ISSN 1 746-7233, England, UK
World Journal of Modelling and Simulation
Vol. 16 (2020) No. 4, pp. 308-320

Mathematical modelling of the control of methane emissions caused by rice paddies and human activities: Effects of mitigation options

Shyam Sundar[1]*, Ashish Kumar Mishra[1], Ram Naresh[2], J B Shukla[3]

[1] Department of Mathematics, Pranveer Singh Institute of Technology, Kanpur-208020, India
[2] Department of Mathematics, School of Basic & Applied Sciences, Harcourt Butler Technical University, Kanpur-208002, India
[3] Innovative Internet University for Research: (A Think Tank), Kanpur-208017, India

(*Received March 25 2020, Accepted July 04 2020*)

Abstract. The rice paddies are the largest anthropogenic sources of methane emissions in the atmosphere causing global warming. In this paper, a nonlinear mathematical model is proposed and analyzed to study the effects of mitigation options on the control of methane emissions in the atmosphere caused by rice paddies and human activities in order to reduce global warming. In the modelling process, five dependent variables namely; the biomass density of rice paddies, the density of human population using rice, the amount of which is assumed to be proportional to the biomass density of rice paddies, the cumulative density of methane formed by various processes involved in the production of rice paddies, the atmospheric concentration of methane emitted by various processes involved in the production of rice paddies as well as natural and human activities and the cumulative density of various mitigation options. It is assumed that both the cumulative biomass density of rice paddies and the density of human population follow logistic models with their respective growth rates and carrying capacities. The growth rate of atmospheric methane concentration is assumed to be directly proportional to the cumulative density of various processes involved in the production of rice paddies as well as the density of human population. This growth rate is also assumed to be depleted by various mitigation options. Further, the cumulative density of mitigation options is assumed to be proportional to the increased level of methane concentration in the atmosphere. The analysis of the proposed model is carried out using stability theory of differential equations and numerical simulation. It is found that the concentration of atmospheric methane increases with increase in the growth rate of cumulative density of methane, formed by various processes involved in the production of rice paddies and subsequently the increased level of mitigation options above its equilibrium value is required to reduce methane concentration. Moreover, increase in the implementation rate coefficient of various mitigation options and depletion rate coefficient due to net effectiveness of mitigation options further reduces the atmospheric methane concentration. The qualitative results obtained are confirmed quantitatively using numerical simulations of the model. The data from model prediction is also compared with actual methane data in the atmosphere and found to be fairly close.

Keywords: Mathematical model, Methane, Rice paddies, Mitigation options, Stability analysis

1 Introduction

The most abundant greenhouse gases are carbon dioxide, methane, nitrous oxide, ozone, CFCs, water vapour etc. but methane is the stronger greenhouse gas than carbon dioxide present in the atmosphere constituting 16% of greenhouse gas emissions. It is the most prevalent anthropogenic greenhouse gas in the atmosphere, produced mainly by rice paddies and is partly responsible for global warming. The effect of global warming can be seen in the form of many undesirable consequences such as decrease in agricultural production, the spread of infectious diseases, melting of glaciers, sea level rise causing migration of coastal population, loss of

---

* Corresponding author. *E-mail address*: ssmishra15@gmail.com

Published by World Academic Press, World Academic Union

biodiversity, drought, flood, etc,. Thus, global warming phenomenon is one of the most challenging problems to be studied by researchers working in the field of environment and climate change [1, 3, 4, 11, 22, 23]. Rice (Oryza sativa) is an important agricultural crop having significant contribution to global warming due to methane emission from the rice fields. It is noted that 20 % of global anthropogenic methane is due to rice production [3, 27]. Rice varieties have also significant effect on methane emission [7, 30]. In rice fields, methane is formed as a result of decomposition of organic matter under anaerobic soil conditions due to presence of methanogenic bacteria [5, 16, 17]. Various agricultural practices, climatic conditions, methods of cultivation, water management, selection of cultivar, use of fertilizer, physical and chemical properties of soil and its temperature, rain, etc. have significant effect on methane emission [2, 12, 15, 24–26, 38].

Since our environment is adversely affected by methane emissions from rice paddies, natural and human sources, various mitigation options are needed to control its emission. Mitigation options to control methane emission from rice paddies must be eco-friendly and cost-effective without depleting crop yields [6, 9, 21, 28]. Several studies have been conducted to show the importance of water management practices such as mid season drainage, intermittent irrigation, etc., to control methane emissions effectively from rice fields [14, 31–33, 36, 37]. A reduction in methane concentration was observed in Japan during mid season drainage and intermittent irrigation near the end of the growing season [35] whereas 30.5% reduction in total methane emission by mid season drainage was observed during growing season [18]. Methane emissions from rice fields can also be controlled by effective use of fertilizers, proper selection of low methane emitting cultivars, effective methods of cultivation [13, 17, 34, 39].

It is noted that various kinds of processes such as transport, ebullition, diffusion, etc., are involved in the formation and emission of methane from paddy fields. To understand the effect of such processes, they are all combined together in the form of one separate variable which is dependent on the cumulative biomass density of rice paddies. This is a novel idea used in the modeling of the formation and emission of methane from rice paddies which has not been studied earlier using mathematical models.

Thus, in order to curtail methane emissions from rice fields and stabilizing it, proper selection of mitigation options with its effective implementation is crucial. Therefore, in this paper, a nonlinear mathematical model is proposed and analyzed to study the impact of mitigation options in reducing methane emissions from rice fields. The organization of the paper is as follows. In the upcoming Section 2, a mathematical model governing the problem is proposed and discussed. In Section 3, the equilibrium analysis of the model system is carried out. In Section 4, stability analysis of the model system is performed using stability theory of differential equations and certain inferences have been drawn by establishing local and nonlinear stability results of the equilibria. Numerical simulation of the model system is also performed to validate analytical results (Section 5). Section 6 contains conclusion.

