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Chaos control of hyper chaotic delay Lorenz system via back stepping method

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Abstract. The problem of controlling chaotic system is studied by using back stepping design method. This technique is applied to achieve chaos control for each state of the nonlinear dynamical system. Based on Lyapunov stability theory, control laws are derived. The same technique is used to enable the stabilization of the chaotic motion to a steady state as well as tracking of any desired trajectory to be achieved in a systematic way. Numerical simulations are shown to verify the theoretical results..

Keywords: Hyper chaotic system, Time delay, Back stepping method, Control

1. Introduction

Chaotic systems are characterized by their extreme sensitivity to small perturbations in their initial conditions [1-2]. The inherent feature, known as the “butterfly effect”, is often troublesome or even unwanted in many cases of practical importance. Chaos controlling is one of the topics in the field of nonlinear science [3-5]. Chaos controlling concludes two following categories: one is suppressing chaotic dynamical behavior and another is generating or enhancing chaos in nonlinear systems. Chaos is generally believed to be harmful, so research has mainly focused on determining ways to remove or lessen the chaos within systems. There are many techniques and methods have been proposed to achieve chaos control, such as adaptive control [6],OGY method [7], feedback control methods [8], backstepping design technique [9], impulsive control [10], etc.

In this work, by employing back stepping method, chaos in hyperchaotic delay Lorenz system is controlled based on Lyapunov stability theory. At the same time the same method is used to enable stabilization of chaotic motion to a steady state as well as tracking of any desired trajectory. Numerical simulations are shown to verify the results.

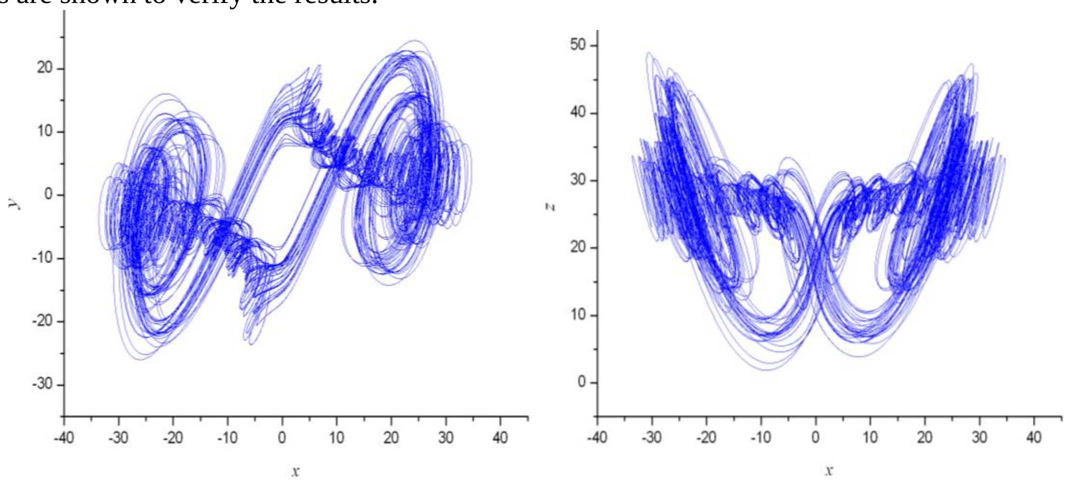

Figure 1. 2D overview hyper chaotic attractor of the system (1) when τ =1(2D overview).

2. Hyper chaotic Lorenz system with time delay

In this paper the considered hyperchaotic Lorenz system with a time delay can be described as

$$\begin{cases} \dot{x} = a(y-x) + w(t-\tau), \\ \dot{y} = cx - y - xz, \\ \dot{z} = xy - bz, \\ \dot{w} = -yz - w. \end{cases} \tag{1}$$

where $\tau > 0$ is the time delay.

