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Identification and Control of MIMO Systems using Time Moments in Delta Domain

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Abstract. Identification and control of continuous time Multi input-Multi output(MIMO) systems from the discrete time input-output data is presented in this work which provides a unified framework for identification and control of continuous time Multi input-Multi output(MIMO) systems. The concept of time moments(TM), a classical tools for system identification ,is recasted using delta operator to formulate the Delta time moments (DTM) .The use of delta operator provides the unified framework as at fast sampling limit the continuous time results are obtained in hand to hand with discrete time results. The DTMs are used for identification and control of MIMO systems in delta domain .Delta time moments (DTM) are computed from real time input out data and utilized to develop adaptive predictive algorithms in a model reference adaptive control framework. The reference model is developed in the complex delta domain from classical time and frequency domain specifications which guarantee both stability and performance of the overall controlled system. Four control schemes namely i) Inverse control using DTM(ICDTM) ii) Pade Adapted Inverse control using DTM with error feedback (PICDTMEF) iii) Plant Delta Time Moment Controller Scheme (PDTMCS) and iv) Plant Delta Time Moment Controller with Error Feedback Scheme (PDTMCEFS) are proposed in this work. Simulation examples are included to demonstrate for the perfect working of the proposed scheme. The stability and performance characteristic of the overall control system closely matches with that of the reference model, demonstrating the efficacy of these control schemes.

Keywords: Adaptive prediction algorithm (APA), Delta time moment (DTM), Model reference adaptive control (MRAC), Multiple input multiple output (MIMO) system.

1 Introduction

For the last few decades more attention was received for discrete time models [17, 28] among the researchers in the field of system identification[2] and parameter estimation. However the situation has been changed a lot. A much more attention has been given toward the identification of continuous time(CT) models. A survey for CT system identification is seen in [37]. The last few years have indeed witnessed considerable development in continuous time approaches to system identification from sampled data [9, 36]. A class of discrete-time models for continuous-time systems has been proposed by s.Mukhapadhyaya et al.[24]. A compilation of different methods related to parameter estimation for CT models methods was published in an edited book [34] in the year 1991 by N. K Sinha and G. P Rao has motivated the background of the work presented in this paper. In case of real time implementation of a controller using finite precision hardware, the user is interested in higher sampling rate since it typically implies better approximation to the continuous time results. The discrete time systems were described using the shift operator [10, 26]. In late 80's Middleton and Goodwin introduced the concept of delta operator [11, 22, 23] as an alternative to the shift operator, so far dominated in the literature of signal processing and system theory. The discrete time signal and system modeling based

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on signal shifting has its limitation as it fails to provide meaningful representation at fast sampling limit. The signal and system modeling using delta operator provides a meaningful representation and converges to its continuous time counter parts at fast sampling limit. The delta operator uses signal shifting in modeling of discrete time signals and systems. The desired frequency response of any system can be extracted by appropriate mathematical modelling of the input output data. With the ever increasing sophistication of model proposed for flow system there was an increasing need for method of evaluating the model parameters with greater accuracy. This calls for renewed interest to use a variant of moment methods so far used in model order reduction literature for parameter estimation, particularly in case of flow process[19]. However, in all these methods, batch data were used making the estimation offline. The application of time moment in real time estimation and control were proposed in [35][29, 32, 33] in which model reference adaptive control(MRAC)[1, 3, 13, 14] framework were used. The theory of time moments [5, 7] has its roots in higher order statistics [21] and finds its application in system theory because of the analogy between impulse response of a linear system, and a probability distribution function. The application of delta operator takes its versatility [4, 6, 8, 12, 15, 16, 18, 20, 25, 27, 30, 31, 38] among different arena of system theory. In this work the concept of time moment as developed in continuous-time signals and systems were recasted using delta operator and renamed as Delta time Moment(DTM). In this paper, online delta time moment (DTM) estimation schemes are developed and used in different control methods for identification and control of MIMO system. The reference model was developed from classical time, frequency and complex domain specifications.

The significant contributions are made in this paper in manifold:

In the earlier work the discrete time systems were represented by the shift operator, but shift operator parameterization fails to provide meaningful information at high sampling rate. For real time implementation of controller in digital domain need very high sampling rate to get better result. In this work the controller design for MIMO systems are done using the delta operator parameterization. The most important part is that at fast sampling limit the discrete domain results resembles to that of the continuous time results in delta operator parameterized system. The earlier method of system estimation and control through the method of moments were offline, in this paper the identification and control of the MIMO systems are done in online basis with the help of model reference adaptive control.So online system identification and control of MIMO system in delta domain is a newer concept and a new direction for further reeearch.

This paper is organized in the following way. The system representation in delta domain is presented in Section 2. In Section 3 the DTM estimation methods are outlined and different control schemes are presented in Section 4. In Section 5 the control algorithms for PDTMCS and PDTMCEFS have been discussed. Finally the Section 6 & Section 7 are devoted for analyzing the result and conclusion respectively.

2 System representation in delta domain

2.1 Definition of delta operator

The δ-operator is defined in the time-domain as:

$$\delta = \frac{\mathrm{q}-1}{\Delta} \tag{1}$$

where Δ the sampling period and q is the forward shift operator. Operating δ function on a deterministic signal $x(t)$, results in:

$$\delta \mathrm{x(t)} = \frac{\mathrm{x(t}+\Delta) - \mathrm{x(t)}}{\Delta} \tag{2}$$

It is straightforward to see that:

$$\lim_{\Delta \to 0} \delta x(t) = \frac{d}{dt} x(t) \tag{3}$$

2.2 State space modeling

The state space model of the linear continuous-time system is represented by following equation:

$$\begin{aligned} \frac{dx(t)}{dt} &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{4}$$

where x is the state, $x\varepsilon R^n$, u is the control input,$u\varepsilon R^m$ and y is the output, $y\varepsilon R^p$.

