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An epidemiological model of corruption with immunity clause in Nigeria

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Abstract. An epidemiological model of corruption in Nigeria with special reference to the role of immunity clause was formulated and analyzed. We calculate the corruption reproduction ratio, R_{co}, the corruption-free and the endemic equilibrium points. Investigating by Lyapunov function, corruption-free equilibrium is stable globally when $R_{c0} < 1$ and unstable otherwise. Moreover, the endemic equilibrium is proved to be stable both locally and globally for the condition $R_{co} > 1$ Numerical simulation shows evidence that removal of immunity clause regime from Nigeria constitution will not only pave way for equality before the law but also reduce corruption burden if control efforts through anti-corruption crusade, conviction and death penalties are sufficiently implemented.

Keywords: Corruption, corruption reproduction ratio, global stability, immunity clause

1 Introduction

The fourth republic of Nigeria fell short of expectations as it is bewitched with maladies such as ethnic sentiments, political violence, unprecedented election rigging, religious crisis, regionalism, nepotism, selfish leaders and corruption. Starting from 1999, the return to democracy was welcomed with high hopes being seen as the best option and escape route from the woes of military rule but these hopes did not last long due to the evolution of corruption. Historically, corruption is derived from the Latin word corruptus which means to break or destroy [1]. In its simplest form, corruption refers to the misuse of entrusted power for private benefit or advantage [2]. In Nigeria, the Corrupt Practices Decree of 1975 defines corruption as the offer, promise or receipt of any gratification as inducement or reward. The Economic and Financial Crimes Commission (EFCC) act in its description of corruption empowers the body to prosecute individuals who engage in money laundering, embezzlement, bribery, illegal oil bunkering, foreign exchange malpractices including counterfeiting of currency, looting, and tax evasion amongst others. In practice, corruption is any form of unethical, dishonest, or illegal conduct by a person in authority, mostly for private gain [3].

Corruption is no doubt an endemic problem that is becoming genetic in Nigeria which underscores the development of the country's economy despite the panoply of laws set against it [4]. This so-called genetic problem swallows about 40% of the Nigeria 20 billion annual oil income and subject Nigerians to chronic poverty and unemployment [5]. The malady is not only a dog in the wheel of development, but it has also tarnished the countrys image among the international community of nations.

Nigeria was ranked the first in 2000, second in 2001-2003 and 21st in 2006 globally in Corruption Perception Index according to Transparency International (TI). The Index scores 177 countries and territories on a scale from 0 (highly corrupt) to 100 (very clean). Two-thirds of countries score below 50 and no country has a perfect score [6]. This shows that corruption is not only a Nigerian problem but also a global issue. However,

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it is more confined in Africa, particularly Nigeria. The situation can be changed if the anti-corrupt agencies such as EFCC, Code of Conduct Bureau (CCB) and Independent Corrupt practices and other related offences, Crimes Commission (ICPC) are allow to operate without interference as well as effective execution of the law on persons with deviant behaviour [7].

Unfortunately, the immunity clause enacted in section 308 of 1999 constitution as well as unstable judiciary system seems to spearhead the canker worm of corruption. The Black's Law Dictionary (as cited in [8] defines immunity as an exemption from a duty, liability or service of process especially such an exemption granted to a public official. Legally speaking, it is an exemption from performing duties, which the law generally requires other citizens to do, or from a penalty or burden that the law generally places on other citizens [3]. Traditionally, sovereign immunity comes from the monarchy system of government where it is of the belief that "the King can do no wrong" The "immunity clause" as currently enshrined in the constitution grants the President and Governors as well as their deputies freedom from prosecution while in office. This provides Governors the liberty to perform constitutional duties without hindrance, embarrassment and the difficulties which may arise if he/she is being constantly pursued and harassed with court processes of a civil or criminal nature [9]. It is a provision designed to protect the dignity of office. However, due to the constant abuses of the clause by the beneficiaries, the average Nigerians regard it as the bullet proof or thick backbone tool against punishment used by selfish leaders to siphon public funds for personal advantage. They further argued that immunity clause gives the public office holders too much power and authority to decide on what to do, when to do and how to do, without being guided by the law. They also are of the opinion that if the law truly is no respecter of persons, as the rule of law clearly stated, then there should be no one exempted from being tried or sued in a competent court of law [8]. Based on the negative implications of the clause on Nigerians, there is need to amend the 1999 constitution and remove the immunity. The removal of the immunity clause, is believed will check executive lawlessness, promote responsive and responsible leadership by the executive branches of government. However, public office holders under this immunity calls for its continuity. To bridge the gap of thoughts among this set of people, there is need to model this phenomenon mathematically since it is found to be useful in controlling infectious diseases and epidemics.

The use of mathematical epidemiology as an alternative weapon in the fight against corruption has gained attention among researchers in recent times. Traditionally, mathematical epidemiology was meant to study the transmission dynamics of infectious diseases [10]. Modelling corruption as a disease has been done by so many authors [11]-[14]. Particularly, Abduirahman [11] developed and analyzed a mathematical model for the transmission dynamics and control of corruption. The four compartmental model (Suseptible-Corrupt-Jailed-Honest individuals) model was formulated epidemiologically based on standard incidence function. This was an extension of Hathroubis model, which has 3 compartments only. Yusuf et al. [12] modelled corruption dynamics with similar characteristics to [11] except that they incorporated social media as an external control tool in fighting corruption at a relatively low cost. Likewise, Sooknanan et al. [13] studied police corruption in terms of predator-prey model among police and gangs interaction. In their model, police officers who are predators of gang members may become corrupted by the gangs in the course of their service delivery. Other models on the control of corruption with anti-corruption awareness among the jailed individuals that are mathematically and statistically formulated can be found in [14, 15]. Unfortunately, the authors do not consider god-fatherism and immunity clause factors as the key determinants forces of corruption in Nigeria, and conviction without death sentence as control measures.