As the dynamical systems given by DDEs have an infinite dimensional state space, usually the attractors of the solutions are high dimensional. The time delay hyper chaotic Lorenz system may exhibit more complicated complex behaviors[11]. When a = 10, b = 8/3, c = 28 and the time delay τ is chosen as 1, system (1) has two positive Lyapunov exponents, i.e., λ_1 = 0.4513, λ_2 = 0.1394, which exhibits hyper chaotic behavior, the hyper chaotic attractors of system (1) are shown in Figure 1 (2D overview).

3. Controlling the time delay hyper chaotic Lorenz system via back stepping control

In the followings, we will explore a single controller to control the chaos of the system via backstepping method.

Theorem 1 If $u_1 = -ay - w(t-\tau)$ is added to the first equation of system (1), the states of the system (2) will be stabilized at the origin point**,** where the controlled system can be written as

$$\begin{cases} \dot{x} = a(y-x) + w(t-\tau) + u_1, \\ \dot{y} = cx - y - xz, \\ \dot{z} = xy - bz, \\ \dot{w} = -yz - w. \end{cases} \tag{2}$$

Proof: Starting from the fourth equation, a stabilizing function $z(w)$, has to be designed for the virtual control x in order to make the derivative of

$$V_1 = \frac{1}{2}w^2,$$

and

$$\dot{V}_1 = w(-yz - w) \\ = -w^2 + z(w),$$

be negative definite. Assume that $z(w)$=0, and define an error variable

$$\bar{z} = z - z(w)\ , \tag{3}$$

we can obtained the $(w, \bar{z})$-subsystem

$$\begin{cases} \dot{w} = -y\bar{z} - w \\ \dot{\bar{z}} = xy - b\bar{z} \end{cases}. \tag{4}$$

We can construct a Lyapunov function as follows:

$$V_2 = V_1 + \frac{1}{2}\bar{z}^2.$$

Calculating the time derivative of V_2 along system (4), we have

$$\dot{V}_2 = \dot{V}_1 + \bar{z}\dot{\bar{z}} \\ = -w^2 + \bar{z}(xy - b\bar{z}) \\ = -w^2 - b\bar{z}^2 + y(w, \bar{z}). \tag{5}$$

When we choose $y(w, \bar{z}) = 0$, $\dot{V}_2$ is negative definite. Define an error variable as

$$\bar{y} = y - y(w, \bar{z}), \tag{6}$$

the $(w, \bar{z}, \bar{y})$-subsystem can be obtained

$$\begin{cases} \dot{w} = -\bar{y}\bar{z} - w, \\ \dot{\bar{z}} = x\bar{y} - b\bar{z}, \\ \dot{\bar{y}} = cx - \bar{y} - x\bar{z}. \end{cases} \tag{7}$$

The Lyapunov function can be constructed as

$$V_3 = V_2 + V_1 + \frac{1}{2}\bar{y}^2.$$

The time derivative of V_3 along the $(w, \bar{z}, \bar{y})$-subsystem can be obtained

$$\dot{V}_3 = \dot{V}_2 + \dot{V}_1 + \bar{y}\dot{\bar{y}} \\ = \dot{V}_2 + \dot{V}_1 + \bar{y}(cx - \bar{y} - x\bar{z})$$

$$= \dot{V}_2 + \dot{V}_1 - \bar{y}^2 + x(w, \bar{z}, \bar{y}).$$

Let $x(w, \bar{z}, \bar{y}) = 0$, $\bar{x} = x - x(w, \bar{z}, \bar{y})$, we have

$$\dot{\bar{x}} = a(\bar{y} - \bar{x}) + w(t - \tau), \tag{8}$$

then $V_4 = V_3 + V_2 + V_1 + \frac{1}{2}\bar{x}^2$, and

$$\begin{aligned} \dot{V}_4 &= \dot{V}_3 + \dot{V}_2 + \dot{V}_1 + \bar{x}\dot{\bar{x}} \\ &= \dot{V}_3 + \dot{V}_2 + \dot{V}_1 + \bar{x}(a(\bar{y} - \bar{x}) + w(t - \tau) + u_1) \\ &= \dot{V}_3 + \dot{V}_2 + \dot{V}_1 - \bar{x}^2 + \bar{x}(a\bar{y} + w(t - \tau) + u_1), \end{aligned} \tag{9}$$

