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Fuzzy Model Identification:A Review and Comparison of Type-1 and Type-2 Fuzzy Systems

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Abstract. Recently, a number of extensions to classical fuzzy logic systems (type-1 fuzzy logic systems) have been attracting interest. One of the most widely used extensions is the interval type-2 fuzzy logic systems. An interval type-2 TSK fuzzy logic system can be obtained by considering the membership functions of its existed type-1 counterpart as primary membership functions and assigning uncertainty to cluster centers, standard deviation of Gaussian membership functions and consequence parameters. This paper presents a review and comparison of type-1 fuzzy logic system and type-2 fuzzy systems in fuzzy modeling and identification. TSK fuzzy model is considered for both type-1 and type-2 fuzzy systems and model parameters are updated using gradient descent method. The experimental study is done on two widely known data, namely chemical plant data and the stock market data.

Keywords: Fuzzy Modeling, Identification, Type-1 Fuzzy Logic, Type-2 Fuzzy Logic.

1. Introduction

With the development of type-2 fuzzy logic systems (T2 FLSs) and their ability to handle uncertainty, interval type-2 FLCs (IT2 FLCs) has attracted a lot of interest in recent years. The concept of type-2 fuzzy sets was first introduced by Zadeh as an extension of the concept of well-known ordinary fuzzy sets, type-1 fuzzy sets. Mendel and Karnik have further developed the theory of type-2 fuzzy sets [1-5]. The type-2 fuzzy system has the capability to handle and minimize the effect of both linguistic and random uncertainties. A wide range of applications related to type-2 fuzzy system show that these systems provide much better solution specially in handling uncertainties. A type-2 fuzzy set [1-5] is characterized by a fuzzy membership function i.e. the membership grade for each element is also a fuzzy set in [0,1], unlike a type-1 fuzzy set, where the membership grade is a crisp number in [0,1]. The membership functions of type-2 fuzzy sets are three dimensional and include a footprint of uncertainty (FOU), which is the new third dimension of type-2 fuzzy sets. The footprint of uncertainty provides an additional degree of freedom to make it possible to directly model and handle uncertainties. The FOU represents the blurring of a type-1 membership function, and is completely described by its two bounding functions, a lower membership function (LMF) and an upper membership function (UMF), both of which are type-1 fuzzy sets. Wu and Tan has shown in [6] that the extra degree of freedom provided by the footprint of uncertainty enables a type-2 FLS to produce outputs that cannot be achieved by type-1 FLSs with the same number of membership functions. Type-2 fuzzy sets are useful especially when it is difficult to determine the exact and precise membership functions.

Fuzzy systems are generally designed using either Mamdani or TSK type IF-THEN rules. In the former type, both the antecedent and consequent parts utilize fuzzy values. The TSK type fuzzy rules utilize fuzzy values in the antecedent part and crisp values or linear functions in the consequent part. The use of type-2 fuzzy systems for system identification is a recent topic and one can find few papers in the literature [7-10]. A recent work [7] presents a type-2 neuro fuzzy system for identification of time-varying systems and equalization of time-varying channels.

The paper is organized as follows. We start (Section 2) with a general overview of the type-1 fuzzy systems. A background of type-2 fuzzy systems is given in Section 3. The experimental results and comparative analysis are given in Section 4 followed by the main conclusions presented in Section 5.

2. Type-1 fuzzy logic systems

Fuzzy inference systems also known as fuzzy rule-based systems or fuzzy models are schematically shown in Fig. 1. They are composed of 5 conventional blocks: a rule-base containing a number of fuzzy IF-THEN rules, a database which defines the membership functions of the fuzzy sets used in the fuzzy rules, a decision-making unit which performs the inference operations on the rules, a fuzzification interface which transforms the crisp inputs into degrees of match with linguistic values, a defuzzification interface which transforms the fuzzy results of the inference into a crisp output.