If the input is sampled by a sampling and a hold device then the corresponding discrete-time model is obtained. In shift operator domain the discrete model is represented as:

$$\begin{aligned} qx(t) &= A_q x(t) + B_q u(t) \\ y(t) &= Cx(t) \end{aligned} \tag{5}$$

Where,$\mathrm{A_q} = \mathrm{e}^{\mathrm{A}\varDelta}$

$$\mathrm{B_q} = \int_0^{\varDelta} \mathrm{e}^{\mathrm{A}(\varDelta-\tau)}\mathrm{B}\mathrm{d}\tau \tag{6}$$

The difficulty with the above representation is that, at fast sampling limit i.e. $\varDelta \to 0$

$$\begin{aligned} \lim_{\varDelta\to 0} A_q &= I \\ \lim_{\varDelta\to 0} B_q &= 0 \end{aligned} \tag{7}$$

Therefore, the discrete-time representation becomes numerically ill conditioned as the sampling interval decreases. The delta operator representation in state-space form is given as below:

$$\begin{aligned} \delta\mathrm{x(t)} &= \mathrm{A}_\delta\mathrm{x(t)} + \mathrm{B}_\delta\mathrm{u(t)} \\ \mathrm{y(t)} &= \mathrm{Cx(t)} + \mathrm{Du(t)} \end{aligned} \tag{8}$$

where,

$$\begin{aligned} &\mathrm{A}_\delta = \frac{\mathrm{e}^{\mathrm{A}\varDelta}-\mathrm{I}}{\varDelta} = \varPsi(\mathrm{A},\varDelta)\mathrm{A} \\ &\mathrm{B}_\delta = \frac{1}{\varDelta}\int_0^{\varDelta} \mathrm{e}^{\mathrm{A}(\varDelta-\tau)}\mathrm{B}\mathrm{d}\tau = \varPsi(\mathrm{A},\varDelta)\mathrm{B} \\ &\varPsi(\mathrm{A},\varDelta) = \left(\mathrm{I} + \frac{\mathrm{A}\varDelta}{2!} + \frac{\mathrm{A}^2\varDelta^2}{3!} + \cdots\right) \\ &\lim \mathrm{A}_\delta = \mathrm{A} \text{ and } \lim \mathrm{B}_\delta = \mathrm{B} \\ &\varDelta \to 0 \qquad \varDelta \to 0 \end{aligned} \tag{9}$$

Therefore, at fast sampling limit the delta operator model in Eq. (8) converges to its continuous-time state-space model in Eq. (4). Thus the use of delta operator ensures better numerical conditioning as compared to the shift operator representation. In addition, the convergence feature of delta representation to its continuous-time counterpart as ensures unified treatment of system analysis, synthesis, and design and control problems in delta formulation.

2.3 Transfer function modelling

The delta transform of the signal $f(\mathrm{k}\varDelta)$ is defined as:

$$\mathrm{F}_\delta(\gamma) = \varDelta\sum_{\mathrm{k}=0}^{\infty} \mathrm{f}(\mathrm{k}\varDelta)(1+\varDelta\gamma)^{-\mathrm{k}} \tag{10}$$

where k is the indexing discrete-time parameter and $t = k\varDelta$ are the independent discrete-time instants for $\mathrm{k} \in [0,\infty]$. The transfer function in γ-domain is represented as:

$$\mathrm{G}_\delta(\gamma) = \mathrm{C}\left(\gamma\mathrm{I} - \mathrm{A}_\delta\mathrm{I}\right)^{-1}\mathrm{B}_\delta \tag{11}$$

The system transfer function of Eq. (4) in s-domain and in z-domain are represented as:

$$\begin{aligned} G_c(s) &= C(sI - A)^{-1}B \\ G_q(z) &= C\,(sI - A_q)^{-1}\,B_q \end{aligned} \tag{12}$$

At fast sampling limit, $\lim G_q(z) = 0\ \Delta \to 0$ and $\lim G_\delta(\gamma) = G_c(s)\ \Delta \to 0$. Thus it is clearly observed that at fast sampling limit the results in z-domain are uninformative, whereas in the limit as $\Delta \to 0$,the delta domain transfer function becomes the original continuous-time transfer function.

3 Estimation of delta time moments(dtm)

3.1 Delta time moment(dtm)

The i^{th} DTM of the distribution is defined as:

$$T_{\delta_i} = \Delta \sum_{k=0}^{\infty} \frac{(k+i-1)!}{(k-1)!}(-\Delta)^i f(k\Delta) \tag{13}$$

3.2 Online delta time moment estimation for mimo system

As a first step toward adaptive control of an unknown plant, a DTM estimation scheme is proposed. Let $g_m(\bullet), g_p(\bullet) and g_{pc}(\bullet)$ respectively denote the impulse response of the reference model ,the unknown plant and the compensated plant respectively, each of which is asymptotically stable. In case of exact model matching (EMM) $g_m(\bullet)g_{pc}(\bullet)$ and this in turn implies:

$$(-1)^i \Delta \sum_{k=0}^{\infty} \frac{(k+i-1)!}{(k-1)!} \Delta^i g_m(k) = (-1)^i \Delta \sum_{k=0}^{\infty} \frac{(k+i-1)!}{(k-1)!} \Delta^i g_{pc}(k) \tag{14}$$

where, i =0, 1, 2...

Approximate model matching is built upon matching the first finite nos. of DTMs, i.e. $i = 0, 1, 2, ... < \eta < \infty$. Let, g(•) denote the impulse response of an asymptotically stable system . Let a MIMO system transfer function be a square matrices with same number of inputs and outputs i.e. a=b, such that $G_\delta(\gamma) = G_{\delta,k}(\gamma); \qquad k, r \in [1, a]$

Expanding $G_\delta(\gamma)$ using the McLaren series expansion gives,

$$G_{\delta,kr}(\gamma) = \sum_{i=0}^{\infty} g_{\delta_{i,kr}} \gamma^i \tag{15}$$

where, $G_{\delta,kr}$ are the delta time-moments(DTM) of $G_{\delta,kr}(\gamma)$, and t $g_{\delta_{i,kr}}$ is defined as:

$$g_{\delta_{i,kr}} = \frac{\Delta}{i!} \sum_{k=0}^{\infty} \frac{(k+i-1)!}{(k-1)!}(-\Delta)^i g_{kr}(k\Delta) \tag{16}$$

If the estimates of the $g(k\Delta)$ are taken up to N terms, the estimates of DTMs shall be:

$$g_{\delta_{ikr}} = \frac{\Delta}{i!} \sum_{k=0}^{N-1} \frac{(k+i-1)!}{(k-1)!} \quad (-\Delta)^i g_{kr}(k\Delta) \tag{17}$$

Consider an asymmetrically stable system $G_{\delta,kr}(\gamma)$ with no zeros on the unit circle, it can be assumed in the form as:

$$G_{\delta,kr}(\gamma) = \frac{Y_{\delta,kr}(\gamma)}{U_{\delta,kr}(\gamma)} \tag{18}$$

Let $\hat{g}_{\delta_{i,kr}}, \hat{y}_{\delta_{i,kr}}$ and $\hat{u}\delta_{i,kr}$ denote the estimates of system $\mathrm{G}_{\delta,\mathrm{kr}}(\gamma)$ and signals $\mathrm{Y}_{\delta,\mathrm{kr}}(\gamma)$ and $\mathrm{U}_{\delta,\mathrm{kr}}(\gamma)$ respectively. Restricting and up to $\eta + 1$ terms, and expanding Eq. (18) we obtain:

$$\sum_{i=0}^{\eta} \hat{g}_{\delta_{i,kr}} \gamma^i = \frac{\sum_{i=0}^{\eta} \hat{y}_{\delta_{i,kr}} \gamma^i}{\sum_{i=0}^{\eta} \hat{u}\delta_{i,kr} \gamma^i} \tag{19}$$

By cross multiplication and equating the coefficients of like powers of γ yields:

$$\hat{\mathrm{y}}_{\delta_{\mathrm{i,kr}}} = \sum_{j=0}^{1} \hat{\mathrm{g}}_{(\mathrm{i-j}),\mathrm{kr}} \hat{\mathrm{u}}\delta_{\mathrm{j,kr}} \tag{20}$$

where, i =0, 1, 2,...