when chose

$$u_1 = -ay - w(t - \tau), \tag{10}$$

then $\dot{V}_4 \leq 0$. It means that the trivial solution of system (2) is globally asymptotically stable when $u_1 = -ay - w(t - \tau)$, namely, system (1) with u_1 will be stabilized at the origin point.

The time response of the states x, y, z for the first equation of system (1) with u_1 are plotted in Figure 2, which can demonstrate the effectiveness of the proposed chaos control scheme.

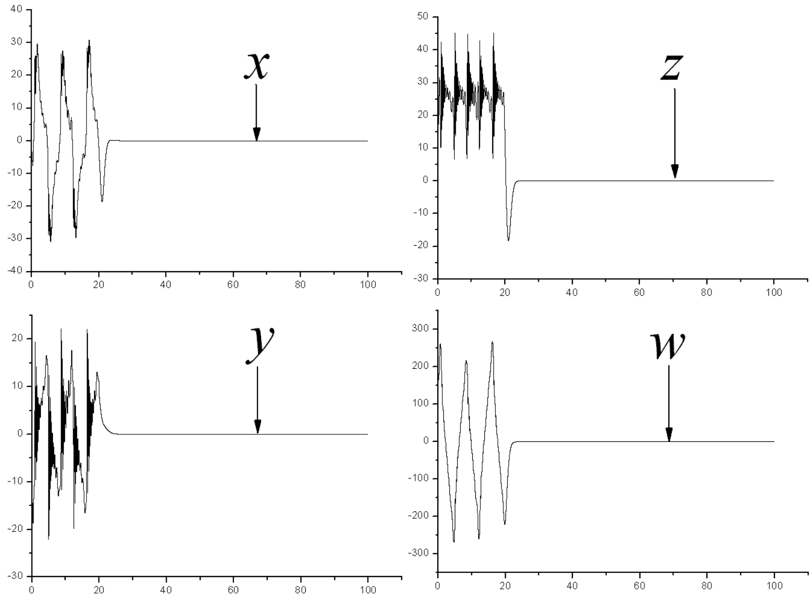

Figure 2. The time response of the states *x*,*y*, *z* for the first equation of system (1) with u_1. The control is activated at about *t* = 20.

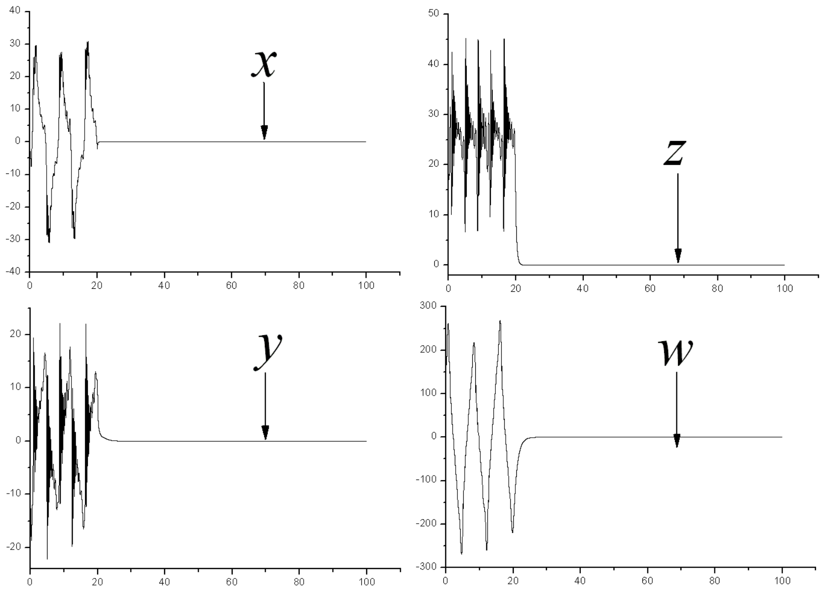

Fig. 3. The time response of the states *x*,*y*, *z* for the second equation of system (1) with u_2. The control is activated at about *t* = 20.