Motivated by these works, in the paper, we propose a more realistic epidemiological model of corruption with immunity clause in Nigeria, which has 3 different categories of corrupt individuals, namely, corrupt individuals with and without immunity clause, and corrupt political god-fathers. These elements of corruption were not explicitly explored in the existing literatures mentioned above. Our model targets the elimination of corruption not only through anti-corruption crusade but also with conviction and death penalties as adopted in China. We concentrate the study on local and global stabilities of equilibria using Lyapunov functional approach and Dulacs multipliers with the aim to examine the role of immunity clause on corruption dynamics. In order to affirm the theoretical results, a numerical simulation is done using Matlab mathematical software. To remove or to not remove: the case of immunity clause dilemma in Nigeria, the authors set to make their recommendations at the end of the paper as this remains the focal point of the research.

The paper is organized as follows: In the next section, we introduce the model construction and equations, and stability analysis presented alongside numerical results and discussion in Section 3. Finally, we conclude the paper in Section 4.

2 Model construction

The formulation of the corruption model closely follows the epidemiological dynamics of infectious diseases.

2.1 Basic framework

The model subdivides the total population at time t denoted by N into five exclusive classes depending on their corruption status: Susceptible public office holders who are not corrupt but are prone to corruption when exposed $(S(t))$, Corrupted class of public office holders under immunity clause $(C_I(t))$, Corrupted class of public office holders without immunity clause $(C_w(t))$, Corrupted class of political god-fathers $(G(t))$ and the class of convicted individuals at time t $(D(t))$ who may not be allowed to hold any public office again. Note that the so-called god-fathers claimed to be the custodians of the Nigerian politics, and most at times impose candidates of their choice on the masses for selfish-demands and based on such selfish-gains, they influence these political office holders negatively. We consider the following assumptions:

- The immunity clause factor can impact negatively on the number of corruption cases.
- All individuals are corrupt but some are more corrupt than others.
- No corrupted person is above the law.
- Death sentence is given only to individuals convicted of corruption

The schematic diagram of the model is depicted in the Fig. 1 below

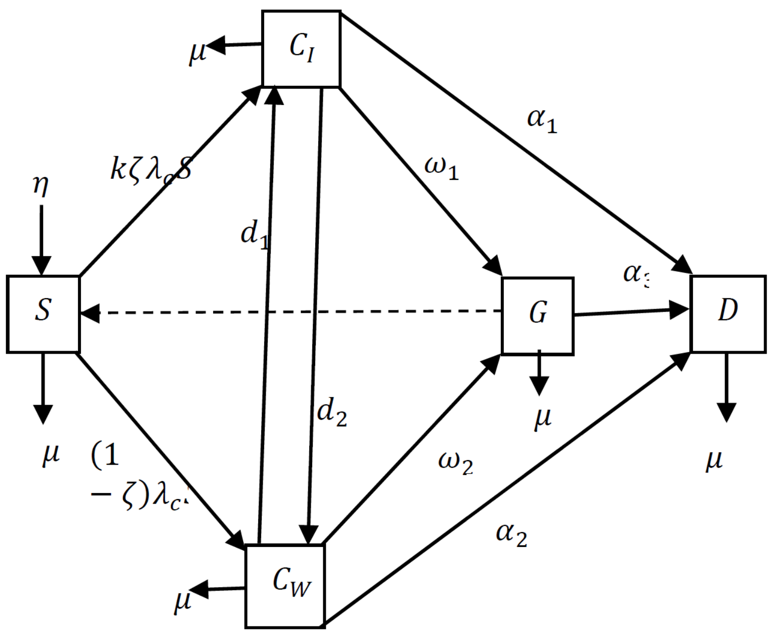

Fig. 1: The schematic diagram of the corruption dynamics

The force of corruption related to the model (2) follows standard incidence function given by

$$\lambda_c = \epsilon(1-u)\frac{\beta_1 C_W + \beta_2 C_I + \beta_3 G}{N} \tag{1}$$

where $\beta_i (i = 1, 2, 3)$ are the effective contact rates that recruits susceptible public office holders into corruption brotherhood by $C_W(t)$, $C_I(t)$ and $G(t)$ respectively. We assume here that $\beta_1 < \beta_2 < \beta_3$, u is the rate of anti-corruption crusade and ϵ refers to the average number of corruption partners.

2.2 Model

Putting the above formulation and assumptions together gives the following system of non-linear differential equations

$$\left.\begin{aligned}
\frac{dS}{dt} &= \eta - (1-\zeta)\lambda_c S - k\zeta\lambda_c S - \mu S \\
\frac{dC_W}{dt} &= (1-\zeta)\lambda_c S + d_2 C_I - (\mu + \alpha_2 + \omega_2 + d_1) C_W \\
\frac{dC_I}{dt} &= k\zeta\lambda_c S + d_1 C_W - (\mu + \alpha_1 + \omega_1 + d_2) C_I \\
\frac{dG}{dt} &= \omega_1 C_I + \omega_2 C_W - (\mu + \alpha_3) G \\
\frac{dD}{dt} &= \alpha_1 C_I + \alpha_2 C_W + \alpha_3 G - (\mu + \delta) D
\end{aligned}\right\} \tag{2}$$

subject to the following initial conditions

$$S(0) \geq 0, C_W(0) \geq 0, C_I(0) \geq 0, G(0)0, D(0) \geq 0 \tag{3}$$

where the meaning of the parameters is summarized in the Table. 1.