By rearranging we also get:

$$\hat{\mathrm{g}}_{\delta_{\mathrm{i,kr}}} = \frac{\hat{\mathrm{y}}_{\delta_{0,\mathrm{kr}}}}{\hat{\mathrm{u}}\delta_{0,\mathrm{kr}}} - \frac{1}{\hat{\mathrm{u}}\delta_{0,\mathrm{kr}}} \sum^{\mathrm{i-1}} \hat{\mathrm{g}}_{\mathrm{j,kr}} \hat{\mathrm{u}}\delta_{(\mathrm{i-j})\mathrm{kr}} \tag{21}$$

Putting the estimated DTM by estimating y (kΔ) and u (kΔ) up to N terms with filtering for noise reduction, we get:

$$\begin{gathered}
\hat{g}\delta_{0,kr} = \operatorname{sgn}\left\{\sum_{k=0}^{N-1} \underline{y}(k\Delta)\underline{u}(k\Delta)\right\} \left[\frac{\sum_{k=0}^{N-1} \underline{y}^2(k\Delta)}{\sum_{k=0}^{N-1} \underline{u}^2(k\Delta)}\right]^{1/2} \quad \& \\
\hat{g}_{\delta i,kr} = \operatorname{sgn}\left\{\sum_{k=0}^{N-1}\left\{\frac{1}{i!}\frac{k+i-1}{k-1}\Delta^i \underline{y}(k\Delta) - \sum_{k=0}^{i-1} \hat{g}_{\delta,kri}\frac{1}{(i-j)!}\frac{(k+i-j-1)!}{(k-1)!}(-\Delta)^{i-j}u(k\Delta)\right\}\mathrm{u}(k\Delta)\right\} - \\
\left[\frac{\sum_{k=0}^{N-1}\left\{\frac{1}{\mathrm{i}!}\frac{(k+i-1)!}{(k-1)!}\Delta^i y(k\Delta) - \sum_{k=0}^{i-1}\hat{g}_{\delta i,kr}\frac{1}{(i-j)!}\frac{(k+i-j-1)!}{(k-1)!}(-\Delta)^{i-j}F^i u(k\Delta)\right\}}{\sum_{k=0}^{N-1}\{F^i u(k\Delta)\}^2}\right]^{1/2}
\end{gathered} \tag{22}$$

4 Control schemes

4.1 Proposed method of dtm matching

By using online time moment estimate algorithm in [29, 32, 33, 35], i^{th} delta time moments $\hat{\mathrm{g}}_{\delta_{\mathrm{i,kr}}}$ for MIMO system were estimated. The DTMs so obtained, are used in control schemes to match the DTMs of the compensated plant with the desired response. The control schemes are i) ICDTM and ii) PICDTMEF iii) PDTMCS and iv) PDTMCEFS as available from [33] are detailed in next four sections.

4.2 Inverse control using dtm(icdtm)

In this scheme the compensator $\mathrm{G_c}(\gamma)_{\mathrm{axb}}$ is cascaded with plant $\mathrm{G_p}(\gamma)_{\mathrm{axb}}$, both together compared with the reference model. Plant follows the reference model output for inverse control when,

$$\mathrm{G_c}(\gamma)_{\mathrm{axb}} \mathrm{G_p}(\gamma)_{\mathrm{axb}} = 1 \tag{23}$$

The structure for MIMO representation is defined as:

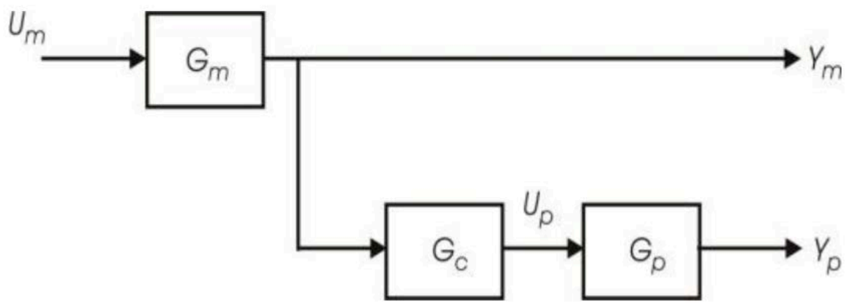

Fig. 1: Block Diagram of ICDTM

$$
\begin{aligned}
G_{\mathrm{p}}(\gamma) &= \begin{bmatrix} g^k_{p,11} & g^k_{p,12} & \cdots & g^k_{p,1b} \\ g^k_{p,21} & g^k_{p,22} & \cdots & g^k_{p,2b} \\ . & . & \cdots & . \\ g^k_{p,a1} & g^k_{p,a2} & \cdots & g^k_{p,ab} \end{bmatrix} \\
G_c(\gamma) &= \begin{bmatrix} g^k_{c,11} & g^k_{c,12} & \cdots & g^k_{c,1b} \\ g^k_{c,21} & g^k_{c,22} & \cdots & g^k_{c,2b} \\ . & . & \cdots & . \\ g^k_{c,a1} & g^k_{c,a2} & \cdots & g^k_{c,ab} \end{bmatrix}
\end{aligned} \tag{24}
$$

Comparing the delta time moments of either side, the coefficients of the compensator are obtained as:

$$
f_{_{0,kr}} = \frac{1}{\hat{g}_{0,p,kr}}, f_{i,kr} = -\frac{\sum_{j=0}^{i-1} \hat{g}_{(i-j),p,kr}\, f_{j,kr}}{\hat{g}_{0,p,kr}} \tag{25}
$$

where, $i = 1, 2 \ldots \eta < \infty$
$\hat{\mathrm{g}}_{\mathrm{ip,kr}}$ is the i^{th} estimated delta time moment of the plant.