It is proved that Takagi-Sugeno fuzzy models are universal approximators of any smooth non-linear systems. In general, there are two approaches for constructing fuzzy models:

1. Identification (Fuzzy modeling using input-output data)
2. Derivation from given non-linear system equations.

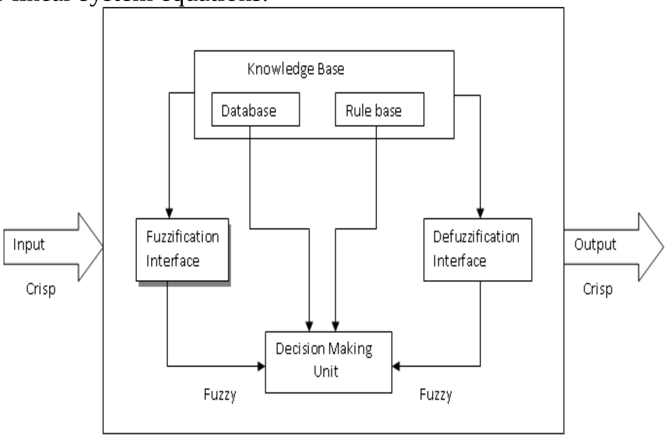

Fig 1: Type-1 Fuzzy Logic System

There has been an extensive literature of fuzzy modeling using input-output data following Takagi-Sugeno and Kang's excellent work. The identification approach to fuzzy modeling is suitable for plants that are unable or too difficult to be represented analytically and/or by physical models. The procedure consists of two parts: structure identification and parameter identification.

A fuzzy model proposed by Takagi and Sugeno [11], is of the following form:

$$\text{Rule } i : \text{IF } x_1 \textit{ is } A_{i1} \textit{ and.....and } x_n \textit{ is } A_{in} \text{ THEN } y_i = c_{io} + c_{i1}x_1 + ...+ c_{in}x_n \tag{1}$$

Where $i = 1,2,........l, l$ is the number of IF-THEN rules, c_{ik} (k = 0,1…n) are consequents parameters. y_i is an output from the IF-THEN rule, and A_{ij} is a fuzzy set.

Given an input $(x_{1,} x_2,..., x_n)$, the final output of the fuzzy model is inferred as follows:

$$y = \sum_{i=1}^{l} w_i y_i \tag{2}$$

where, y_i is calculated for the input by the consequent equation of the i^{th} implication, and the weight w_i implies the overall truth value of the premise of the implication for the input, and calculated as:

$$w_i = \prod_{k=1}^{n} A_{ik}(x_k) \tag{3}$$

where $A_{ik}(x_k) = \exp(-\frac{(x_k - a_{ik})^2}{b_{ik}^2})$ is the Gaussian membership function. Here, a_{ik} and b_{ik} are parameters of the membership functions. We used gradient descent technique to modify the parameters a_{ik} , b_{ik} and c_{ik} .

From Eqns (2) and (3), the overall output is given as:

$$y = \sum_{k=0}^{n} \sum_{i=1}^{l} w_i c_{ik} x_k \tag{4}$$

where, x_0 =1.

The performance of the model is measured by the following index :

$$E = \frac{1}{2}(y^* - y)^2$$

where, y and y^* denote outputs of a fuzzy model and a real system, respectively. By partially differentiating E with respect to each parameter of a fuzzy model, we obtain:

$$\frac{\partial E}{\partial c_{ik}} = \frac{\partial E}{\partial y} \times \frac{\partial y}{\partial c_{ik}}$$
$$= -(y^* - y) w_i x_k = -\delta w_i x_k \tag{5}$$

$$\frac{\partial E}{\partial a_{ik}} = \frac{\partial E}{\partial y} \times \frac{\partial y}{\partial a_{ik}}$$
$$= -(y^* - y) \frac{2(x_k - a_{ik})}{b_{ik}} w_i \sum_{k=0}^{n} c_{ik} x_k \, , = -\delta \, \frac{2(x_k - a_{ik})}{b_{ik}} w_i \sum_{k=0}^{n} c_{ik} x_k \, , \tag{6}$$

$$\frac{\partial E}{\partial b_{ik}} = \frac{\partial E}{\partial y} . \frac{\partial y}{\partial b_{ik}}$$
$$= -(y^* - y) \frac{(x_k - a_{ik})^2}{b^2_{ik}} w_i \sum_{k=0}^{n} c_{ik} x_k \, ,$$
$$= -\delta \, \frac{2(x_k - a_{ik})^2}{b^2_{ik}} w_i \sum_{k=0}^{n} c_{ik} x_k \tag{7}$$

where, $\delta = (y^* - y)$.