Table 1: Variables and parameters of the model (2)

Variables	Definition
$S(t)$	Number of susceptible public office holders at time t
$C_I(t)$	Number of corrupted class of public office holders with immunity at time t
$C_W(t)$	Number of corrupted class of public office holders without immunity at time t
$G(t)$	Number of corrupted class of political god-fathers at time t
$D(t)$	Number of disciplined and convicted individuals at time t
Parameters	**Definition**
η	Recruitment number of susceptible public office holders either by election or appointment
ζ	Fraction of susceptible public office holders that are corrupted by $C_I(t)$.
$\omega_i (i = 1, 2)$	The rate at which $C_I(t)$ and $C_W(t)$ joins $G(t)$ respectively after expiration of their tenure in office.
$d_i (i = 1, 2)$	The rate at which $C_W(t)$ joins $C_I(t)$ and $C_I(t)$ joins $C_W(t)$ respectively by new appointment or re-election.
$\alpha_i (i = 1, 2, 3)$	The respective rates of convicting the corrupted public office holders with(out) immunity and political god-fathers.
μ	Natural death rate
δ	Death sentence due to corruption for those convicted.
k	Immunity clause enhancement factor
u	The rate of anti-corruption crusade
ϵ	Average number of corruption partners
$\beta_i (i = 1, 2, 3)$	Effective contact rates that recruits Susceptible public office holders into corruption brotherhood by $C_W(t)$, $C_I(t)$ and $G(t)$ respectively.

3 Analysis of the model

3.1 Mathematical well-posedness of the model

In this sub-section, we adopt the approach of Omondi et al. [16] to prove that the model (2) is mathematically well structured and biologically sensible.

Lemma 3.1. Provided that the initial conditions of the model (2) are as defined in Eq. (2), then, the solutions $S(t), C_W(t), C_I(t), G(t)$ and $D(t)$ are non-negative for all $t > 0$.

Proof

Assume that

$$\hat{t} = \sup\{t > 0 : S(t) > 0, C_W(t) > 0, C_I(t) > 0, G(t) > 0, D(t) > 0\} \in [0, t]$$

Thus, $\hat{t} > 0$, and its follows immediately from the first equation of model (2) that

$$\frac{dS}{dt} = \eta - (\tau\lambda_c + \mu)S$$

with $\tau = 1 + \zeta(k - 1)$.

Thus we get

$$\frac{d}{dt}\Big(S(t)\exp\Big\{\mu t + \int_0^t \lambda_c(S)dS\Big\}\Big) = \eta\exp\Big(\mu t + \int_0^t \lambda_c(S)dS\Big)$$

Therefore

$$S(t)\exp\Big(\mu t + \int_0^t \lambda_c(S)dS\Big) - S(0) = \int_0^t \eta\exp\Big(\mu t + \lambda_c(\varsigma)d\varsigma\Big)$$

so that

$$S(t) = \exp\Big\{-\Big(\mu t + \int_0^t \lambda_c(S)dS\Big)\Big\}\Big\{S(0) + \int_0^t \eta\exp\Big(\mu t + \int_0^t \lambda_c(\varsigma)d\varsigma\Big)\Big\} > 0$$

In a similar fashion, it can be prove that $C_W(t) > 0, C_I(t) > 0, G(t) > 0$,and $D(t) > 0$ for $t > 0$.

Lemma 3.2. Let the feasible domain $\bar{\omega}$ be defined by

$$\bar{\omega} = \Big\{(S(t) > 0, C_W(t) > 0, C_I(t) > 0, G(t) > 0, D(t) > 0) \in \mathfrak{R}^5_+ | 0 \leq N \leq \frac{\eta}{\mu}\Big\},$$

with the initial conditions given in Eq. (2). The domain $\bar{\omega}$ is positively invariant and attracting with respect to the model (2) for all $t > 0$.

Proof. The evolution derivative of the total population is given by

$$\frac{dN}{dt} = \eta - \mu N$$

$$N(t) = \frac{\eta}{\mu} + \Big(N(0) - \frac{\eta}{\mu}\Big)e^{-\mu t} \tag{4}$$

We can deduce from Eq. (4) that $N(t) \leq \max\Big\{N(0), \dfrac{\eta}{\mu}\Big\}$. Therefore, $\bar{\omega}$ is positively invariant under model (2). Thus, in $\bar{\omega}$, the model (2) is well structured epidemiologically and sufficient to be studied in $\bar{\omega}$.

3.2 Stability of the corruption-free equilibrium (cofe)

The corruption model (2) has a cofe state given by

$$C^0 = (S^0, C_W^0, C_I^0, G^0, D^0) = \Big(\frac{\eta}{\mu}, 0, 0, 0, 0\Big)$$

. The linear stability of C^0 is governed by the corruption reproduction number, R_{co} which will be computed using the next generation matrix proposed by Van den Driesche and Watmough [17]. Using their notations on the Eq. (2), F and V matrices, for the new infection terms and transfer terms are respectively given

$$F = \epsilon(1-\mu)\begin{pmatrix}(1-\zeta)\beta_1 & (1-\zeta)\beta_2 & (1-\zeta)\beta_3 \\ k\zeta\beta_1 & k\zeta\beta_2 & k\zeta\beta_3 \\ 0 & 0 & 0\end{pmatrix} \quad \text{and} \quad V = \begin{pmatrix}k_1 & -d_2 & 0 \\ -d_1 & k_2 & 0 \\ -\omega_2 & -\omega_2 & k_3\end{pmatrix}$$

Note that $k_1 = \mu + \alpha_2 + \omega_2 + d_1, k_2 = \mu + \alpha_1 + \omega_1 + d_2 \quad \text{and} \quad k_3 = \mu + \alpha_3$.
It follows from the above that the corruption reproduction number is

$$R_{co} = \rho(FV^{-1}) = R_{c1} + R_{c2} + R_{c3} \tag{5}$$

where

$$R_{c1} = \epsilon(1-u)\frac{\beta_1(1-\zeta)k_2 + k\eta d_2)}{k_1k_2 - d_1d_2}, \quad R_{c2} = \epsilon(1-u)\frac{\beta_2(1-\zeta)d_1 + k\eta k_1)}{k_1k_2 - d_1d_2}$$

and

$$R_{c3} = \epsilon(1-u)\frac{\beta_3\Big((1-\zeta)(\omega_1 d_1 + \omega_2 k_2) + k\zeta(\omega_2 d_2 + \omega_1 k_1)\Big)}{k_3(k_1k_2 - d_1d_2)}$$

The following cases can be deduced directly from the above reproduction ratio as thus