2 Mathematical model

To model the problem of the effect of mitigation options on the emission of methane in the atmosphere, the following variables are considered in the modeling process,

1. The biomass density of rice paddies.

2. The density of human population using rice, the amount of which is assumed to the proportional to the biomass density of rice paddies.

3. The cumulative density of methane in the atmosphere caused by various practices involved in the production of rice paddies.

4. The atmospheric concentration of methane formed by various processes involved in the production of rice paddies as well as natural and other human sources.

5. The cumulative density of mitigation options to control methane.

Let $B(t)$ be the biomass density of rice paddies, $N(t)$ be the density of human population, $P_B(t)$ be the cumulative density of methane formed by various processes involved in the production of rice paddies, $C(t)$ be the atmospheric concentration of methane caused by various processes mentioned above. Let $M(t)$ be the cumulative density of various mitigation options, assumed to be proportional to the increased level of atmospheric methane concentration from its equilibrium level C_0. It is also assumed that the increased level of various mitigation options from its equilibrium level M_0 is applied to control the concentration of methane in the atmosphere.

In view of the above considerations, the problem is governed by the following system of nonlinear ordinary differential equations[10, 19, 20, 29],

$$\frac{dB}{dt} = s\left(B - \frac{B^2}{L}\right) - s_1 BN \tag{1}$$

$$\frac{dN}{dt} = r\left(N - \frac{N^2}{K}\right) + r_1 BN \tag{2}$$

$$\frac{dP_B}{dt} = \alpha B - \alpha_0 P_B \tag{3}$$

$$\frac{dC}{dt} = Q_0 + \lambda_B P_B + \lambda_N N - \lambda_0 C - \lambda C(M - M_0) \tag{4}$$

$$\frac{dM}{dt} = \varphi(C - C_0) - \varphi_0(M - M_0) \tag{5}$$

where

$$C_0 = \frac{Q_0}{\lambda_0}$$

In the Eq. (1) above, the dynamics of biomass density of rice paddies is assumed to be governed by the logistic equation where the constants s and L denote intrinsic growth rate and carrying capacity of rice paddies respectively. It is noted that the biomass density of rice paddies decreases due to its consumption by human population while the density of human population increases due to consumption of rice paddies. Thus, the decrease in biomass density of rice paddies is assumed to be proportional to the density of human population as well as biomass density of rice paddies (i.e. $s_1 BN$). The constant s_1 represents the consumption rate coefficient of rice paddies due to human population.

Since the cumulative density of human population increases due to use of rice paddies and therefore it is reasonable to assume that the growth rate of density of human population is proportional to the biomass density of rice paddies as well as density of human population (i.e. $r_1 BN$) where r_1 is its growth rate coefficient due to consumption of rice paddies. Thus, the density of human population (N) is also assumed to grow logistically with its intrinsic growth rate r and the carrying capacity K, as in Eq. (2) above.

In Eq. (3), the growth rate of cumulative density of methane formed by various processes involved in the production of rice paddies (i.e. P_B) is assumed to be proportional to the biomass density of rice paddies where the constant α denotes its growth rate coefficient and the constant a_o is its natural depletion rate coefficient.

In Eq. (4), the growth of atmospheric methane is assumed to be increased by cumulative density of methane formed by various processes involved in the production of rice paddies with a growth rate coefficient λ_B and by human population density with a growth rate coefficient λ_N with constant input of methane Q_o from natural sources. The constant λ_o is the depletion rate coefficient of atmospheric methane due to natural factors. The decrease in the concentration of atmospheric methane is assumed to be proportional to the increased level of mitigation options from its equilibrium level M_0 as well as the concentration of atmospheric methane (i.e. $\lambda C(M - M_0)$). The constant M_0 defines the basic level of mitigation options applied at all time in order to maintain the atmospheric methane concentration at the level C_0. This implies that $C = C_0$ when $M = M_0$.The constant λ is the depletion rate coefficient of atmospheric methane due to net effectiveness of mitigation options.

The cumulative density of mitigation options M, as in Eq. (5), is assumed to be proportional to the increased level of atmospheric methane concentration from its equilibrium level C_0 where ϕ denoted its growth rate coefficient and ϕ_0 denotes its natural depletion rate coefficient. It is noted that the increased level of various mitigation options from its equilibrium level M_0 is applied to control the concentration of methane in the atmosphere.

A particular case of the model (1)-(5) can be obtained by assuming that the processes forming methane are instantaneous i.e. $\frac{dP_B}{dt} = 0$.

In such a case the model (1)-(5) reduces to the following form,

$$\frac{dB}{dt} = s\left(B - \frac{B^2}{L}\right) - s_1 BN \tag{6}$$

$$\frac{dN}{dt} = r\left(N - \frac{N}{K}\right) + r_1 BN \tag{7}$$

$$P_B = \frac{\alpha}{\alpha_0} B \tag{8}$$

$$\frac{dC}{dt} = Q_0 + \lambda_B \frac{\alpha}{\alpha_0} B + \lambda_N N - \lambda_0 C - \lambda C(M - M_0) \tag{9}$$

$$\frac{dM}{dt} = \varphi(C - C_0) - \varphi_0(M - M_0) \tag{10}$$

This particular model, governed by Eq. (6)-Eq. (10) is four dimensional and its qualitative properties are the same as that of the model (1)- (5), as regards to the effects of mitigation options on the control of methane are concerned. Therefore, we analyze only the model (1)- (5) and the properties of the model (6)- (10) can be derived from the analysis by making use of the Eq. (8).

In the following we find the bounds of different variables governing the model system (1)- (5) required for stability analysis of the model equilibria. For this, we state a lemma without proof, which gives region of attraction.