The final learning law can be defined as:

$$c_{ik}^{NEW} = c_{ik}^{OLD} + \varepsilon_1 \delta w_j x_k \tag{8}$$

$$a_{ik}^{NEW} = a_{ik}^{OLD} + \varepsilon_2 \delta \frac{2(x_k - a_{ik}^{OLD})}{b_{ik}^{OLD}} w_j \sum_{k=0}^{n} c_{ik}^{OLD} x_k \tag{9}$$

$$b_{ik}^{NEW} = b_{ik}^{OLD} + \varepsilon_3 \delta \frac{2(x_k - a_{ik}^{OLD})^2}{(b_{ik}^{OLD})^2} w_j \sum_{k=0}^{n} c_{ik}^{OLD} x_k \tag{10}$$

where, $\varepsilon_1, \varepsilon_2$ and ε_3 are learning coefficients and $\varepsilon_1, \varepsilon_2, \varepsilon_3 > 0$. By using Eqns (8)-(10), we can successively update the parameters, a_{ik}, b_{ik} and c_{ik}, until the value of the summation of δ for all data points is small enough.

3. Type-II fuzzy Logic systems

An interval type-2 fuzzy logic system (IT2FLS) shown in Fig. 2 is also characterized by fuzzy IF-THEN rules, but the membership functions of the ITFLSs are now interval type-2 fuzzy sets. From Fig. 2, it can be seen that the structure of an IT2FLS is very similar to the structure of a T1FLS and the only difference exists in the output processing block. For a type-1 fuzzy logic system (T1FLS), the output processing block only contains a defuzzifier, but for an IT2FLS, the output processing block includes a type-reducer, which maps a T2FS into a TIFS and a defuzzifier.

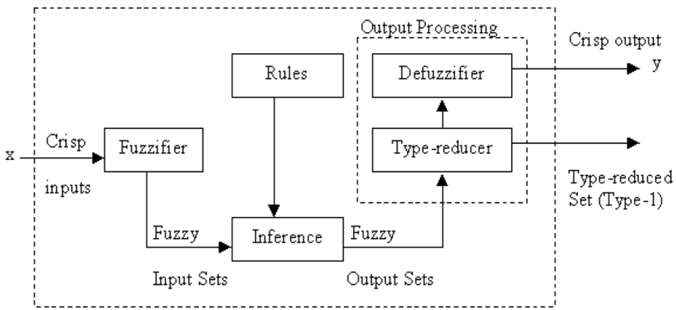

Fig 2: Type-2 fuzzy logic system

The interval type-2 FLC works as follows: the crisp inputs from the input sensors are first fuzzified into type-2 fuzzy sets. The input type-2 fuzzy sets then activate the inference engine and the rule base to produce output type-2 fuzzy sets. The type-2 FLC rules will remain the same as in type-1 FLC, but the antecedents and/or consequents will be represented by interval type-2 fuzzy sets. The inference engine combines the fired rules and gives a mapping from input type-2 fuzzy sets to output type-2 fuzzy sets. The type-2 fuzzy outputs of the inference engine are then processed by the type-reducer, which combines the output sets and performs a centroid calculation that leads to type-1 fuzzy sets called the type-reduced sets. After the type-reduction process, the type-reduced sets are then defuzzified to obtain crisp outputs. Type-2 fuzzy systems are characterized by fuzzy IF-THEN rules, the parameters in the antecedent and the consequent parts of the rules include type-2 fuzzy values. In Gaussian type-2 fuzzy sets, uncertainties can be associated to the mean and/or the standard deviation (STD). In Fig. 3(a) and 3(b), Gaussian type-2 fuzzy sets with uncertain STD and uncertain mean are shown respectively. The mathematical expression for Gaussian membership function is expressed as

$$\mu(x) = \exp\left(\frac{-(x-c)^2}{2\sigma^2}\right);$$

Here c and σ are the centre and width of membership function, x is the input vector. In this paper, only one of these parameters is assumed to be uncertain, i.e. uncertain STD and fixed mean or fixed STD and uncertain mean. Here we consider fixed STD and uncertain mean as in Fig. 3(b). It is to be noted that the fixed values are also subject to parameter adjustment.