4.3 Pade adapted inverse control using dtm with error feedback (picdtmef)

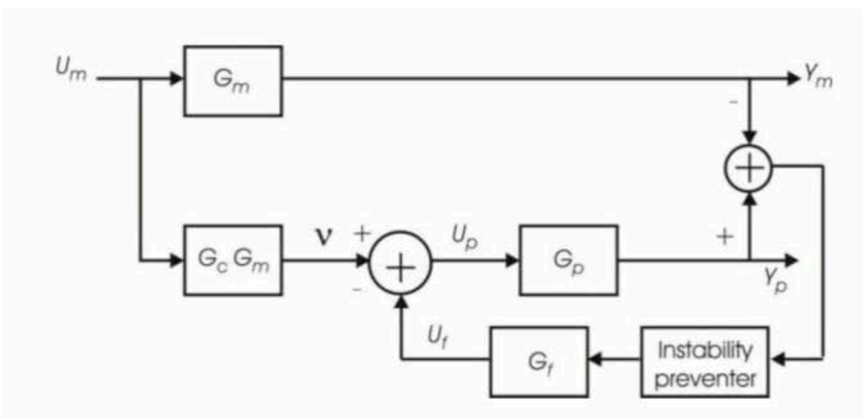

Fig. 2: Block Diagram of PICDTMEF

In this scheme error feedback is generated & fed to the plant and an instability preventer is provided to protect the system from instability. The rest of the other details are same as ICDTM scheme.

4.4 Plant delta time moment controller scheme (pdtmcs)

It is desirable to get G_pH_c such that approximates G_m in the sense of matching the first few delta time moments. H_c must be such that G_pH_c to be stable. In that case $\mathrm{G_p}(\gamma)\mathrm{H_c}(\gamma) = \mathrm{G_m}(\gamma)$ and the compensator may be written like

$$H_c(\gamma) = h_0 + h_1\gamma + ... + h_\eta\gamma^n = \frac{B(\gamma)}{A(\gamma)} = \frac{b_0 + b_1\gamma + ... + b_\nu\gamma^\nu}{a_0 + a_1\gamma + ... + a_\mu\gamma^\mu} \tag{26}$$

through $\gamma^{\mu+\nu}$

Set $a_\mu = 1$. Cross multiplying and equating the coefficients of like powers of γ yields,

Ha=b

Where, $H = \begin{bmatrix} h_0 & 0 & 0 & ... & 0 \\ h_1 & h_0 & 0 & ... & 0 \\ h_2 & ... & ... & ... & ... \\ ... & ... & & ... & ... \\ h_\eta & h_{\eta-1} & ... & ... & h_0 \end{bmatrix}$

$a = \begin{bmatrix} a_0 & a_1 & ... & a_{\eta-1} & 1 \end{bmatrix}$ and $b = \begin{bmatrix} b_0 & b_1 & ... & b_{\eta-1} & b_\eta \end{bmatrix}$

The structure of the compensator representation is defined as:

$$H_c(\gamma) = \begin{bmatrix} h^k_{c,11} & h^k_{c,12} & ... & h^k_{c,1b} \\ h^k_{c,21} & h^k_{c,22} & ... & h^k_{c,2b} \\ . & . & ... & . \\ h^k_{c,a1} & h^k_{c,a2} & ... & h^k_{c,ab} \end{bmatrix} \tag{27}$$

Comparing the delta time moments of either side, the coefficients of the compensator are obtained as:

$$h_{0,kr} = \frac{1}{\hat{g}_{0,p,kr}}, \quad h_{i,kr} = -\frac{\sum_{j=0}^{i-1} \hat{g}_{(i-j),p,kr}\, h_{j,kr}}{\hat{g}_{0,p,kr}} \tag{28}$$

Where, $i = 1, 2 \ldots \eta < \infty$

$\hat{g}_{ip,kr}$ is the i^{th} estimated delta time moment of the plant. The block diagram of this scheme is presented in Fig. 3.

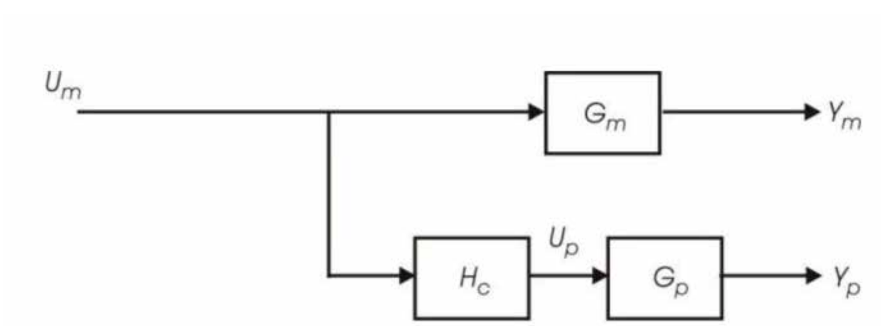

Fig. 3: Block Diagram of PDTMCS

4.5 Plant delta time moment controller with error feedback scheme(pdtmcefs)

In this scheme error feedback is generated & fed to the plant. An instability preventer is provided to protect the system from instability. The rest of the other details are same as PDTMCS scheme.

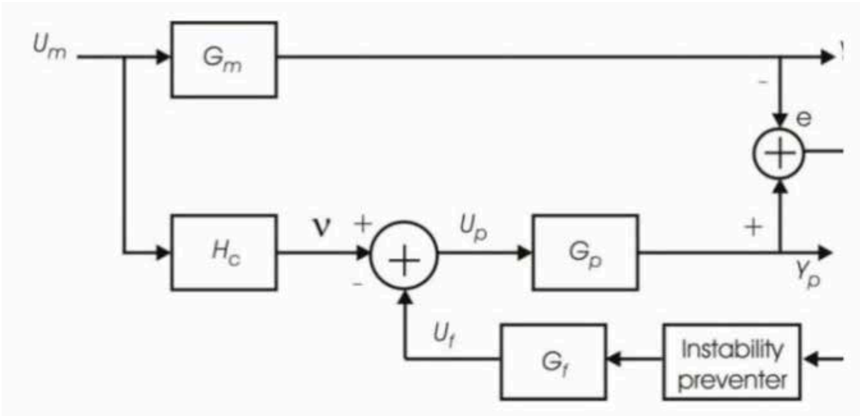

Fig. 4: Block Diagram of PDTMCEFS

5 Algorithms for pdtmcs and picdtmef control schemes

5.1 Algorithm for pdtmcs

Step1: Design the reference model (DRM) in the complex γ-domain from the classical time, frequency and complex domain specification.

Step2: Excite the DRM by u_m and obtain y_m.

Step3: Assume structure of the compensator $H_c(\gamma) = h_0 + h_1\gamma + ... + h_\eta\gamma^\eta = \frac{b_0 + b_1\gamma + ...+b_\nu\gamma^\nu}{a_0 + a_1\gamma + ...+a_\mu\gamma^\mu} = \frac{B(\gamma)}{A(\gamma)} = I\ (only\,1st\,case)$ Set $a = 1, \upsilon = \mu = \eta$ and after cross multiplication, equating co-efficient up to γ^η. Specify A such that B/A is stable and compute b through a so obtained.