2.1 Lemma

The set $\Omega = \{ (B,\, N,\, P_B,\, C, M) \in R^5_+ : 0 \le B \le L,\, 0 \le N \le N_{\max},\, 0 \le P_B \le P_{B\max},\, ,\; C_0 \le C \le C_{\max}, M_0 \le M \le M_{\max}\}$

where, $N_{\max} = \frac{K(r+r_1 L)}{r}, P_{B\max} = \frac{\alpha}{\alpha_0}L, C_{\max} = \frac{Q_0+\lambda_B P_{B\max}+\lambda_N N_{\max}}{\lambda_0}, M_{\max} = M_0 + \frac{\varphi}{\varphi_0}(C_{\max} - C_0)$ and $C_0 = \frac{Q_0}{\lambda_0}$, attracts all solutions initiating in the interior of positive octant.

3 Equilibrium analysis

The model (1)- (5) has the following four equilibria,

1. $E_0(0, 0, 0,\, C_0,\, M_0), C_0 = \frac{Q_0}{\lambda_0}$
2. $E_1(L, 0, \bar{P}_B,\, \bar{C},\, \bar{M})$
3. $E_2(0, K,\, 0, \tilde{C},\, \tilde{M})$
4. $E^*(B^*, N^*, {P_B}^*,\, C^*,\, M^*)$

The equilibrium $E_0(0, 0, 0,\, C_0,\, M_0)$ always exists. It implies that the absence of rice paddies and human population makes no contribution to the atmospheric methane emissions and therefore the atmospheric methane is always maintained at its equilibrium level C_0 with mitigation options at its basic level.

3.1 Existence of $E_1(L, 0, \bar{P}_B, \bar{C}, \bar{M})$

Where, $\bar{P}_B = \frac{\alpha}{\alpha_0}L$, $\bar{C} = \frac{-\left(\lambda_0 - Q_0\frac{\lambda\varphi}{\lambda_0\varphi_0}\right) + \sqrt{\left(\lambda_0 - Q_0\frac{\lambda\varphi}{\lambda_0\varphi_0}\right)^2 + 4\lambda\frac{\varphi}{\varphi_0}\left(Q_0 + \lambda_B\frac{\alpha}{\alpha_0}L\right)}}{2\lambda\frac{\varphi}{\varphi_0}}$, $\bar{M} = M_0 + \frac{\varphi}{\varphi_0}(\bar{C} - C_0)$ and $C_0 = \frac{Q_0}{\lambda_0}$.

The equilibrium $E_1(L, 0, \bar{P}_B, \bar{C}, \bar{M})$ always exists. It implies the presence of rice paddies at its carrying capacity with no human intervention in such a way that the concentration of atmospheric methane exceeds its equilibrium level due to its emission from various processes involved in the production of rice paddies and therefore mitigation options need to be applied above its equilibrium value.

3.2 Existence of $E_2(0, K, 0, \tilde{C}, \tilde{M})$

Where, $\tilde{C} = \frac{-\left(\lambda_0 - Q_0\frac{\lambda\varphi}{\lambda_0\varphi_0}\right) + \sqrt{\left(\lambda_0 - Q_0\frac{\lambda\varphi}{\lambda_0\varphi_0}\right)^2 + 4\lambda\frac{\varphi}{\varphi_0}(Q_0 + \lambda_N K)}}{2\lambda\frac{\varphi}{\varphi_0}}$, $\tilde{M} = M_0 + \frac{\varphi}{\varphi_0}(\tilde{C} - C_0)$ and $C_0 = \frac{Q_0}{\lambda_0}$.

The equilibrium $E_2(0, K, 0, \tilde{C}, \tilde{M})$ always exists. It signifies the presence of human population with no availability of rice paddies and associated formation of methane by other processes involved in the production of rice paddies. In this case also the concentration of atmospheric methane exceeds its equilibrium level C_0 due to its emission from human activities and hence the mitigation options need to be applied above its basic level.

3.3 Existence of $E^*(B^*, N^*, {P_B}^*, C^*, M^*)$

The existence and uniqueness of nontrivial equilibrium is carried out as follows. The variables in $E^*(B^*, N^*, {P_B}^*, C^*, M^*)$ are given by the following algebraic equations,

$$B = \frac{L}{s}(s - s_1 N) \tag{11}$$

$$N = \frac{K}{r}(r + r_1 B) \tag{12}$$

$$P_B = \frac{\alpha}{\alpha_0}B \tag{13}$$

$$Q_0 + \lambda_B\frac{\alpha}{\alpha_0}B + \lambda_N N - \lambda_0 C - \lambda C(M - M_0) = 0 \tag{14}$$

$$M = M_0 + \frac{\varphi}{\varphi_0}(C - C_0) \tag{15}$$

From Eq. (11) and Eq. (12), we get

$$B = \frac{Lr(s - s_1 K)}{sr + s_1 r_1 L K_a} = B^*, \text{ for } s > s_1 K. \tag{16}$$

$$N = \frac{Ks(r + r_1 L)}{sr + s_1 r_1 L K} = N^* \tag{17}$$

Using Eqs. (15)- (17) in Eq. (14), we get

$$Q_0 + \lambda_B\frac{\alpha}{\alpha_0}B^* + \lambda_N N^* - \lambda_0 C - \lambda\frac{\varphi}{\varphi_0}C(C - C_0) = 0 \tag{18}$$

From which we get,

$$C = \frac{-\left(\lambda_0 - C_0\frac{\lambda\varphi}{\varphi_0}\right) + \sqrt{\left(\lambda_0 - C_0\frac{\lambda\varphi}{\varphi_0}\right)^2 + 4\lambda\frac{\varphi}{\varphi_0}\left(Q_0 + \lambda_B\frac{\alpha}{\alpha_0}B^* + \lambda_N N^*\right)}}{2\lambda\frac{\varphi}{\varphi_0}} = C^*\text{(Let)}$$

Using the value of B^* and C^* , we can find the equilibrium value of ${P_B}^*$ and M^* from Eq. (13) and Eq. (15) respectively.

3.4 Variations of C with different parameters

From Eq. (18), we can easily calculate $\frac{dC}{d\varphi} = -\frac{\lambda C(C-C_0)}{\lambda_0\varphi_0+\lambda\varphi(2C-C_0)} < 0 \, as \, C > C_0 = \frac{Q_0}{\lambda_0}$.