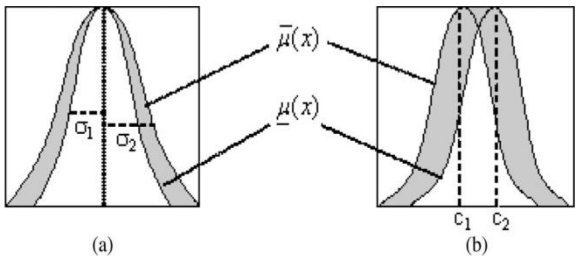

Fig 3: Gaussian type-2 fuzzy sets with (a) uncertain spread and (b) uncertain mean

Assuming that there are M rules in the rule base, each of which has the following form

Rule k :

IF x_1 is $\tilde{A}_1^k$ and x_p is $\tilde{A}_p^k$,

THEN y_j is $\sum_{i=1}^{p} w_{ij} x_i + b_j$ (11)

Where $k = 1,2,...M$, p is the number of input variables in the antecedent part, $j = 1,2,...,n$ are the output variables, $\tilde{A}_i^k$ $(i = 1,2,...p, k = 1,2...M)$ are IT2FLS of the IF-part, w_{ij} and b_j are the parameters in the consequent part of rules.

Once a crisp input $X = (x_1, x_2,...., x_p)^T$ is applied to the IT2FLS, through the singleton fuzzifier and the inference process, the firing strength of the k^{th} rule which is an interval type-1 set can be obtained as:

$$F^k = [\underline{f}^k, \bar{f}^k]$$

Where

$$\underline{f}^k = \underline{\mu}_{\tilde{A}_1^k}(x_1) * \underline{\mu}_{\tilde{A}_2^k}(x_2) * .... * \underline{\mu}_{\tilde{A}_p^k}(x_p) \quad (12) \qquad \bar{f}^k = \bar{\mu}_{\tilde{A}_1^k}(x_1) * \bar{\mu}_{\tilde{A}_2^k}(x_2) * .... * \bar{\mu}_{\tilde{A}_p^k}(x_p) \quad (13)$$

In which $\underline{\mu}(\ )$ and $\bar{\mu}(\ )$ denote the grades of the lower and upper membership functions of IT2FSs and * denotes minimum or product t-norm. To generate a crisp output, the outputs of the inference engine should be type reduced and then defuzzified.

$$u = \frac{q\sum_{j=1}^{N} f_{-j} y_j}{\sum_{j=1}^{N} f_{-j}} + \frac{(1-q)\sum_{j=1}^{N} \overline{f_j} y_j}{\sum_{j=1}^{N} \overline{f_j}} \tag{14}$$

$$y_j = \sum_{i=1}^{p} x_i w_{ij} + b_j \tag{15}$$

Here N is number of active rules, $\overline{f_j}$ and f_{-j} are determined using (12)-(13), y_i is determined using (15), q is a design factor indicating the share of lower and upper values in the final output. The parameter q enables to adjust the lower or the upper portions depending on the level of certainty of the system.

The performance of the model again is measured by the following index :

$$E = \frac{1}{2}\left(u^* - u\right)^2 \tag{16}$$

where, y and y^* denote outputs of a fuzzy model and a real system, respectively. Now the fuzzy parameters are updated using gradient descent algorithm in order to minimize the error obtained in (16). The updation formulas are as follows:

$$w_{ij}(t+1) = w_{ij}(t) + \lambda \frac{\partial E}{\partial w_{ij}}; b_j(t+1) = b_j(t) + \lambda \frac{\partial E}{\partial b_j}$$

$$c1_{ij}(t+1) = c1_{ij}(t) + \lambda \frac{\partial E}{\partial c1_{ij}};$$

$$c2_{ij}(t+1) = c2_{ij}(t) + \lambda \frac{\partial E}{\partial c2_{ij}}$$

$$\sigma_{ij}(t+1) = \sigma_{ij}(t) + \lambda \frac{\partial E}{\partial \sigma_{ij}} \tag{17}$$

Now by partially differentiating E with respect to each parameter of fuzzy model, we obtain

$$\frac{\partial E}{\partial w_{ij}} = \frac{\partial E}{\partial u}\frac{\partial u}{\partial y_j}\frac{\partial y_j}{\partial w_{ij}} = (u-u^*).\left(\frac{q.\underline{f}_j}{\sum_{j=1}^{n}\underline{f}_j} + \frac{(1-q).\overline{f}_j}{\sum_{j=1}^{n}\overline{f}_j}\right)x_i \tag{18}$$

$$\frac{\partial E}{\partial b_j} = \frac{\partial E}{\partial u}\frac{\partial u}{\partial y_j}\frac{\partial y_j}{\partial b_j} = (u-u^*).\left(\frac{q.\underline{f}_j}{\sum_{j=1}^{n}\underline{f}_j} + \frac{(1-q).\overline{f}_j}{\sum_{j=1}^{n}\overline{f}_j}\right) \tag{19}$$

