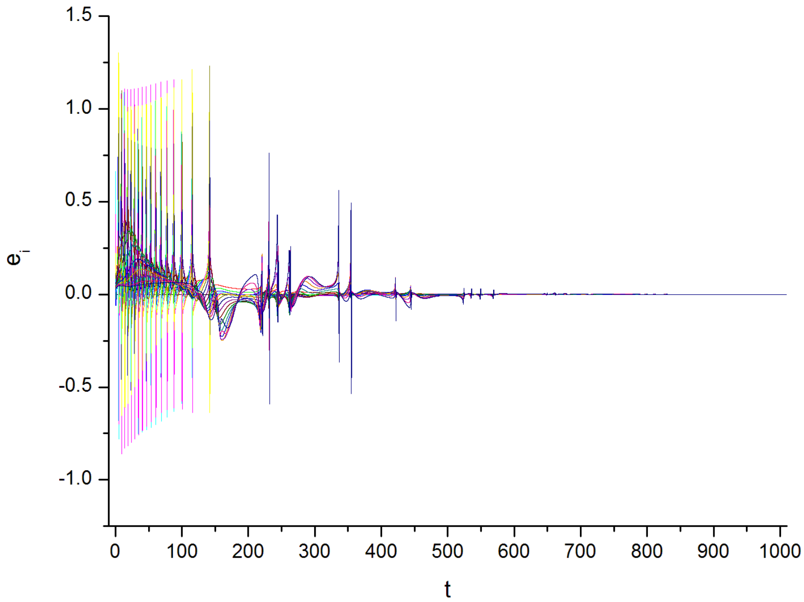

Fig.2. Evolution of errors $e_i(i = 2,3,\cdots,N)$, with the controller in (4).

5. Conclusions

In this paper, based on Lyapunov stability theory, the pinning synchronization of unilateral coupled hyperchaotic network with time delay is achieved. The proposed scheme is not only tested by theoretical analysis, but also verified by numerical simulation.

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