Case 1: Corruption in the absence of immunity clause, ant-corruption crusade and conviction ($\beta_2 = \zeta = \omega_i = d_i = u = \alpha_j = 0, k = 1, j = 1,2,3, i = 1,2$). Using Eq. (5) we have the basic corruption reproduction ratio denoted by R_{00} as thus:

$$R_{00} = \frac{\epsilon}{\mu}\left[\frac{\mu\beta_1 + \beta_3\omega_2}{(\mu + \omega_2)}\right]$$

Case 2: Corruption reproduction ratio in the absence of immunity clause with anti-corruption crusade and conviction ($\beta_2 = \zeta = \omega_i = d_i = 0, u, \alpha_j \neq 0, k = 1, i, j = 1,2$) can be presented as

$$R_{01} = \frac{\epsilon(1-u)}{\mu + \alpha_3}\left[\frac{(\mu + \alpha_3)\beta_1 + \beta_3\omega_2}{(\mu + \omega_2 + \omega_2)}\right]$$

The first two cases are obtained by setting all parameters related to immunity clause to zero.

Case 3: Corruption reproduction ratio in the presence of immunity clause without anti-corruption crusade and conviction ($\beta_2, \zeta \neq 0, k > 1, u, \alpha_j = 0, j = 1,2,3$) can be presented as

$$\begin{aligned}R_{02} = \epsilon\Big[&\frac{\beta_1\Big((1-\zeta)(\mu+\omega_1+d_2) + k\zeta d_2\Big) + \beta_2[(1-\zeta)d_1 + k\zeta(\mu+\omega_2+d_2)]}{(\mu+\omega_1)(\mu+\omega_2) + d_1(\mu+\omega_1) + d_2(\mu+\omega_2)}\Big] \\ &+ \frac{\epsilon}{\mu}\Big[\frac{\beta_3\Big((1-\zeta)(\omega_1 d_1 + \omega_2(\mu+\omega_1+d_2)\Big) + k\zeta(\omega_2 d_2 + \omega_1(\mu+\omega_2+d_1))\Big)}{(\mu+\omega_1)(\mu+\omega_2) + d_1(\mu+\omega_1) + d_2(\mu+\omega_2)}\Big]\end{aligned}$$

Case 4: Corruption in the presence of immunity clause with anti-corruption crusade and conviction ($\beta_2, \zeta \neq 0, k > 1, u, \alpha_j \neq 0, j = 1,2,3$). This scenario gives the general form of the control corruption reproduction ratio as given in Eq. (5).
Using Theorem 2 in [17], the following result is established.

Lemma 3.3. The Cofe state C^0 of the system (2) is locally asymptotically stable if $R_{co} < 1$ and unstable when $R_{co} > 1$.
Remark 1. The results in lemma 3.3 shows that corruption can be eliminated from the community when $R_{co} < 1$ provided the initial sizes of the sub-groups in the model are in the basin of attraction of the corruption-free equilibrium, C^0. Thus, with few corrupted persons in the country, cases of corruption cannot spread in

the presence of effective anti-corruption services. However, to ensure that the eradication of corruption in any nation is independent of the number of corruption cases, it is necessary to prove that the Cofe state admits a global asymptotic stability. Thus, we have the following lemma.

Lemma 3.4. The Cofe state C^0 of the system (2) is globally asymptotically stable if $R_{co} < 1$ and unstable when $R_{co} > 1$.

Proof. The lemma is proved based on the Lyapunov function of the form

$$L = A_1 C_W + A_2 C_I + A_3 G$$

with the time derivative given as

$$\begin{aligned}\frac{dL}{dt} &= (k\zeta A_2 + (1-\zeta)A_1)\lambda_c S + (d_1 A_2 + \omega_2 A_3 - k_1 A_1)C_W \\ &+ (d_2 A_1 + \omega_1 A_3 - k_2 A_2)C_I - k_3 A_3 G \end{aligned} \tag{6}$$

Setting the coefficients of $\lambda_c S$ to the numerator of R_{co} (excluding $\beta_i, i = 1, 2, 3$) and G to the denominator, we get

$$A_1 = k_3(k_2 + d_2) + \omega_2 d_2 + \omega_1 k_2 > 0, A_2 = k_3(k_1 + d_1) + \omega_1 d_1 + \omega_2 k_1 > 0 \quad \text{and}$$

$$A_3 = k_1 k_2 - d_1 d_2 > 0 \quad \text{since} d_1 d_2 \in k_1 k_2.$$

Now replacing the constants together with the expression for the force of infection into Eq. (6) leads to

$$\begin{aligned}\frac{dL}{dt} &= (k\zeta(k_3(k_1 + d_2) + \omega_2 d_2 + \omega_1 k_1) \\ &= +(1-\zeta)(k_3(k_2 + d_1) + \omega_1 d_1 + \omega_2 k_2))\Big(\epsilon(1-u)\frac{\beta_1 C_I + \beta_2 C_W + \beta_3 G\Big)}{N}\Big)S \\ &- k_3(k_1 k_2 - d_1 d_2)(C_I + C_W + G) \\ &\leq k_3(k_1 k_2 - d_1 d_2)(C_I + C_W + G) \\ &\Big(\epsilon(1-u)\hat{\beta}\frac{(k\zeta(k_3(k_1 + d_2) + \omega_2 d_2 + \omega_1 k_1) + (1-\zeta)(k_3(k_2 + d_1) + \omega_1 d_1 + \omega_2 k_2)}{k_3(k_1 k_2 - d_1 d_2)} - 1\Big)\end{aligned}$$

provided that $\hat{\beta} = \max(\beta_1, \beta_2, \beta_3)$ and $S \leq N$.

Thus,

$$\frac{dL}{dt} \leq \epsilon(1-u)k_3(k_1 k_2 - d_1 d_2)(C_I + C_W + G)(R_{co} - 1).$$

It is obvious that $\frac{dL}{dt} \leq 0$ whenever $R_{co} \leq 1$. Equality holds at the point $C_I = C_W = G = 0$.