$$\frac{\partial E}{\partial \sigma_{ij}} = \sum_j \frac{\partial E}{\partial u}\left(\frac{\partial u}{\partial \underline{f}_j}\frac{\partial \underline{f}_j}{\partial \underline{\mu}_{ij}}\frac{\partial \underline{\mu}_{ij}}{\partial \sigma_{ij}} + \frac{\partial u}{\partial \overline{f}_j}\frac{\partial \overline{f}_j}{\partial \overline{\mu}_{ij}}\frac{\partial \overline{\mu}_{ij}}{\partial \sigma_{ij}}\right) \tag{20}$$

$$\frac{\partial E}{\partial c1_{ij}} = \sum_j \frac{\partial E}{\partial u}\left(\frac{\partial u}{\partial \underline{f}_j}\frac{\partial \underline{f}_j}{\partial \underline{\mu}_{ij}}\frac{\partial \underline{\mu}_{ij}}{\partial c1_{ij}} + \frac{\partial u}{\partial \overline{f}_j}\frac{\partial \overline{f}_j}{\partial \overline{\mu}_{ij}}\frac{\partial \overline{\mu}_{ij}}{\partial c1_{ij}}\right) \tag{21}$$

$$\frac{\partial E}{\partial c2_{ij}} = \sum_j \frac{\partial E}{\partial u}\left(\frac{\partial u}{\partial \underline{f}_j}\frac{\partial \underline{f}_j}{\partial \underline{\mu}_{ij}}\frac{\partial \underline{\mu}_{ij}}{\partial c2_{ij}} + \frac{\partial u}{\partial \overline{f}_j}\frac{\partial \overline{f}_j}{\partial \overline{\mu}_{ij}}\frac{\partial \overline{\mu}_{ij}}{\partial c2_{ij}}\right) \tag{22}$$

Here

$$\frac{\partial E}{\partial u} = u - u^*; \frac{\partial u}{\partial \underline{f}_j} = q \frac{y_j - \underline{u}}{\sum_{j=1}^{n} \underline{f}_j}; \frac{\partial u}{\partial \overline{f}_j} = (1-q)\frac{y_j - \overline{u}}{\sum_{j=1}^{n} \overline{f}_j}$$

$$\underline{u} = \frac{\sum_{j=1}^{n} \underline{f}_j y_j}{\sum_{j=1}^{n} \underline{f}_j}; \overline{u} = \frac{\sum_{j=1}^{n} \overline{f}_j y_j}{\sum_{j=1}^{n} \overline{f}_j} \tag{23}$$

Upper and lower membership functions can be written as follows:

$$\underline{\mu}_{ij}(x) = \begin{cases} G(c2_{ij}, \sigma_{ij}, x_i), & x_i \le \frac{c1_{ij} + c2_{ij}}{2} \\ G(c1_{ij}, \sigma_{ij}, x_i), & x_i > \frac{c1_{ij} + c2_{ij}}{2} \end{cases}$$

$$\overline{\mu}_{ij}(x) = \begin{cases} G(c1_{ij}, \sigma_{ij}, x_i), & x_i < c1_{ij} \\ 1, & c1_{ij} \le x_i \le c2_{ij} \\ G(c2_{ij}, \sigma_{ij}, x_i), & x_i > c2_{ij} \end{cases} \tag{24}$$

Here, $G(c_{ij}, \sigma_{ij}, x_i)$ is determined as

$$G(c_{ij}, \sigma_{ij}, x_i) = \exp\left(-\frac{1}{2}\frac{(x_i - c_{ij})^2}{\sigma_{ij}^{\ 2}}\right) \tag{25}$$

Then

$$\frac{\partial \overline{\mu}_j(x_i)}{\partial c1_{ij}} = \begin{cases} G(c1_{ij}, \sigma_{ij}, x_i)\left(\frac{x_i - c1_{ij}}{\sigma_{ij}^{\ 2}}\right), & x_i < c1_{ij} \\ 0, & c1_{ij} \le x_i \le c2_{ij} \\ 0, & x_i > c2_{ij} \end{cases}$$

$$\frac{\partial \underline{\mu}_j(x_i)}{\partial c1_{ij}} = \begin{cases} 0, & x_i \le \frac{c1_{ij} + c2_{ij}}{2} \\ G(c1_{ij}, \sigma_{ij}, x_i)\left(\frac{x_i - c1_{ij}}{\sigma_{ij}^{\ 2}}\right), & x_i > \frac{c1_{ij} + c2_{ij}}{2} \end{cases} \tag{26}$$