This implies that the concentration of atmospheric methane decreases with increase in the growth rate coefficient of the cumulative density of mitigation options ϕ.

Similarly, we can show $\frac{dC}{d\lambda_B} > 0$, as $C > C_0$, which implies that the concentration of atmospheric methane increases as the growth rate coefficient λ_B representing the growth of cumulative density of methane formed by various processes involved in the production of rice paddies increases.

4 Stability analysis

The local stability of equilibria can be investigated by determining the sign of the eigenvalues of Jacobian matrix evaluated at each equilibrium. The Jacobian matrix for the model system (1)- (5) is given as follows:

$$J = \begin{bmatrix} s\left(1-\frac{2B}{L}\right)-s_1N & -s_1B & 0 & 0 & 0 \\ r_1N & r\left(1-\frac{2N}{K}\right)+r_1B & 0 & 0 & 0 \\ \alpha & 0 & -\alpha_0 & 0 & 0 \\ 0 & \lambda_N & \lambda_B & -[\lambda_0+\lambda(M-M_0)] & -\lambda C \\ 0 & 0 & 0 & \varphi & -\varphi_0 \end{bmatrix}$$

Let J_0, J_1 and J_2 be the Jacobian matrices evaluated at equilibria $E_0(0,0,0,C_0,M_0)$, $E_1(L,0,\bar{P}_B,\bar{C},\bar{M})$ and $E_2(0,K,0,\tilde{C},\tilde{M})$ respectively. It can easily be checked that E_0, E_1 and E_2 are unstable.

4.1 Local stability analysis

To check the local stability of $E^*(B^*,N^*,{P_B}^*,C^*,M^*)$, we consider the following positive definite function,

$$V = \frac{1}{2}k_1{B_1}^2 + \frac{1}{2}k_2{N_1}^2 + \frac{1}{2}k_3{P_{B1}}^2 + \frac{1}{2}k_4{C_1}^2 + \frac{1}{2}k_5{M_1}^2 \tag{19}$$

where $k_i (i = 1 \text{ to } 5)$ are positive constants to be chosen suitably and B_1, N_1, P_{B1}, C_1, M_1 are small perturbations about $E^*(B^*,N^*,{P_B}^*,C^*,M^*)$ such that $B_1 = B - B^*$, $N_1 = N - N^*$, $P_{B1} = P_B - {P_B}^*$, $C_1 = C - C^*$ and $M_1 = M - M^*$. Differentiating V with respect to $'t'$ we get,

$$\frac{dV}{dt} = k_1B_1\frac{dB_1}{dt} + k_2N_1\frac{dN_1}{dt} + k_3P_{B1}\frac{dP_{B1}}{dt} + k_4C_1\frac{dC_1}{dt} + k_5M_1\frac{dM_1}{dt} \tag{20}$$

Putting the values of the linearized form of derivatives and simplifying, we get,

$$\begin{aligned} \frac{dV}{dt} &= -k_1\frac{s}{L}B^*{B_1}^2 - k_2\frac{r}{K}N^*{N_1}^2 - k_3\alpha_0{P_{B1}}^2 - k_4(\lambda_0+\lambda(M^*-M_0)){C_1}^2 \\ &-k_5\varphi_0{M_1}^2 + (k_2r_1N^* - k_1s_1B^*)B_1N_1 + k_3\alpha B_1P_{B1} + k_4\lambda_BP_{B1}C_1 \\ &+k_4\lambda_NC_1N_1 + (k_5\varphi - k_4\lambda C^*)C_1M_1 \end{aligned} \tag{21}$$

After some algebraic manipulations and by choosing, $k_1 = 1$, $k_2 = \frac{s_1B^*}{r_1N^*}$, $k_3 < \frac{s\alpha_0B^*}{\alpha^2L}$, $k_4 < \frac{2}{3}[\lambda_0 + \lambda(M^*-M_0)]\min\left\{\frac{rN^*k_2}{K{\lambda_N}^2}, \frac{\alpha_0k_3}{{\lambda_B}^2}\right\}$, $k_5 = \frac{\lambda C^*k_4}{\varphi}$

It is found that $\frac{dV}{dt}$ is negative definite. Hence E^* is locally asymptotically stable without any condition.

This result can be stated in the form of the following theorem.

Theorem 1. *The equilibria E_0, E_1 and E_2 are unstable and the equilibrium E^* is locally asymptotically stable without any condition.*

4.2 Nonlinear stability analysis

In the following, the nonlinear stability behavior of the equilibrium $E^*(B^*, N^*, {P_B}^*, C^*, M^*)$ is established.

Consider a positive definite function,

$$\begin{aligned} U = m_1\left(B - B^* - B^* \log \tfrac{B}{B^*}\right) + m_2\left(N - N^* - N^* \log \tfrac{N}{N^*}\right) + \tfrac{1}{2}m_3(P_B - {P_B}^*)^2 \\ +\tfrac{1}{2}m_4(C - C^*)^2 + \tfrac{1}{2}m_5(M - M^*)^2 \end{aligned} \tag{22}$$

where $m_i (i = 1\ to\ 5)$ are positive constants to be chosen suitably.