Theorem 2 If the second equation of system (1) added a control input $u_2 = xz - cx$, **the states of the system will be stabilized at** the origin point**.**

The process of proving is similar to theorem 1, and omitted here. The numerical simulation results are shown in Figure 3, which can demonstrate the effectiveness of the proposed chaos control scheme.

Theorem 3 If the second equation of system (1) added a control input $u_3 = \dot{r}(t) - r(t) + xz - cx$, **the** second **state of the system will be tracking** a desired trajectory state $r(t)$**.**

Proof: The controlled system can be written as

$$\begin{cases} \dot{x} = a(y - x) + w(t - \tau), \\ \dot{y} = cx - y - xz + u_{3,} \\ \dot{z} = xy - bz, \\ \dot{w} = -yz - w. \end{cases} \tag{11}$$

Starting from the fourth equation, a stabilizing function $z(w)$ has to be designed. Let $V_1 = \frac{1}{2}w^2$, we have

$$\begin{aligned} \dot{V}_1 &= w(-yz - w) \\ &= -w^2 + z(w), \end{aligned}$$

be negative definite. Assume that $z(w)$=0, and define an error variable

$$\bar{z} = z - z(w). \tag{12}$$

Then we obtained the $(w, \bar{z})$-subsystem

$$\begin{cases} \dot{w} = -y\bar{z} - w \\ \dot{\bar{z}} = xy - b\bar{z} \end{cases}. \tag{13}$$

We can construct a Lyapunov function as follows:

$$V_2 = V_1 + \frac{1}{2}\bar{z}^2.$$

Calculating the time derivative of V_2 along system (13), we have

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + \bar{z}\dot{\bar{z}} \\ &= -w^2 + \bar{z}(xy - b\bar{z}) \\ &= -w^2 - b\bar{z}^2 + y(w, \bar{z}) . \end{aligned} \tag{14}$$

When we choose $y(w, \bar{z}) = 0$, $\dot{V}_2$ is negative definite. Define an error variable as

$$\bar{y} = y - r(t) \tag{15}$$

the $(w, \bar{z}, \bar{y})$-subsystem can be obtained

$$\begin{cases} \dot{w} = -\bar{y}\bar{z} - w, \\ \dot{\bar{z}} = x\bar{y} - b\bar{z}, \\ \dot{\bar{y}} = cx - \bar{y} - x\bar{z} + r(t) - \dot{r}(t) + u_3. \end{cases} \tag{16}$$

The Lyapunov function can be constructed as

$$V_3 = V_2 + V_1 + \frac{1}{2}\gamma\bar{y}^2.$$

The time derivative of V_3 along the $(w, \bar{z}, \bar{y})$-subsystem can be obtained

$$\begin{aligned} \dot{V}_3 &= \dot{V}_2 + \dot{V}_1 + \gamma\bar{y}\dot{\bar{y}} \\ &= \dot{V}_2 + \dot{V}_1 + \gamma\bar{y}(cx - \bar{y} - x\bar{z} + r(t) - \dot{r}(t) + u_3) \\ &= \dot{V}_2 + \dot{V}_1 - \gamma\bar{y}^2 + x(w, \bar{z}, \bar{y}). \end{aligned}$$