$$\frac{\partial \overline{\mu}_j(x_i)}{\partial c2_{ij}} = \begin{cases} 0, & x_i < c1_{ij} \\ 0, & c1_{ij} \le x_i \le c2_{ij} \\ G(c2_{ij}, \sigma_{ij}, x_i)\left(\frac{x_i - c2_{ij}}{\sigma_{ij}^{\ 2}}\right), & x_i > c2_{ij} \end{cases}$$

$$\frac{\partial \underline{\mu}_j(x_i)}{\partial c2_{ij}} = \begin{cases} G(c2_{ij}, \sigma_{ij}, x_i)\left(\frac{x_i - c2_{ij}}{\sigma_{ij}^{\ 2}}\right), & x_i < \frac{c1_{ij} + c2_{ij}}{2} \\ 0, & x_i \ge \frac{c1_{ij} + c2_{ij}}{2} \end{cases} \tag{27}$$

$$\frac{\partial \overline{\mu}_j(x_i)}{\partial \sigma_{ij}} = \begin{cases} G(c1_{ij}, \sigma_{ij}, x_i)\left(\dfrac{(x_i - c1_{ij})^2}{\sigma_{ij}^{\ 3}}\right), & x_i < c1_{ij} \\ 0, & c1_{ij} \le x_i \le c2_{ij} \\ G(c2_{ij}, \sigma_{ij}, x_i)\left(\dfrac{(x_i - c2_{ij})^2}{\sigma_{ij}^{\ 3}}\right), & x_i > c2_{ij} \end{cases}$$

$$\frac{\partial \underline{\mu}_j(x_i)}{\partial \sigma_{ij}} = \begin{cases} G(c2_{ij}, \sigma_{ij}, x_i)\left(\dfrac{(x_i - c2_{ij})^2}{\sigma_{ij}^{\ 3}}\right), & x_i \le \dfrac{c1_{ij} + c2_{ij}}{2} \\ G(c1_{ij}, \sigma_{ij}, x_i)\left(\dfrac{(x_i - c1_{ij})^2}{\sigma_{ij}^{\ 3}}\right), & x_i > \dfrac{c1_{ij} + c2_{ij}}{2} \end{cases} \quad (28)$$

The parameters of type-2 fuzzy system can thus be updated using the above equations.

4. Experimental Results

Fuzzy model identification is implemented on the two dynamic nonlinear plants which are discussed in this section. The input and the output data in both of the cases is available and the fuzzy model is identified using the data given for these plants. The parameters of the membership function (c_{ij}, σ_{ij}) and consequent parameters are randomly initialized and then gradient descent method is used for these parameters for both type-1 and type-2 fuzzy models. We use the error function SE = $\frac{1}{2}e^2$, as a performance index (PI) of the fuzzy model, where $e = y^* - y$, y and y^* denote outputs of the fuzzy model and the real system, respectively. The approximate power of the identified models is then compared based on this performance index.

Both plants are discussed below:

4.1 Human operation at a chemical plant

Here, we deal with a model of an operator's control action at the startup of a chemical plant, which is meant for producing a polymer by the polymerization of some monomers. Since the start-up of the plant is very complicated, a man has to make the manual operations in the plant.

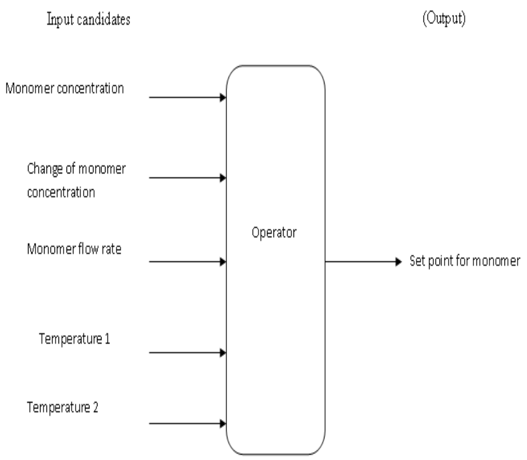

Fig 4: Control action of an operator

As shown in Fig. 4 there are five input candidates which the human operator might refer to for control, and one output, i.e., his control action given as under:

u_1 : monomer concentration

u_2 : change in monomer concentration

u_3 : monomer flow rate

u_4, u_5 : local temperatures inside the plant

y : set point for monomer flow rate

The operator determines the set point for the monomer flow rate and the actual value of the monomer flow rate for the plant controlled by the PID controller.