Differentiating U with respect to $'t'$ we get

$$\begin{aligned} \tfrac{dU}{dt} = \tfrac{m_1}{B}(B - B^*)\tfrac{dB}{dt} + \tfrac{m_2}{N}(N - N^*)\tfrac{dN}{dt} + m_3(P_B - {P_B}^*)\tfrac{dP_B}{dt} \\ +m_4(C - C^*)\tfrac{dC}{dt} + m_5(M - M^*)\tfrac{dM}{dt} \end{aligned} \tag{23}$$

Putting the values of derivatives from Eq. (1)-Eq. (5) and simplifying, we get,

$$\begin{aligned} \tfrac{dU}{dt} = -m_1\tfrac{s}{L}(B - B^*)^2 - m_2\tfrac{r}{K}(N - N^*)^2 - m_3\alpha_0(P_B - {P_B}^*)^2 \\ -m_4\left\{\lambda_0 + \lambda_1(M - M_0)\right\}(C - C^*)^2 - m_5\varphi_0(M - M^*)^2 + (m_2 r_1 - m_1 s_1)(B - B^*)(N - N^*) \\ +m_3\alpha(B - B^*)(P_B - {P_B}^*) + m_4\lambda_B(P_B - {P_B}^*)(C - C^*) \\ +m_4\lambda_N(N - N^*)(C - C^*) + (m_5\varphi - m_4\lambda C^*)(C - C^*)(M - M^*) \end{aligned} \tag{24}$$

After some algebraic manipulations and by choosing, $m_1 = 1$, $m_2 = \frac{s_1}{r_1}$, $m_3 < \frac{s\alpha_0}{\alpha^2 L}$, $m_4 < \frac{2}{3}[\lambda_0 + \lambda(M_m - M_0)] \min\left\{\frac{rm_2}{K{\lambda_N}^2}, \frac{\alpha_0 m_3}{{\lambda_B}^2}\right\}$, $m_5 = \frac{\lambda C^* m_4}{\varphi}$

It is noted that $\frac{dU}{dt}$ is negative definite. Hence E^* is nonlinearly asymptotically stable without any condition inside the region of attraction Ω.

This result can be stated in the form of the following theorem.

Theorem 2. *The equilibrium E^* is nonlinearly asymptotically stable without any condition inside the region of attraction Ω.*

5 Numerical simulation

In this section, numerical simulation is performed to check the feasibility of analytical results obtained for the model system (1)- (5) by taking into account the following set of parameter values involved in the model system,

$$Q_0 = 70, \lambda_0 = 0.1, s = 0.5, s_1 = 2 \times 10^{-6}, L = 2000, r = 0.2, r_1 = 2 \times 10^{-6}, K = 10000$$

$$\alpha = 0.01, \alpha_0 = 0.05, \lambda_B = 0.01, \lambda_N = 0.02, \lambda = 0.0001, \varphi = 0.01,\ \varphi_0 = 0.025, C_0 = 700, M_0 = 100$$

The equilibrium values corresponding to different variables in E^* are obtained as:

$$B^* = 1918.4652, N^* = 10191.8465, {P_B}^* = 383.6930, C^* = 1884.2138,\ M^* = 573.6855$$

The eigenvalues of the Jacobian matrix corresponding to equilibrium E^* are $-0.4793, -0.2041, -0.05, -0.0570$ and -0.0870. Since all the eigenvalues are negative and hence equilibrium E^* is locally asymptotically stable.

To present nonlinear stability of E^* for the model system (1)- (5), trajectories with different initial starts have been plotted in $P_B - C - M$ plane as shown in Fig. 1. It is apparent from Fig. 1 that all trajectories approach the equilibrium showing that the equilibrium E^* is nonlinearly asymptotically stable.

To visualize the variations of various model variables for different values of parameters, these variables are plotted with time as shown in Figs. 2-5. In Fig. 2, the variation of atmospheric methane concentration (C) with time $'t'$ is shown for different values of growth rate coefficient of the cumulative density of methane (i.e. $\lambda_B = 0.01,\ 0.05,\ 0.09$). From the figure, it is seen that the concentration of atmospheric methane increases as the growth rate of the cumulative density of methane, formed by various processes involved in the production of rice paddies (λ_B), increases. In Fig. 3, the variation of the cumulative density of various mitigation options (M) with time $'t'$ is shown for different values of growth rate of the cumulative density of methane (i.e. $\lambda_B = 0.01,\ 0.05,\ 0.09$). From this figure, it is seen that the cumulative density of mitigation options increases as the growth rate of the cumulative density of methane formed by various processes involved in the production of rice paddies (λ_B) increases. Thus, the increase in the concentration of atmospheric methane due to methane formation by the various processes involved in the production of rice paddies necessitates the increase in the cumulative density of mitigation options. In Fig. 4, the variation of the concentration (C) of atmospheric methane with time $'t'$ is shown for different values of the implementation rate coefficient of the various mitigation options (i.e. $\varphi = 0.010,\ 0.012,\ 0.014$). It is seen that the concentration of atmospheric methane decreases as the implementation rate coefficient of the various mitigation options (φ) increases. The atmospheric methane concentration also decreases with increase in the depletion rate coefficient of atmospheric methane due to net effectiveness of mitigation options (i.e. $\lambda = 0.00010,\ 0.00015,\ 0.00020$), as shown in Fig. 5. Further, regarding concentration of methane in the atmosphere, model data is compared with actual data taken from European Environmental Agency[8] as shown in Fig. 6. It is noted from the figure that the trend of increase of model data for the atmospheric concentration of methane is approximately similar to actual data.