Step3: Excite H_c by u_m and obtain u_p.

Step5: Excite G_p(assume unknown) by u_p and obtain y_p.

Step6: Estimate DTM series of G_p from the estimated u_p and y_p using Eq. (28).

Step7: Excite G_m by up and obtain y_{m1}.

Step8: Estimate DTM series of G_m from the estimated up and ym1 using Eq. (28).

Step9: update the parameter of the compensator (H_c) from the estimated DTMs of G_p and G_m with scalar correction using following relationship: $\mathrm{g}_\mathrm{p}^* h = \hat{\mathrm{g}}_\mathrm{m}$ where,

$$g_p^* = \begin{bmatrix} g^*_{0,p} & 0 & 0 & ------- & 0 \\ g^*_{1,p} & g^*_{0,p} & 0 & ------- & 0 \\ | & & | & & \\ | & & | & & \\ | & & | & & \\ g^*_{\eta,p} & g^*_{\eta-1,p} & & ------- & g^*_{0,p} \end{bmatrix}$$

$h = \begin{bmatrix} h_0 & h_1 & h_2 & ... & h_{\eta-1} \end{bmatrix}$ and $\hat{g}_m = \begin{bmatrix} \hat{g}_{0,m} & \hat{g}_{1,m} & ... & ... & ... & \hat{g}_{\eta-1,m} & \hat{g}_{\eta,m} \end{bmatrix}^T$

Step10: According to adaptation time repeat step 3 to step 9 then go step 11.

Step11: If adaptation is complete ($y_p \cong y_m$) stop, else go to step 5, after adjusting adaptation time.

5.2 Algorithm for picdtmefs

Step1: Design the reference model (DRM) in the complex γ-domain from the classical time, frequency and complex domain specification.

Step2: Excite DRM by u_m and obtain y_m.

Step3: Compute exact delta time moments (DTM) from DRM.

Step4: Compute the parameter of the controller ($f_0, f_1, f_2, ..., f_\eta$) by using Eq. (25).

Step5: Assume structure of the compensator $G_c(\gamma) = f_0 + f_1\gamma + ...+f_\eta\gamma^\eta = \frac{b_0 + b_1\gamma + ...+b_\nu\gamma^\nu}{a_0 + a_1\gamma + ...+a_\mu\gamma^\mu} = \frac{B(\gamma)}{A(\gamma)} = I\ (only\,1st\,case)$ Set $a = 1, \upsilon = \mu = \eta$ and after cross multiplication, equating co-efficient up to γ^η. Specify A such that B/A is stable and compute co-efficient b through a so obtained.

Step6: Develop the structure of G_cG_m (proper) by u_m and obtain u_p.

Step7: Excite G_p by u_p and obtain y_p.

Step8: Estimate DTM series of G_p from the estimated u_p and y_p by using Eq. (25).

Step9: Excite G_m by u_p and obtain y_{m1}.

Step10: Estimate DTM series ofG_m from the estimated up and y_{m1} by using Eq. (25).

Step11: update the parameter of the compensator (G_c) from the estimated DTMs of G_p and G_m with scalar correction by the same equation as used in ICDTM.

Step12: Repeat step 5 to step 11 two times, then go step 13.

Step13: Assume structure of the compensator as in step 5.

Step14: Develop the structure of G_cG_m (proper) and excite it by u_m and obtain v.

Step15: Take error e($y_p - y_{m1}$), pass it through instability preventer, by the output of the instability preventer execute $P_f(= G_cG_m)$ to obtain u_f.

Step16: Substitute u_f from v to obtain up (v-u_f) and by u_p execite plant (G_p) and obtain y_p.

Step17: Estimate DTM series of G_p from the estimated u_p and y_p by using Eq. (25).

Step18: Excite G_m by u_p and obtain y_{m1}.

Step19: Estimate DTM series of G_m from the estimated u_p and y_{m1} by using Eq. (25).

Step20: update the parameter of the compensator (G_c) from the estimated DTMs of G_p and G_m with scalar correction as in step 11.

Step21: According to adaptation time repeat step 13 to step 20 then go step 22.

Step22: If adaptation is complete ($y_p \cong y_m$) stop, else go to step 5, after adjusting adaptation time.

6 Simulation and result analysis

6.1 Input signal generation

The reference model input is a vector which comprises of a step signal with a zero shift and amplitude unity up to 10sec and then minus unity up to 20sec and another step signal with zero shift and amplitude of two up to 10 sec. and then minus two up to 20 sec. It serves as the good measure of effectiveness of adaptive control. In case of MIMO system analysis both the reference model and unknown plants have the identical no of inputs and outputs .The results are obtained with two DTM matching.

6.2 Results obtained

Sampling interval Δ = 0.05sec; Reference model input: $u_m = \begin{bmatrix} u & 2u \end{bmatrix}^T$

Reference model close loop model:$\mathrm{Gm}(\gamma) = \begin{bmatrix} \frac{1.90325}{\gamma+1.90325} & \frac{0.0929}{\gamma+12.7858} \\ \frac{0.094002}{\gamma+2.3500} & \frac{0.9754}{\gamma+0.9754} \end{bmatrix}$

Unknown plant close loop model: $\mathrm{Gp}(\gamma) = \begin{bmatrix} \frac{0.97704\gamma+0.57777}{1.0000\gamma^2+1.56543\gamma+2.4072} & \frac{0.01150\gamma+0.4417}{1.0000\gamma^2+4.777\gamma+7.0666} \\ \frac{0.17278}{1.00\gamma+5.18363} & \frac{1.8557\gamma+1.8108}{1.0000\gamma^2+3.8517\gamma+4.570} \end{bmatrix}$

Period of adaptation of controller NΔ = 1.7sec.