The desired output for the plant is shown in Fig. 5. The main aim is to identify this output using the fuzzy model obtained using the algorithms discussed above. The number of rules taken in identification of the chemical plant using type-1 fuzzy system and type-2 fuzzy system are five.

Using the type-2 fuzzy logic systems, fuzzy model for the above plant was obtained and output was traced. The achieved output with respect to the desired one for type-2 fuzzy systems is shown in Fig. 6. Here in the Fig. 6, the blue graph corresponds to the output achieved whereas the red graph represents the desired output. The error between the desired and the identified output in the type-2 fuzzy identification scheme was lesser than that using type-1 fuzzy identification scheme. The minimization of error using type-1 and type-2 fuzzy identification scheme is shown in Fig. 7. In table 1, the value of performance index i.e. square of error, is listed. The type-1 TSK fuzzy model achieved a performance index of $PI = 6.1 \times 10^{5.}$ Type-2 TSK fuzzy gives better results with $PI = 5.886 \times 10^5$.

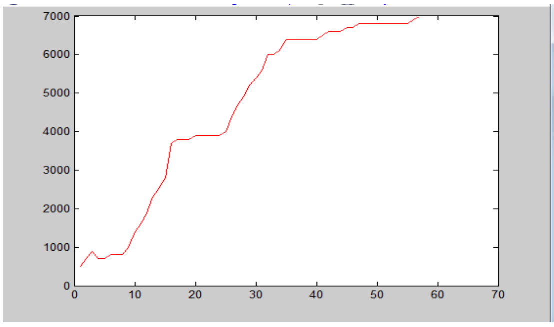

Fig.5: Desired output of chemical plant

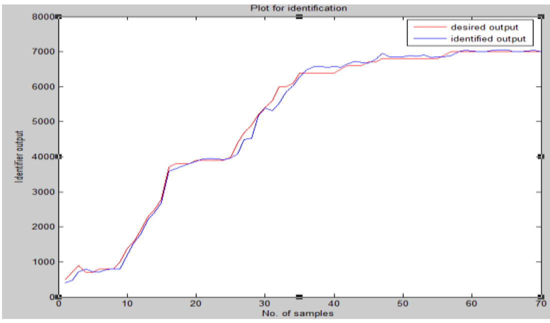

Fig 6: Desired vs. Actual Identified output using type-2 fuzzy logic systems for chemical plant data

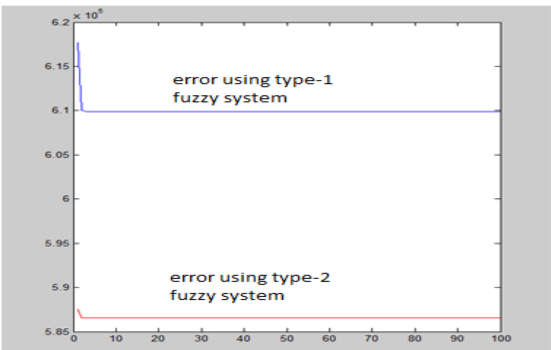

Fig 7: Comparison of performance index

Table 1: Performance of fuzzy models of the chemical plant

IDENTIFICATION SCHEME	NUMBER OF RULES	NUMBER OF ITERATIONS	ERROR $PI = \frac{1}{2} * e^2$
FUZZY TYPE-1	5	100	6.1×10^5
FUZZY TYPE-2	5	100	5.886×10^5

4.2 Daily Stock Price

Next, we take up the trend data of stock prices. Here, we use the daily data of a stock market. There are 100 data points available here. The data consist of ten inputs and one output.
The desired output for the plant is shown in Fig. 8.

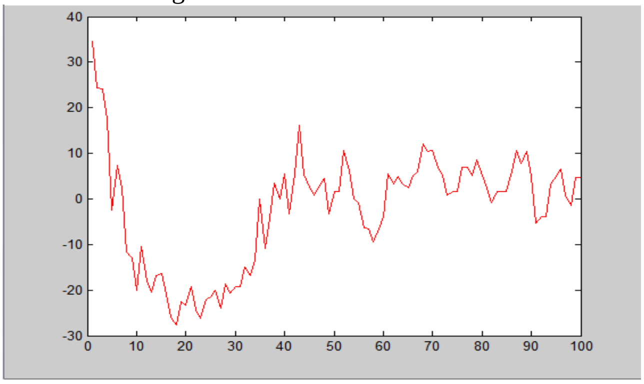

Fig 8: Desired Output

The achieved output with respect to the desired one for type-2 fuzzy system is shown in Fig. 9. Here in the Fig. 9, the blue graph corresponds to the output achieved whereas the red graph represents the desired output.