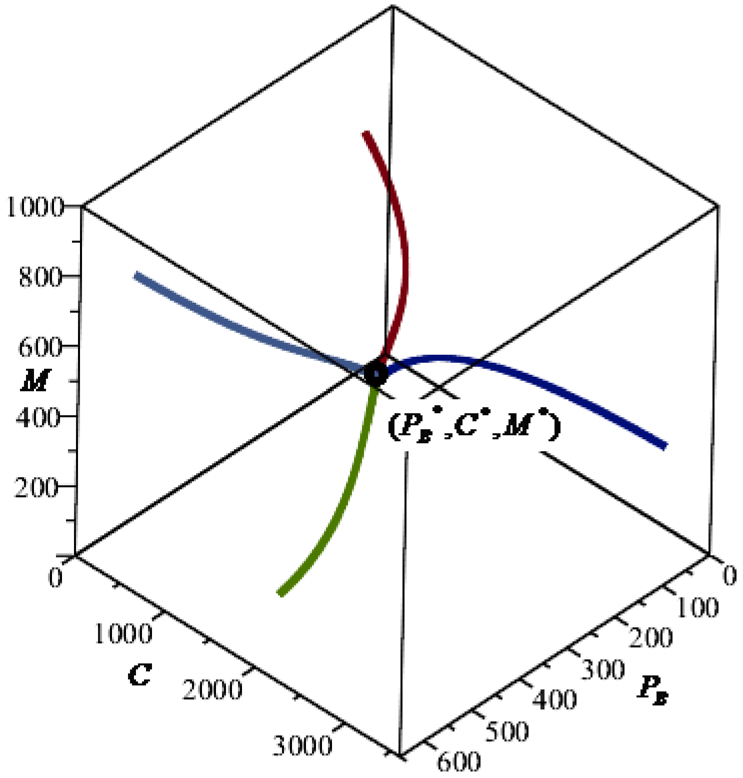

Fig. 1: Nonlinear stability in $P_B - C - M$ plane

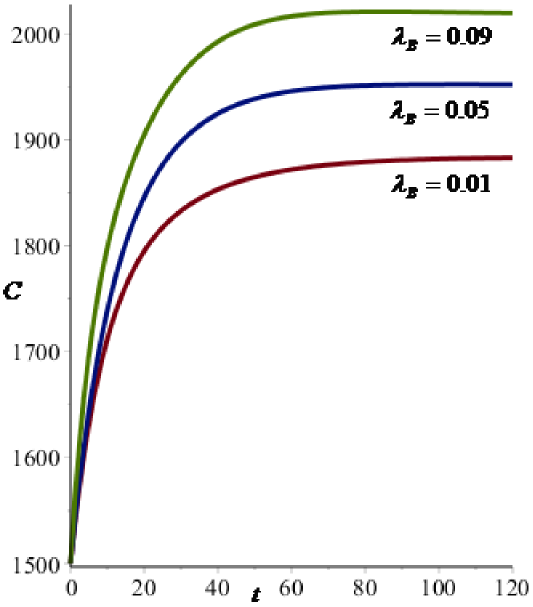

Fig. 2: Variation of atmospheric methane concentration C with time t for different values of the growth rate coefficient of the cumulative density of methane in the production of rice paddies (i.e. $\lambda_B = 0.01,\ 0.05,\ 0.09$)

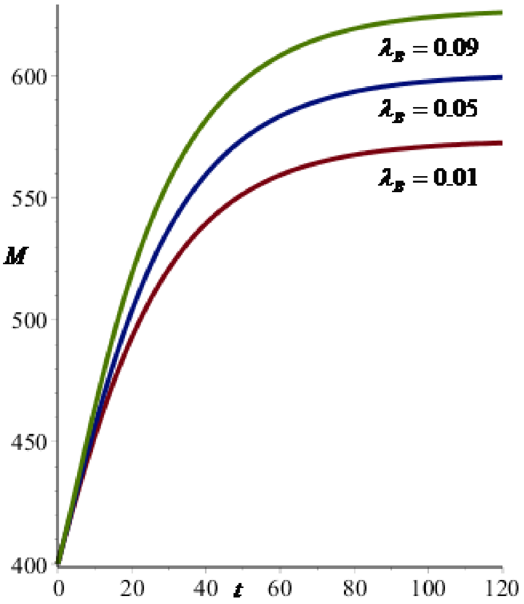

Fig. 3: Variation of the cumulative density of various mitigation options m with time t for different values of growth rate of the cumulative density of methane in the production of rice paddies (i.e. $\lambda_B = 0.01,\ 0.05,\ 0.09$)

6 Conclusion

In this paper, a nonlinear mathematical model has been proposed to study the effect of mitigation options on the control of methane emissions due to rice paddies and other human activities. The model consists of five nonlinearly interacting variables namely, the biomass density of rice paddies, the density of human population using rice, the amount of which is assumed to be proportional to the biomass density of rice paddies, the cumulative density of methane formed by various processes involved in the production of rice paddies, the

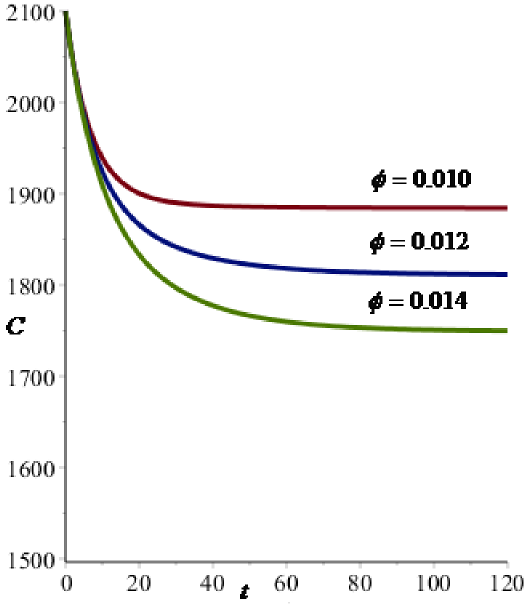

Fig. 4: Variation of atmospheric methane concentration C with time t for different values of the implementation rate coefficient of the various mitigation options (i.e. $\varphi = 0.010,\ 0.012,\ 0.014$)