Let $u_3 = x\bar{z} - cx - r(t) + \dot{r}(t)$, $\bar{x} = x - x(w, \bar{z}, \bar{y})$, we have $x(w, \bar{z}, \bar{y}) = 0$, and

$$\begin{cases} \dot{w} = -\bar{y}\bar{z} - w, \\ \dot{\bar{z}} = \bar{x}\bar{y} - b\bar{z}, \\ \dot{\bar{y}} = c\bar{x} - \bar{y} - \bar{x}\bar{z} + r(t) - \dot{r}(t) + u_{3,} \\ \dot{\bar{x}} = a(\bar{y} - \bar{x}) + w(t - \tau). \end{cases} \tag{17}$$

let $V_4 = V_3 + V_2 + V_1 + \frac{1}{2}\bar{x}^2 + \lambda\int_{t-\tau}^{t} w^2(\theta)\, d\theta$, then

$$\dot{V}_4 = \dot{V}_3 + \dot{V}_2 + \dot{V}_1 + \bar{x}\dot{\bar{x}} + \lambda w^2 - \lambda w^2(t - \tau) \le -\frac{1}{2}(a - 1)\bar{x}^2 - \left(\gamma - \frac{1}{2}a\right)\bar{y}^2 - 2b\bar{z}^2 - (3 - \lambda)w^2 - \left(\lambda - \frac{1}{2}\right)w^2(t - \tau).$$

For suitable values of γ, λ, $\dot{V}_4$ will be negative semi-definite and the zero solution of the error dynamical system (17) can be globally asymptotically stable. So, **the** second **state of the system (11) will be tracking** a desired trajectory state $r(t)$**.**

The output *y*(t) of system (11) tracks the trajectory *r*(*t*) = *sin*(*t*) with u_3 is plotted in Figure 4, which can demonstrate **the** second **state of the system will be tracking** a desired trajectory state $r(t)$.

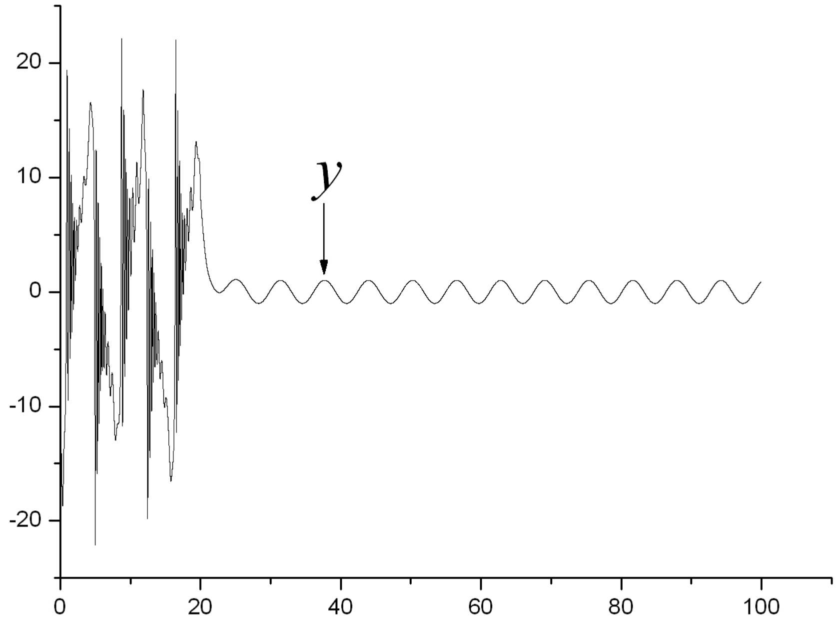

Figure 4. Output *y*(*t*) of system (11) tracks the trajectory *r*(*t*) = sin(*t*) with u_3. The control is activated at about *t* = 20.

4. Conclusion

In this paper, back stepping method is used to control delay hyper chaotic system. The correctness of the proposed methods is verified by theoretical analysis. Based on Lyapunov stability theory, a single controller is derived. Furthermore, the technique can be used to enable the stabilization of the chaotic motion to a steady state as well as tracking of any desired trajectory to be achieved in a systematic way. Numerical simulations are also provided to show the effectiveness of the developed methods.

Acknowledgements

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