Therefore, by La Salle invariance principle [18], the cofe state is globally asymptotically stable

3.3 Existence and stability of corruption endemic equilibrium

3.3.1 Existence

Let

$$\begin{aligned}0 &= \eta - (\tau\lambda_c^{**} + \mu)S^{**} \\ 0 &= (1-\zeta)\lambda_c^{**}S^{**} + d_2 C_I^{**} - k_1 C_W^{**} \\ 0 &= k\zeta\lambda_c^{**}S^{**} + d_1 C_W^{**} - k_2 C_I^{**} \\ 0 &= \omega_1 C_I^{**} + \omega_2 C_W^{**} - k_3 G^{**} \\ 0 &= \alpha_1 C_I^{**} + \alpha_2 C_W^{**} + \alpha_3 G^{**} - k_4 D^{**}\end{aligned}$$

Then the corruption endemic equilibrium point (CEEP) denoted by C^{**} in terms of λ_c^{**} is given as

$$C^{**} = (S^{**}, C_W^{**}, C_I^{**}, G^{**}, D^{**}) \tag{7}$$

where

$$\left.\begin{aligned} S^{**} &= \frac{\eta}{\tau\lambda_c^{**} + \mu} \\ C_W^{**} &= g_1\lambda_c^{**} S^{**} \\ C_I^{**} &= g_2\lambda_c^{**} S^{**} \\ G^{**} &= g_3\lambda_c^{**} S^{**} \\ D^{**} &= g_4\lambda_c^{**} S^{**} \end{aligned}\right\} \tag{8}$$

with

$$g_1 = \frac{k\zeta d_2 + (1-\zeta)k_2}{k_1k_2 - d_1d_2}, \quad g_2 = \frac{k\zeta k_1 + (1-\zeta)d_1}{k_1k_2 - d_1d_2}, \quad g_3 = \frac{g_1\omega_2 + g_2\omega_2}{k_3}$$

and

$$g_4 = \frac{\alpha_2 g_1 + \alpha_1 g_2 + \alpha_3 g_3}{k_4}$$

with

$$\lambda_c^{**} = \epsilon(1-u)\frac{\beta_1 C_W^{**} + \beta_2 C_I^{**} + \beta_3 G^{**}}{N^{**}} = \epsilon(1-u)\Big(\sum_{i=0}^{4} g_i\Big)^{-1}(R_{co} - 1).$$

Thus, we claim the following result

Lemma 3.5. The corruption endemic equilibrium point (CEEP) exists whenever $R_{co} > 1$.

3.3.2 Local stability

Now that we have shown the existence of the unique positive endemic corruption equilibrium point (CEEP), we proceed to prove that C^{**} is locally asymptotically stable. The Jacobian matrix of the model (2) being evaluated at C^{**} is

$$J_{C^{**}} = \begin{pmatrix} -\mu - \tau\lambda_c^{**} & -a_1 & -a_2 & -a_3 & 0 \\ (1-\zeta)\lambda_c^{**} & (1-\zeta)a_1 - k_1 & (1-\zeta)a_2 + d_2 & (1-\zeta)a_3 & 0 \\ k\zeta\lambda_c^{**} & k\zeta a_1 + d_1 & k\zeta a_2 - k_2 & k\zeta a_3 & 0 \\ 0 & \omega_2 & \omega_1 & -k_3 & 0 \\ 0 & \alpha_2 & \alpha_1 & \alpha_3 & -k_4 \end{pmatrix} \tag{9}$$

where $a_1 = \beta_1\frac{S^{**}}{N^{**}}, a_2 = \beta_2\frac{S^{**}}{N^{**}}$ and $a_3 = \beta_3\frac{S^{**}}{N^{**}}$ with $\frac{S^{**}}{N^{**}} = \frac{1}{R_{oc}}$

The characteristics equation of the Eq. (9) is

$$\det(J_{C^{**}} - \lambda I) = (-k_4 - \lambda)\cdot\det(B - \lambda I) = 0 \tag{10}$$

giving that

$$B = \begin{pmatrix} -\mu - \tau\lambda_c^{**} & -a_1 & -a_2 & -a_3 \\ (1-\zeta)\lambda_c^{**} & (1-\zeta)a_1 - k_1 & (1-\zeta)a_2 + d_2 & (1-\zeta)a_3 \\ k\zeta\lambda_c^{**} & k\zeta a_1 + d_1 & k\zeta a_2 - k_2 & k\zeta a_3 \\ 0 & \omega_2 & \omega_1 & -k_3 \end{pmatrix}$$

The corruption endemic equilibrium is locally asymptotically stable if and only if all the eigenvalues of the polynomial Eq. (10) above have negative real parts. Clearly, $\lambda = -k_4 = -(\mu + \delta)$ is a negative eigenvalue. To investigate the other four eigenvalues, we expand $\det(B - \lambda I)$ to obtain the equation

$$p_4\lambda^4 + p_3\lambda^3 + p_2\lambda^2 + p_1\lambda + p_0 = 0 \tag{11}$$

with

$$
\begin{aligned}
p_4 &= 1\\
p_3 &= \mu + k_1 + k_2 + k_3 + \epsilon(1-u)\Big(\sum_{i=1}^{4} g_i\Big)^{-1}(R_{co}-1) - \frac{1}{R_{co}}(k\zeta\beta_2 + (1-\zeta)\beta_1)\\
p_2 &= (k_1k_2 - d_1d_2)\Big(1 - \frac{R_{c1}+R_{c2}}{R_{co}}\Big) + \Big(\mu + \epsilon(1-u)\Big(\sum_{i=1}^{4} g_i\Big)^{-1}(R_{co}-1)\Big)(k_1+k_2+k_3)\\
&\quad + k_3(k_1+k_3) - \frac{1}{R_{co}}\Big((\mu+k_3)(k\zeta\beta_2 + (1-\zeta)\beta_1) + \beta_3(k\zeta\omega_1 + (1-\zeta)\omega_2)\Big)\\
p_1 &= (\mu+k_3)(k_1k_2 - d_1d_2)\Big(1 - \frac{\mu}{\mu+k_3}\Big(\frac{R_{c1}+R_{c2}}{R_{co}}\Big) - \frac{k_3}{\mu+k_3}\Big(\frac{R_{c3}}{R_{co}}\Big)\Big)\\
&\quad + \epsilon(1-u)\Big(\sum_{i=1}^{4} g_i\Big)^{-1}(R_{co}-1)[k_3(k_1+k_2) + k_1k_2 - d_1d_2]\\
&\quad + \mu k_3(k_1+k_2) - \frac{k_3}{R_{co}}\Big(\beta_1(k\zeta(\mu+k_2) + (1-\zeta)(d_1+k_1) + \beta_2(k\zeta d_2 + (1-\zeta)\mu)\Big)\\
&\quad - \frac{\mu}{R_{co}}\beta_3(k\zeta\omega_1 + (1-\zeta)\omega_2)\\
p_0 &= k_3(k_1k_2 - d_1d_2)\epsilon(1-u)\Big(\sum_{i=1}^{4} g_i\Big)^{-1}(R_{co}-1)
\end{aligned}
$$