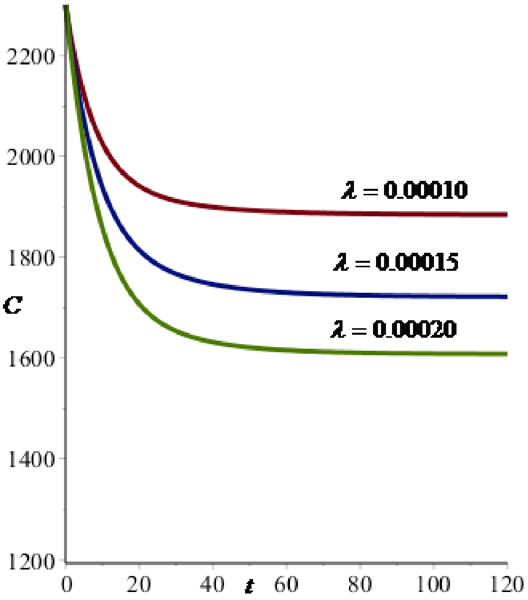

Fig. 5: Variation of atmospheric methane concentration C with time t for different values of depletion rate coefficient due to net effectiveness of mitigation options (i.e. $\lambda = 0.00010,\ 0.00015,\ 0.00020$)

atmospheric concentration of methane and the cumulative density of various mitigation options. The model analysis has been carried out using stability theory of ordinary differential equations. The analysis of the model reveals that the concentration of atmospheric methane increases as the growth rate of cumulative density of methane, formed by various processes involved in the production of rice paddies, increases. This increase in the atmospheric methane concentration requires the increased level of cumulative density of mitigation options above its equilibrium value to curtail methane concentration. Moreover, with increase in the implementation rate coefficient of various mitigation options, the atmospheric methane concentration also decreases which further declines with increase in the depletion rate coefficient due to net effectiveness of mitigation options. The trend of model data for atmospheric methane concentration is also found to be approximately similar to actual data taken from European Environmental Agency (EEA 2015).

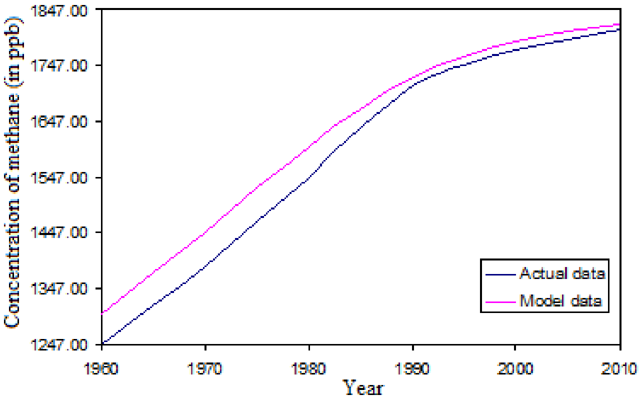

Fig. 6: Comparison between actual data and model data of atmospheric methane concentration

Acknowledgements

Authors are thankful to the anonymous reviewers for their fruitful suggestions and comments which helped us a lot to finalize the manuscript.

References

[1] N. K. Arora. Impact of climate change on agriculture production and its sustainable solutions. *Environmental Sustainability*, (2019), **2**:95-96.

[2] K. K. Boateng, G. Y. Obeng, E. Mensah. Rice Cultivation and Greenhouse Gas Emissions: A review and conceptual framework with reference to Ghana. *Agriculture*, 2017, **7**(1), 7. https://doi:10.3390/agriculture7010007.

[3] M. Cao, K. Gregson, S. Marshall, J. B. Dent, O. W. Heal. Global methane emissions from rice paddies. *Chemospher*, 1996, **33**(5): 879-897.

[4] J. A. Church, N. J. White. Sea-level rise from the late 19th to the early 21st Century. *Surveys in Geophysics*, 2011, **32**(4-5): 585-602. http://doi.org/10.1007/s10712-011-9119-1.

[5] R. Conrad. Control of microbial methane production in wetland rice fields. *Nutr. Cycl. Agroecosyst*, 2002, **64**: 59-69.

[6] V. Davamani, E. Parameswari, S Arulmani. Mitigation of methane gas emission in flooded paddy soil through the utilization of methanotrophs. *Science of the Total Environment*, 2020, 726: 138570, https://doi.org/10.1016/j.scitotenv.2020.138570

[7] A. Ding, C. Willis, R. Sass, F. Fisher. Methane emissions from rice fields: effect of plant height among several rice cultivars. *Global Biogeochem Cy*. 1999, **13**: 1045-1052.

[8] EEA. Atmospheric concentration of methane (ppb). *European Environment Agency*, Accessed 1 june 2018 (2015). http://www.eea.europa.eu/data-and-maps/figures/atmospheric-concentration-of-ch4-ppb-1.

[9] S. Faiz-ul Islam, J. van Groenigen, L. S. Jensen, B. O. Sander, A. de Neergaard. The effective mitigation of greenhouse gas emissions from rice paddies without compromising yield by early-season drainage. *Science of the Total Environment*, 2018, **612**: 1329-1339.

[10] A. Goyal, J. B. Shukla. Can methane oxidizing bacteria reduce global warming? A modeling study. *International Journal of Global Warming*, 2018, **15**: 82-97.

[11] P. Ghosh, P. Das, D.Mukherjee. Effect of environmental variation in a prey-predator fishery with harvesting in presence of toxicity. *World Journal of Modelling and Simulation*, 2018, **14**(3): 181-191.

[12] P. Hou, Y. Yu, L. Xue, E. Petropoulos, S. He, Y. Zhang, A. Pandey, L. Xue, L. Yang, D. Chen. Effect of long term fertilization management strategies on methane emissions and rice yield. *Science of the Total Environment,* 2020, **725**:138261, https://doi.org/10.1016/j.scitotenv.2020.138261

[13] IPCC Agriculture, In: B. Metz, O. R. Davidson, P. R. Bosch, R. Dave, L. A. Meyer, (eds). Climate change 2007: mitigation. Contribution of working group III to the fourth assessment report of the intergovernmental panel on climate change, *CambridgeUniversity Press, Cambridge, United Kingdom and New York NY, USA* (2007).

[14] M. Itoh, S. Sudo, et al. Mitigation of methane emissions from paddy fields by prolonging midseason drainage. *Agric. Ecosyst. Environ.* 2011, **141**: 359-372.

[15] M. K. Khosa, B. S. Sidhu, D. K. Benbi. Methane emission from rice fields in relation to management of irrigation water. *J. Environ. Biol.* 2011, **32**: 169-172.