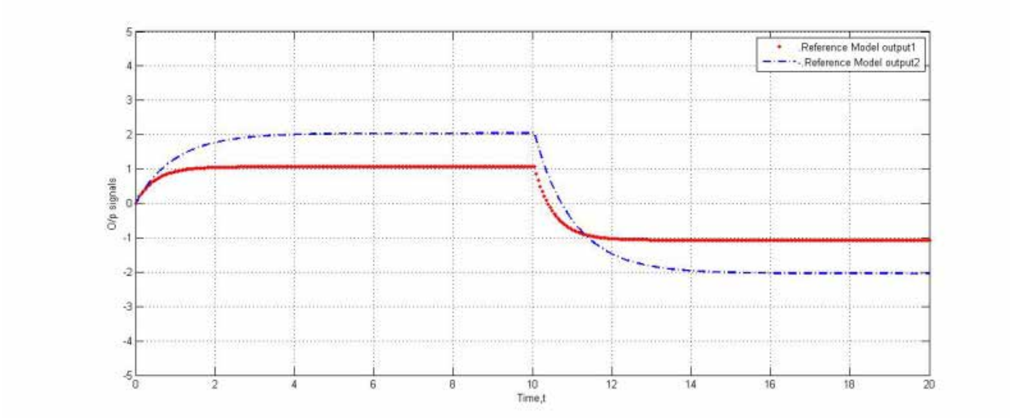

Fig. 5: Reference model outputs

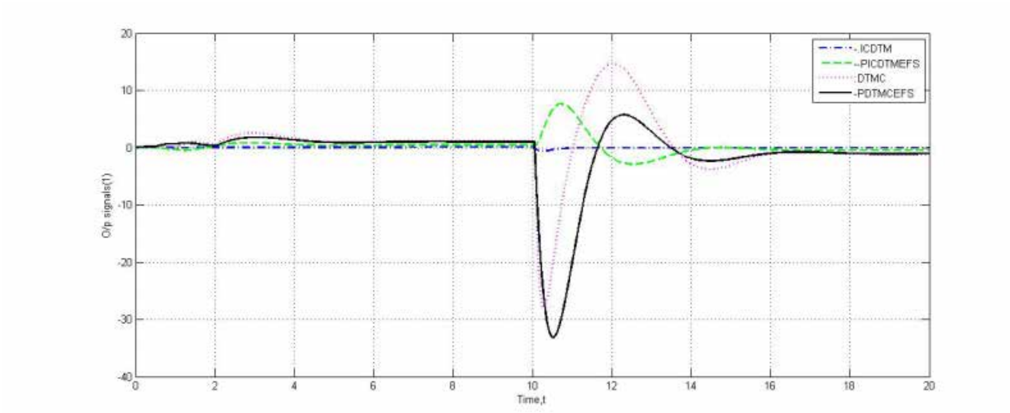

Fig. 6: MIMO output signals I

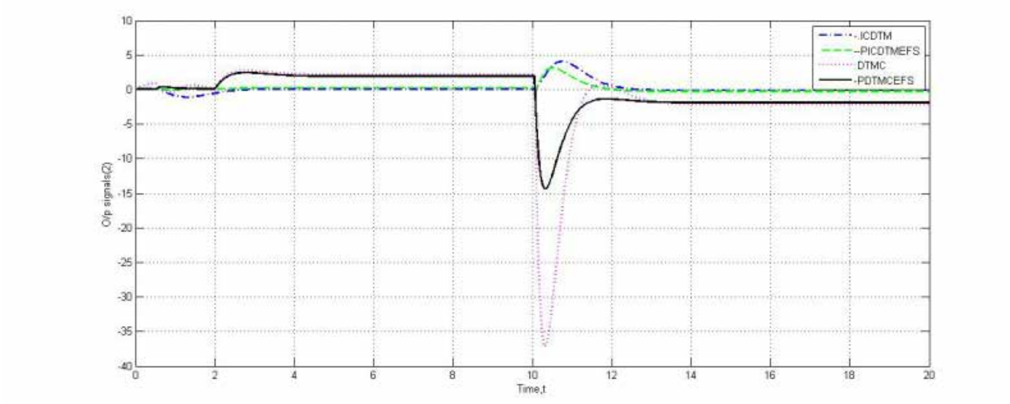

Fig. 7: MIMO output signals II

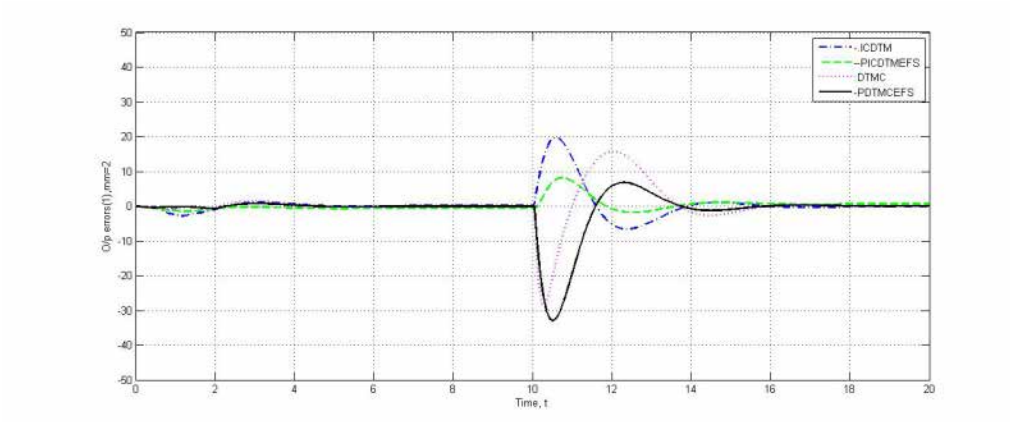

Fig. 8: MIMO output error I

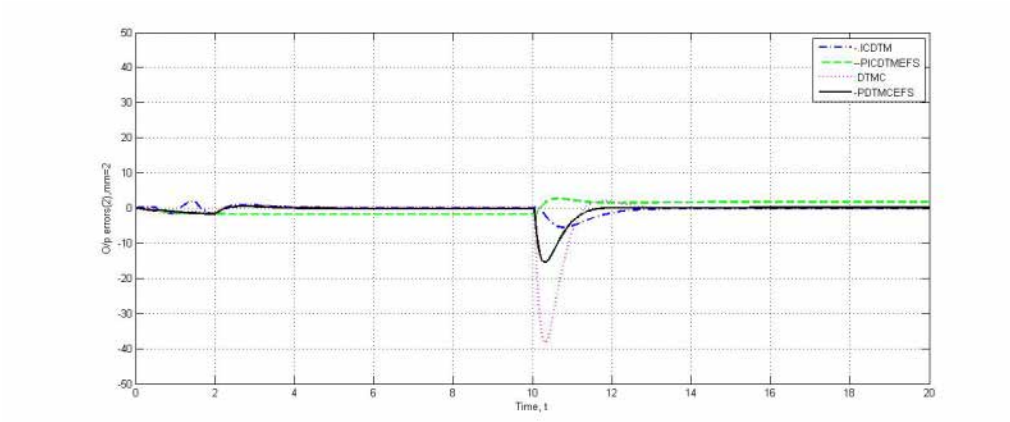

Fig. 9: MIMO output error II

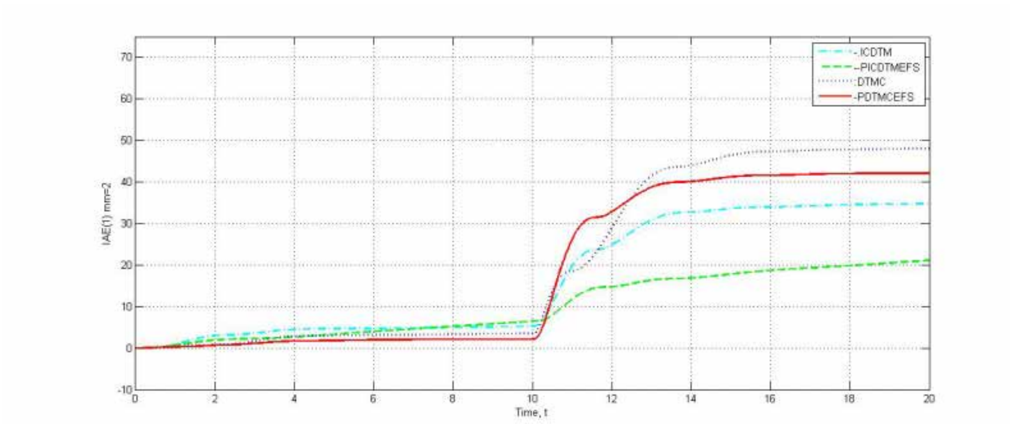

Fig. 10: MIMO Plant output signal I

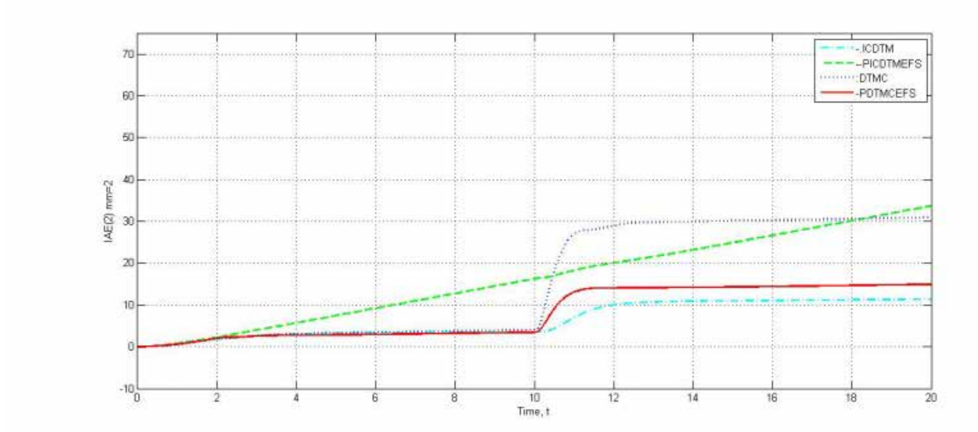

Fig. 11: MIMO Plant output signal II

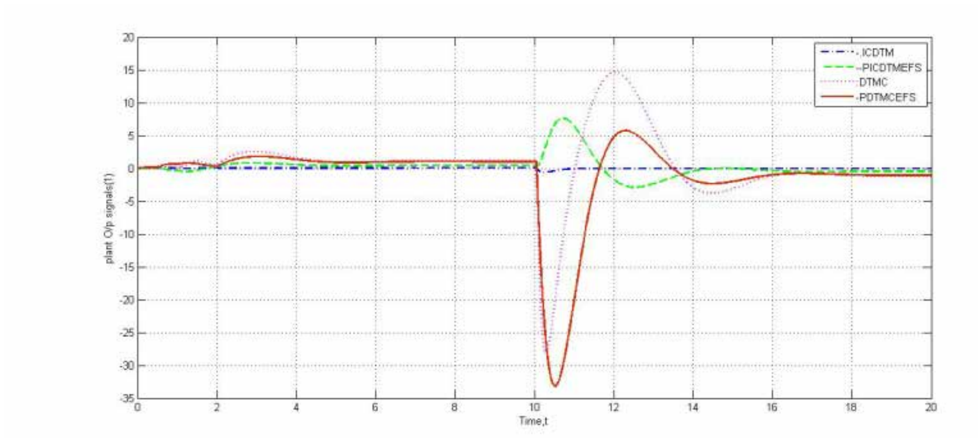

Fig. 12: Integral Absolute Error(IAE) for signalI

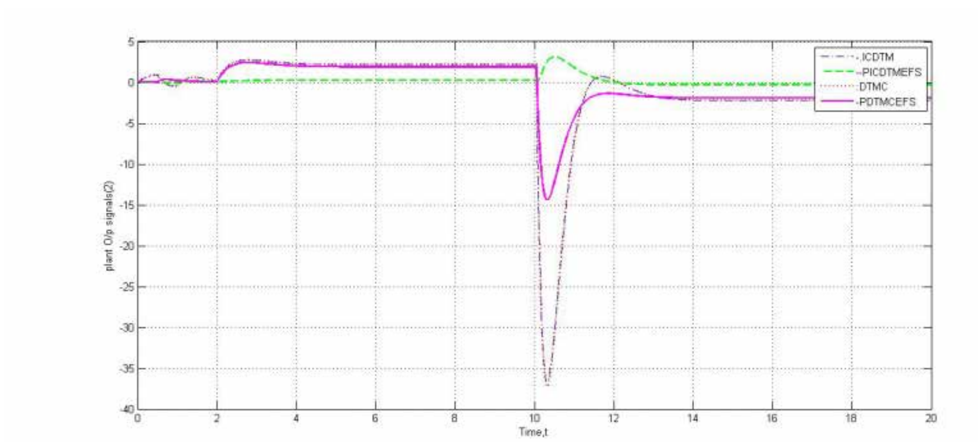

Fig. 13: Integral Absolute Error (IAE) for signal II

7 Conclusion

The four different methods of controller design of MIMO systems are presented in this paper. The aim of the paper was to get the overall closed loop system output as same as the output of the refrence model so developed in complex delta domain. If the output of the closed loop system matches the the output of the reference model then the philosophy of MRAC is justified. In the result section it is clear that, in all the four control schemes the errors between the MIMO system output and reference model are very close to zero as shown in Fig. 8 and Fig. 9. One of the error criterion known as the Integral Absolute Error (IAE) is used to find out the error between the closed loop system output and the reference model output. The IAE of all the four schemes are presented as shown in Fig. 12 & Fig. 13. From the Fig. 8, Fig. 9, Fig. 12 and Fig. 13 it is very much clear that the aim of the proposed work is satisfied. As the complete system is developed using the delta operator, hence the online parameter identification technique for a discrete-data MIMO system in delta domain has been successful by matching delta time moment in adaptive control framework. The sampling rate considered was 0.05 second which is very small, so at high sampling rate the discrete-data system model in delta domain is very close to its continuousCtime form, leading to unified treatment of this approach for discrete and continuous-time systems.