To ensure that all eigenvalues of the Eq. (2) have negative real parts, the Routh-Hurwitz stability criterion [10] requires

$$p_4 > 0, p_3 > 0, p_2 > 0, p_1 > 0, p_0 > 0 \quad \text{and} \quad H_1 = p_3 > 0$$

$$H_2 = \begin{vmatrix} p_3 & 1\\ p_1 & p_2 \end{vmatrix} > 0, \quad H_3 = \begin{vmatrix} p_3 & 1 & 0\\ p_1 & p_2 & p_3\\ 0 & p_0 & p_1 \end{vmatrix} > 0, \quad H_4 = \begin{vmatrix} p_3 & 1 & 0 & 0\\ p_1 & p_2 & p_3 & 1\\ 0 & p_0 & p_1 & p_2\\ 0 & 0 & 0 & p_0 \end{vmatrix} > 0$$

Clearly, $H_4 = p_0H_3$. Provided $R_{co} > 1$, it follows that $p_j > 0, j = 1,2,3$ and $p_0 > 0$. Thus, it is enough to prove that $H_2 > 0$ and $H_3 > 0$. Obviously, $H_2 = p_2p_3 - p_1$ and $H_3 = p_1(p_2p_3 - p_1) - p_0p_3^2$.
It can be seen that all the eigenvalues have negative real parts and therefore, the corruption equilibrium point is locally asymptotically stable.

3.3.3 Global stability

We derive motivation from the work of Omondi et al. [16] to establish that corruption endemic equilibrium is globally stable.
Lemma 3.6. The CEEP of the model (2) attains a global asymptotic stability if and only if $R_{co} > 1$.
Proof. It is obvious that $N = \frac{\eta}{\mu}$ as $t \to \infty$. Thus,using the relation $S = \dfrac{\eta}{\mu} - C_W - C_I - G - D$ and replacing it in model (2) yields the limiting system below

$$
\left.
\begin{aligned}
\frac{dC_W}{dt} &= (1-\zeta)\lambda_c\Big(\frac{\eta}{\mu} - C_W - C_I - G - D\Big) + d_2C_I - k_1C_W\\
\frac{dC_I}{dt} &= k\zeta\lambda_c\Big(\frac{\eta}{\mu} - C_W - C_I - G - D\Big) + d_1C_W - (\mu + \alpha_1 + \omega_1 + d_2)k_2C_i\\
\frac{dG}{dt} &= \omega_1C_I + \omega_2C_W - k_3G\\
\frac{dD}{dt} &= \alpha_1C_I + \alpha_2C_W + \alpha_3G - k_4D
\end{aligned}
\right\} \tag{12}
$$

Using the Dulacs multiplier $\Phi(C_W, C_I) = \dfrac{1}{C_W C_I}$ (see Omondi et al., [16] and references therein), it suffices that

$$\begin{aligned}
\partial\Phi &= \frac{\partial}{\partial C_W}\Big\{\mu(1-\zeta)\epsilon(1-u)\Big(\frac{\beta_1 C_W+\beta_2 C_I+\beta_3 G}{\eta C_W C_I G D}\Big)\Big(\frac{\eta}{\mu}-C_W-C_I-G-D\Big)+\frac{d_2}{C_W G D}-\frac{k_1}{C_I G D}\Big\}\\
&+\frac{\partial}{\partial C_W}\Big\{\mu(1-\zeta)\epsilon(1-u)\Big(\frac{\beta_1 C_W+\beta_2 C_I+\beta_3 G}{\eta C_W C_I G D}\Big)\Big(\frac{\eta}{\mu}-C_W-C_I-G-D\Big)+\frac{d_2}{C_W G D}-\frac{k_1}{C_I G D}\Big\}\\
&= -\Big[\frac{Q}{\eta C_W C_I G D}\Big] < 0
\end{aligned}$$

where

$$\begin{aligned}
Q &= \mu\tau C_W C_I(\beta_1 C_W+\beta_2 C_I+\beta_3 G)+d_1 C_W^2+d_2 C_I^2+(\eta-\mu(C_W+C_I+G+D)))\\
&= [(1-\zeta)\epsilon(1-u)C_I(\beta_2 C_I+\beta_3 G)+k\zeta\epsilon(1-u)C_W(\beta_1 C_W+\beta_3 G)].
\end{aligned}$$

By Dulacs stability criterion, no periodic orbits exist in $\bar{\omega}$. Also, since $\bar{\omega}$ is positively invariant, and corruption persists with the condition that $R_{co} > 1$, then it follows from the usual Poincare-Bendixson Theorem [10] that all solutions of the limiting system originating in $\bar{\omega}$ remain in $\bar{\omega}$ for all times. In addition, the non-existence of periodic orbits in that domain shows that the above lemma holds.

3.4 Numerical results and discussion

Numerical simulations of the model (2) with immunity clause factor are carried using a set of reasonable parameter values given in Table. 2. We adopt a fourth-order Runge Kulta numerical scheme coded in Matlab for the numerical simulations.