[16] B. A. Linquist, K. J. Van. Groenigen, M. A. Adviento-Borbe, C. Pittelkow, C. Van. Kessel. An agronomic assessment of greenhouse gas emissions from major cereal crops. *Glob. Chang Biol.* 2012a, **18**: 194-209.

[17] B. A. Linquist, M. A. Adviento-Borbe, C. M. Pittelkow, C. Van. Kessel, K. J. Van. Groenigen. Fertilizer management practices and greenhouse gas emissions from rice systems: A quantitative review and analysis. *Field Crops Res.* 2012b, **135**: 10-21.

[18] K. Minamikawa, T. Fumoto, M. Itoh, M. Hayano, S. Sudo, K. Yagi. Potential of prolonged midseason drainage for reducing methane emission from rice paddies in Japan: a long-term simulation using the DNDC-Rice model. *Biol. Fertil. Soils*, 2014, **50**: 879-889.

[19] A. K. Misra, M. Verma. Modeling the impact of mitigation options on methane abatement from rice fields. *Mitig. Adapt. Strateg. Glob. Change*, 2014, **19**(7): 927-945.

[20] A. K. Misra, M. Verma. Modeling the impact of mitigation options on abatement of methane emission from livestock. *Nonlinear Analysis: Modelling and Control*, 2017, **22**: 210-229.

[21] T. Parthasarathi, K. Vanitha, S. Mohandass, E. Vered. Mitigation of methane gas emission in rice by drip irrigation F1000Research 2019, **8**:2023. (https://doi.org/10.12688/f1000research.20945.1)

[22] V. R. Pereira, G. C. Blain, A. M. H. de Avila , R. C. de Matos Pires, H. S. Pinto. Impacts of climate change on drought: changes to drier conditions at the beginning of the crop growing season in southern Brazil. *Bragantia*, 2018, **77**(1): 201-211, DOI: http://dx.doi.org/10.1590/1678-4499.2017007

[23] P. Raja Sekhara Rao, K. Venkata Ratnam, M. Sita Rama Murthy. Predictive dynamics of infectious diseases-a new technique. *World Journal of Modelling and Simulation*, 2019, **15**(2): 128-139.

[24] V. Ramanathan, Y. Feng. Air pollution, greenhouse gases and climate change: Global and regional perspectives. *Atmospheric Environment,* 2009, **43**: 37-50.

[25] R. L. Sass, F. M. Fisher, S. T. Lewis, M. F. Jund, F. T. Turner. Methane emissions from rice fields: Effect of soil properties. *Glob. Biogeochem. Cycles,* 1994, **8**(2): 135-140.

[26] H. Schutz, W. Seiler, R. Conrad. Influence of soil temperature on methane emission from rice paddy fields. *Biogeochemistry,* 1990, **11**(2): 77-95.

[27] W. Seiler, A. Holzapfel-Pschorn, R. Conard, D. Scharffe. Methane emission from rice paddies. *Journal of Atmospheric Chemistry,* 1984, **1**(3): 241-268.

[28] P. Setyanto, A. Pramono, T. A. Adriany, H. L. Susilawati, T. Tokida, A.T. Padre, K. Minamikawa. Alternate wetting and drying reduces methane emission from a rice paddy in Central Java, Indonesia without yield loss. *Soil Sci Plant Nut.*, 2018, **64**(1): 23-30.

[29] J. B. Shukla, M. S. Chauhan, S. Sundar, R. Naresh. Removal of carbon dioxide from the atmosphere to reduce global warming: A modeling study. *International Journal of Global Warming*, 2015, **7**: 270-292.

[30] J. Su, C. Hu, X. Yan, Y. Jin, Z. Chen, Q. Guan, Y. Wang, D. Zhong, C. Jansson, F. Wang. Expression of barley SUSIBA2 transcription factor yields high-starch low-methane rice. *Nature*, 2015, **523**: 602-606.

[31] S. Towprayoon, K. Smakgahn, S. Poonkaew. Mitigation of methane and nitrous oxide emissions from drained irrigated rice fields. *Chemosphere*, 2005, **59**: 1547-1556.

[32] L. Tyagi, B. Kumari, S. N. Singh. Water management - a tool for methane mitigation from irrigated paddy fields. *Sci. Total Environ.* 2010, **408**: 1085-1090.

[33] R. Wassmann, H. Papen, H. Rennenberg. Methane emission from rice paddies and possible mitigation strategies. *Chemosphere,* 1993, **26**: 201-217.

[34] B. Xie, X. Zheng, Z. Zhou, J. Gu, B. Zhu, X. Chen, Y. Shi, Y. Wang, Z. Zhao, C. Liu, Z. Yao, J. Zhu. Effects of nitrogen fertilizer on CH_4 emission from rice fields: multi-site field observations. *Plant Soil,* 2010, **326**: 393-401.

[35] K. Yagi, K. Minami. Effects of organic matter application on methane emission from some Japanese paddy fields. *Soil Sci. Plant Nutr.*, 1990, **36**: 599-610.

[36] K. Yagi, H. Tsuruta, K. Kanda, K. Minami. Effect of water management on methane emission from a Japanese rice paddy field: Automated methane monitoring. *Glob. Biogeochem.* Cycles, 1996, **10**(2): 255-267.

[37] K. Yagi, H. Tsuruta, K. Minami. Possible options for mitigating methane emission from rice cultivation. *Nutr. Cycl. Agroecosyst.* 1997, **49**: 213-220.

[38] H. Yao, R. Conrad, R. Wassmann, H. Neue. Effect of soil characteristics on sequential reduction and methane production in sixteen rice paddy soils from China, the Philippines, and Italy. *Biogeochemistry*, 1999, **47**: 269-295.

[39] G. Zhang, H. Yu, X. Fan, Y. Yang, J. Ma, H. Xu. Drainage and tillage practices in the winter fallow season mitigate CH4 and N2O emissions from a double-rice field in China. *Atmos. Chem. Phys.*, 2016, **16**: 11853-11866.