References

[1] K. J. Astrom. Adaptive feedback control. *Proceedings of the IEEE*, 1987, **75**(2): 185–217.

[2] K. J. Åström, P. Eykhoff. System identificationła survey. *Automatica*, 1971, **7**(2): 123–162.

[3] K. J. Åström, B. Wittenmark. *Adaptive control*. Courier Corporation, 2013.

[4] A. Chotai, P. Young, P. McKenna, W. Tych. Proportional-integral-plus (pip) design for delta (delta) operator systems part 2. mimo systems. *International Journal of Control*, 1998, **70**(1): 149–168.

[5] P. Coirault, J. Trigeassou, J. Gaubert, G. Champenois. Parameter estimation of an induction machine using reinitialized partial moments. **in:** *Proceedings of International Conference on Control Applications*, IEEE, 1995, 979–984.

[6] X.-z. Dong. Controllability analysis of linear singular delta operator systems. **in:** *2012 12th International Conference on Control Automation Robotics & Vision (ICARCV)*, IEEE, 2012, 1199–1204.

[7] E. Etien, J. Gabano, L. Rambault, J. Trigeassou, D. Mehdi. Frequency moments: application to controller reduction. **in:** *Proceedings of the 2000 American Control Conference. ACC (IEEE Cat. No. 00CH36334)*, vol. 4, IEEE, 2000, 2869–2873.

[8] H. Fan. Delta-operator based efficient stability tests. **in:** *Proceedings of the 1997 American Control Conference (Cat. No. 97CH36041)*, vol. 4, IEEE, 1997, 2508–2512.

[9] H. Garnier, M. Mensler, A. Richard. Continuous-time model identification from sampled data: implementation issues and performance evaluation. *International journal of Control*, 2003, **76**(13): 1337–1357.

[10] R. Gessing. Identification of shift and delta operator models for small sampling periods. **in:** *Proceedings of the 1999 American Control Conference (Cat. No. 99CH36251)*, vol. 1, IEEE, 1999, 346–350.

[11] G. C. Goodwin, R. H. Middleton, H. V. Poor. High-speed digital signal processing and control. *Proceedings of the IEEE*, 1992, **80**(2): 240–259.

[12] H. Hu, B. Jiang, H. Yang. Fault-tolerant control for delta operator non-linear switched systems. **in:** *Proceedings of the 33rd Chinese Control Conference*, IEEE, 2014, 1778–1782.

[13] D. Huang. Adaptive-feedback control algorithm. *Physical Review E*, 2006, **73**(6): 066204.

[14] D. Huang. Adaptive-feedback control algorithm. *Physical Review E*, 2006, **73**(6): 066204.

[15] D. Janecki. Model reference adaptive control using the delta operator. *IEEE transactions on automatic control*, 1988, **33**(8): 771–775.

[16] J. Kauraniemi, T. I. Laakso, I. Hartimo, S. J. Ovaska. Delta operator realizations of direct-form iir filters. *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing*, 1998, **45**(1): 41–52.

[17] B. C. Ku. *Digital control systems*. HRW, 1980.

[18] M. B. Lauritsen, M. Rostgaard. Delta-operator predictive control. **in:** *Proceedings of the 36th IEEE Conference on Decision and Control*, vol. 1, IEEE, 1997, 884–889.

[19] O. Levenspiel. Chemical reaction engineering. *Industrial & engineering chemistry research*, 1999, **38**(11): 4140–4143.

[20] H. Li, Y. Gao, P. Shi, X. Zhao. Output-feedback control for t–s fuzzy delta operator systems with time-varying delays via an input–output approach. *IEEE Transactions on Fuzzy systems*, 2014, **23**(4): 1100–1112.

[21] J. M. Mendel. *Lessons in estimation theory for signal processing, communications, and control*. Pearson Education, 1995.

[22] R. Middleton, G. Goodwin. Improved finite word length characteristics in digital control using delta operators. *IEEE transactions on automatic control*, 1986, **31**(11): 1015–1021.

[23] R. H. Middleton, G. C. Goodwin. *Digital control and estimation: a unified approach*. Prentice-Hall International, 1990.

[24] S. Mukhopadhyay, A. Patra, G. Rao. New class of discrete-time models for continuous-time systems. *International Journal of Control*, 1992, **55**(5): 1161–1187.

[25] C. P. Neuman. Properties of the delta operator model of dynamic physical systems. *IEEE transactions on systems, man, and cybernetics*, 1993, **23**(1): 296–301.

[26] C. P. Neuman. Transformations between delta and forward shift operator transfer function models. *IEEE transactions on systems, man, and cybernetics*, 1993, **23**(1): 295–296.

[27] R. Neumann, D. Dumur, P. Boucher. Delta-operator generalized predictive control (dgpc). **in:** *[1992] Proceedings of the 31st IEEE Conference on Decision and Control*, IEEE, 1992, 2224–2225.

[28] J. R. Ragazzini, G. F. Franklin. Sampled-data control systems. 1958.

[29] K. Rajagopal, H. Shankar. Adaptive cascade and feedback compensators for stable minimum and non-minimum phase plants through refined time moments estimators. **in:** *Intl. Conf. on Communications, Control & Signal Processing (CCSP-2000), IEEE Bangalore Section, Bangalore*, 2000.

[30] P. Sarkar. *Reduced order modeling and controller design in delta domain*. Ph.D. Thesis, IIT, Kharagpur, 2001.

[31] P. Sarkar, R. Kumar, A. Mondal, S. Banerjee. A unified approach for parametric robust control of electromagnetic levitation system. **in:** *2016 International Conference on Intelligent Control Power and Instrumentation (ICICPI)*,

IEEE, 2016, 1–6.

[32] H. Shankar. Model reference adaptive control of unknown non-minimum phase plants. *M. Sc.(Engg.) dissertation, Dept. Elect. Eng., Indian Inst. Science, Bangalore, India*, 1993.

[33] H. Shankar, et al. Adaptive control of a general class of finite dimensional stable lti systems. 2006.

[34] N. K. Sinha, G. P. Rao. *Identification of continuous-time systems: Methodology and computer implementation*, vol. 7. Springer Science & Business Media, 2012.

[35] G. Sivararnakumar. Model matching and adaptive control of finite and infinite dimensional systems. *Department of Electrical Engg., IISC*, 1991.

[36] E. Tohme. Initialization of output error identification algorithms. *Université de Poitiers*, 2008.

[37] H. Unbehauen, G. Rao. A review of identification in continuous-time systems. *Annual reviews in Control*, 1998, **22**: 145–171.

[38] H. Yang, Y. Xia, P. Shi, M. Fu. A novel delta operator kalman filter design and convergence analysis. *IEEE Transactions on Circuits and Systems I: Regular Papers*, 2011, **58**(10): 2458–2468.