Table 2: Parameter values used for the numerical simulations

Parameter	Value	Source	Parameter	Value	Source
η	30,000	[11]	$(\alpha_1, \alpha_2, \alpha_3)$	$(0.0001, 0.0001, 0.0001)$	[11]
ζ	0.5	Assumed	k	$k \geq 1$	Variable
(ω_1, ω_2)	$\left(\frac{1}{4}, \frac{1}{4}\right)$	Assumed	δ	0.0001	[11]
(d_1, d_2)	$\left(\frac{1}{8}, \frac{1}{4}\right)$	Assumed	μ	0.02/year	[11]
			u	$[0, 1]$	Variable
$(\beta_1, \beta_2, \beta_3)$	$(0.02, 0.036, 0.036)$	[11]	ϵ	$\epsilon \geq 1$	Variable

It is conceivable from the plots Fig. 2(a)-(c) that an increase in immunity clause factor can cause a huge volume of corruption cases in Nigeria and the entire world. Also, we notice from Fig. 2(b) and (c) that corrupt individuals under immunity clause and god-fathers attain the same peak of corruption, however, the latter are most corrupted at the beginning. Fig. 2(d) on the other hand, gives an unreasonable fractional number of convicted cases since less or no persons are sanctioned for criminal offenses. The plots shows that, an increase in conviction rates limits the number of corrupted people and a corresponding increase in conviction cases. Therefore, anti-corruption crusade need to be intensify across all sectors in order to eliminate the consequences of corruption.

In Fig. 4, $R_{o1} < R_{co} < R_{oo} < R_{o2}$ implying that corruption is more prevalent in the presence of immunity clause than when it is removed ($R_{o2} > R_{oo}$). Also, with the present day anti-corruption crusade and conviction measures, corruption burden can still not be easily reduced if the protection clause for the corrupted people is sustained in the constitution since $R_{o1} < R_{co}$.

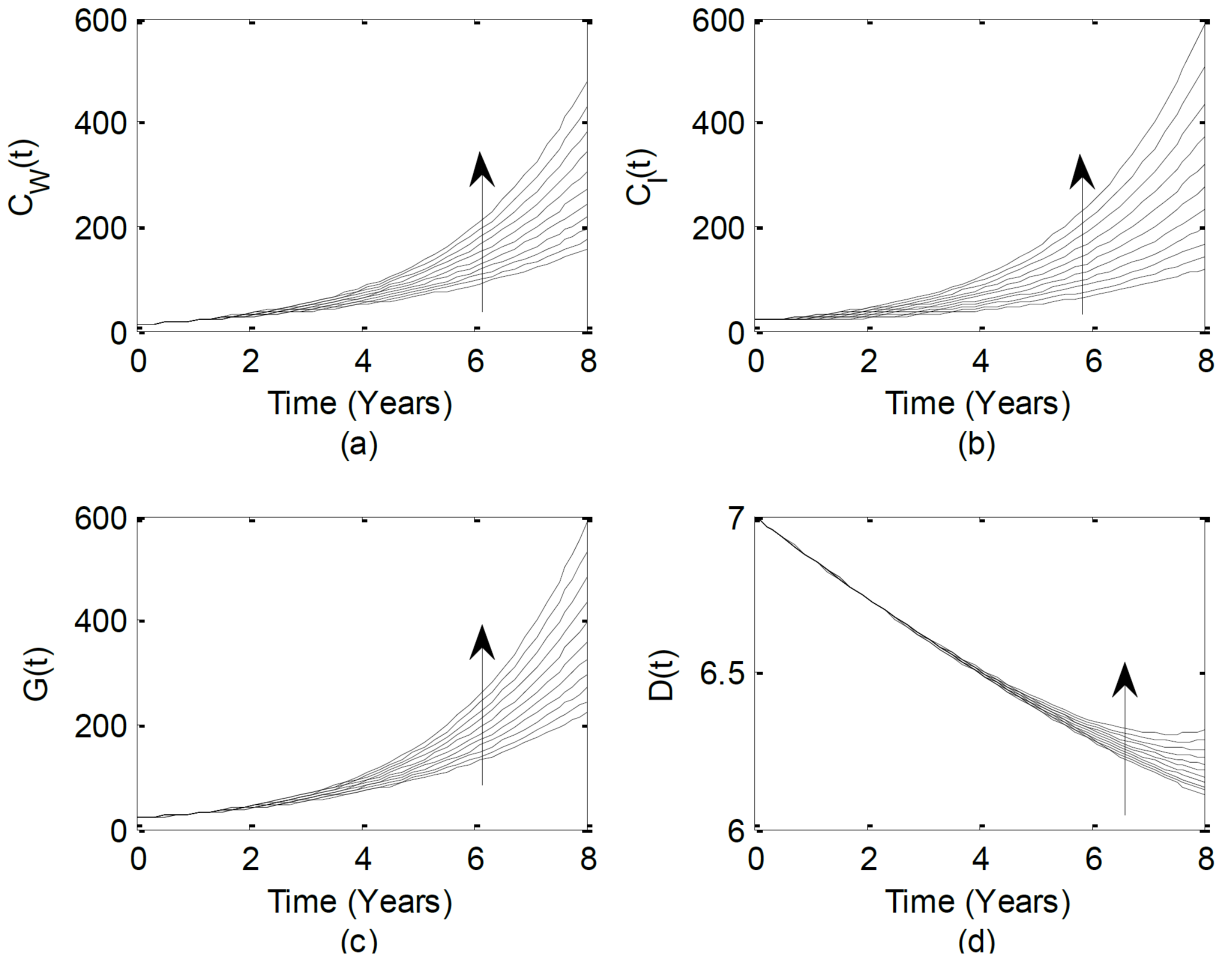

Fig. 2: Simulation of model (2) showing plots of Corrupted individuals without immunity clause (C_W), Corrupted individuals with immunity clause (C_I), Corrupted political god-fathers (G) and Convicted individuals (D) with immunity enhancement factor increasing from 1 to 2 in step 0.1. Parameters used are from Table. 2 with $(\beta_1, \beta_2, \beta_3) = (0.25, 0.35, 0.4), u = 0.1, \epsilon = 1$. The arrow in the plots illustrates the direction of increase in immunity clause factor, with assumed initial conditions $S = 100, C_W = 10, C_I = 20, G = 20$ and $D = 7$