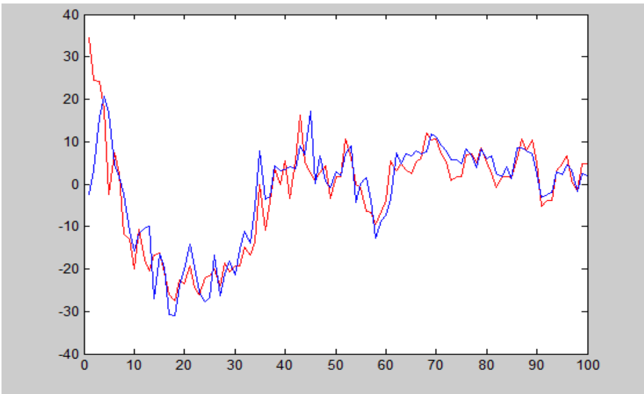

Fig.9: Identification of stock market data using Type-2 fuzzy system

The number of rules taken in identification using type-1 fuzzy system and type-2 fuzzy system are five. The error between the desired and the identified output in the type-2 fuzzy identification scheme was lesser than that using type-1 fuzzy identification scheme. The plot of error using type-2 fuzzy identification scheme is shown in Fig. 9. The comparison of the error in both the identification scheme using type-1 and type-2 system is plotted in Fig. 10. Table 2 compares the performance of the model identified with the two methods.

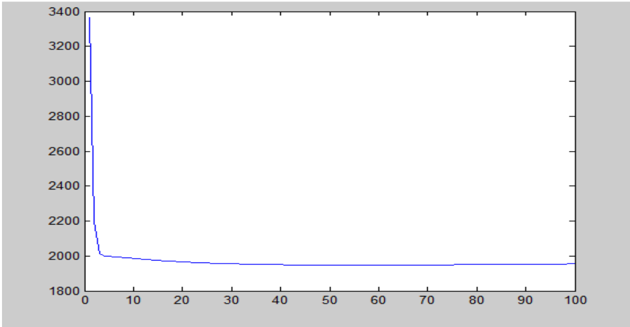

Fig. 9: Error minimization of stock market data using type-2 fuzzy identification scheme

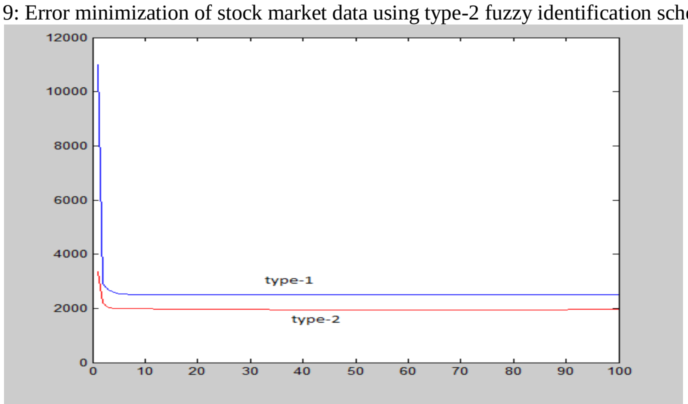

Fig 10: Comparison of error

Table 2: Performance of fuzzy models for the stock market data

IDENTIFICATION SCHEME	NUMBER OF RULES	NUMBER OF ITERATIONS	ERROR $PI = \frac{1}{2} * e^2$
FUZZY TYPE-1	5	100	2500
FUZZY TYPE-2	5	100	2000

5. Conclusion

Better results for fuzzy modeling are reached using interval type-2 fuzzy sets in all analyzed cases. The better performance of type-2 fuzzy model is due to: i) Type-2 fuzzy models allow uncertainty associated with antecedent fuzzy sets ii) Type-2 fuzzy model has more degree of freedom than type-1 FLS. The results obtained for type-2 fuzzy model are better than type-1 using the same number of membership function for each input in both cases.

6. References

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