4 Conclusion

In this paper, a system of ordinary differential equations with immunity clause in the presence of anti-corruption crusade and conviction as correctional services was developed to assess the impact on dynamical system of corruption in Nigeria. First and foremost, we examined the positivity and boundedness of solution of the system 2. With respect to the initial conditions (3), we showed that the model is mathematically well-posed and biologically reasonable. Also, by means of next generation matrix, we obtain corruption reproduction ratio, R_{co}, which plays a critical role on the dynamics of corruption in terms of reducing its cases when the value is less than unity. By using a Lyapunov function and Dulac's stability criteria, we prove the global stability of both corruption-free and persistent equilibria under certain conditions. Additionally, simulation results have shown that treating corruption in the presence of immunity clause regime will be a hard-war to be won if the clause continued to exist. The study points out evidently that immunity clause in Nigeria constitution contributes heavily in the repeated episodes of corruption than being the mere sore protector of office dignity. Based on these analysis, the study concludes that the holistic use of anti-corruption mechanisms in non-immunity clause regime is more hopeful to stopping corruption and reducing its burden in Nigeria and the rest of the world. In view of that the clause should be removed along with the practice of god-fatherism in Nigerian politics, and

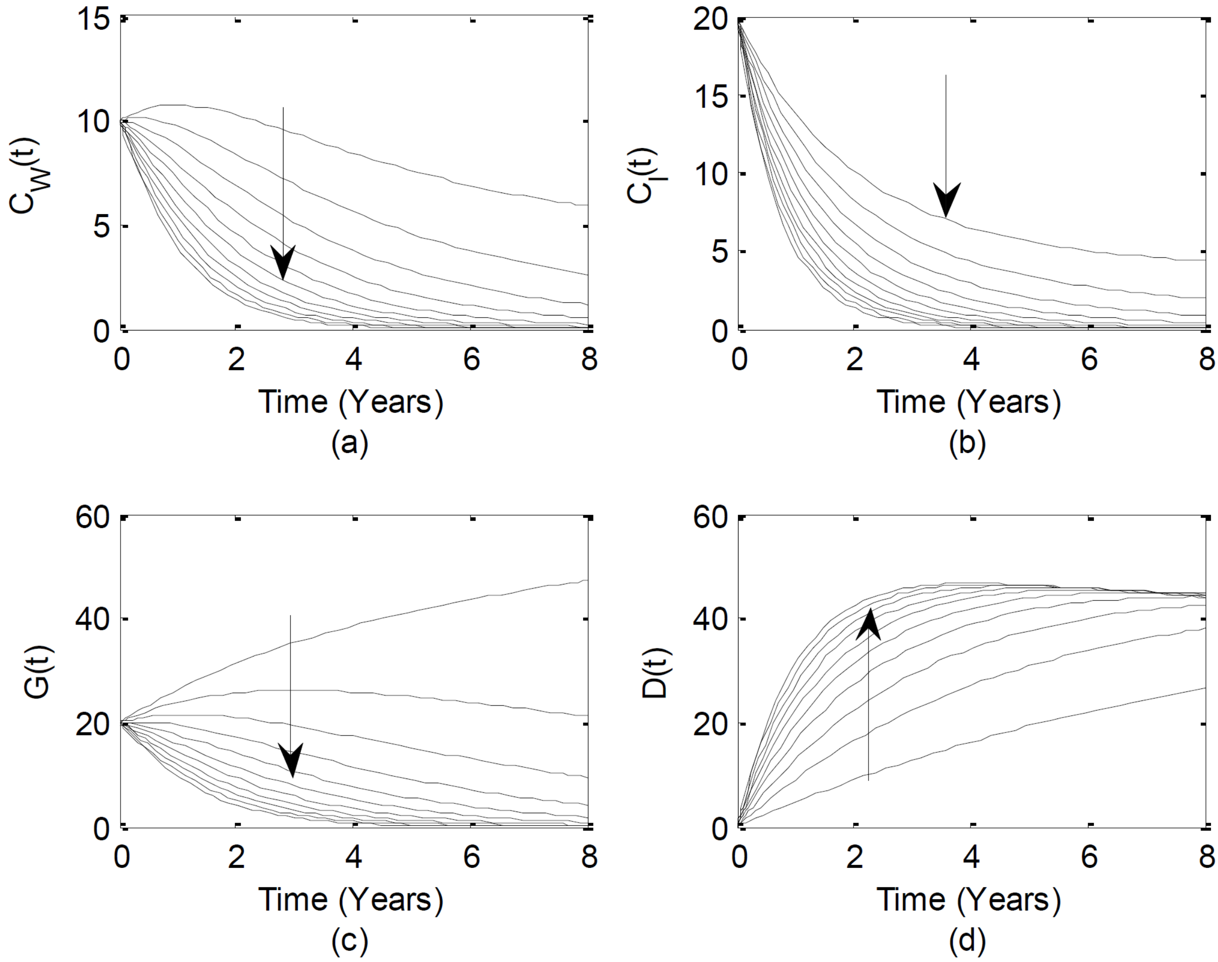

Fig. 3: Simulation of model (2) showing plots of Corrupted individuals without immunity clause (C_w), Corrupted individuals with immunity clause (C_I), Corrupted political god-fathers (G) and Convicted individuals (D) with conviction increasing from 0 to 1 in step 0.1. Parameters used are from Table. 2 with $u = 0.1, \epsilon = 1$. The arrow in the plots illustrates the direction of increase in immunity clause factor, with assumed initial conditions $S = 100, C_w = 10, C_I = 20, G = 20$ and $D = 7$.

further suggested that conviction with death sentence be enacted in the constitution of Nigeria. Lastly, the cost effectiveness of these control measures can be pursed in future works.

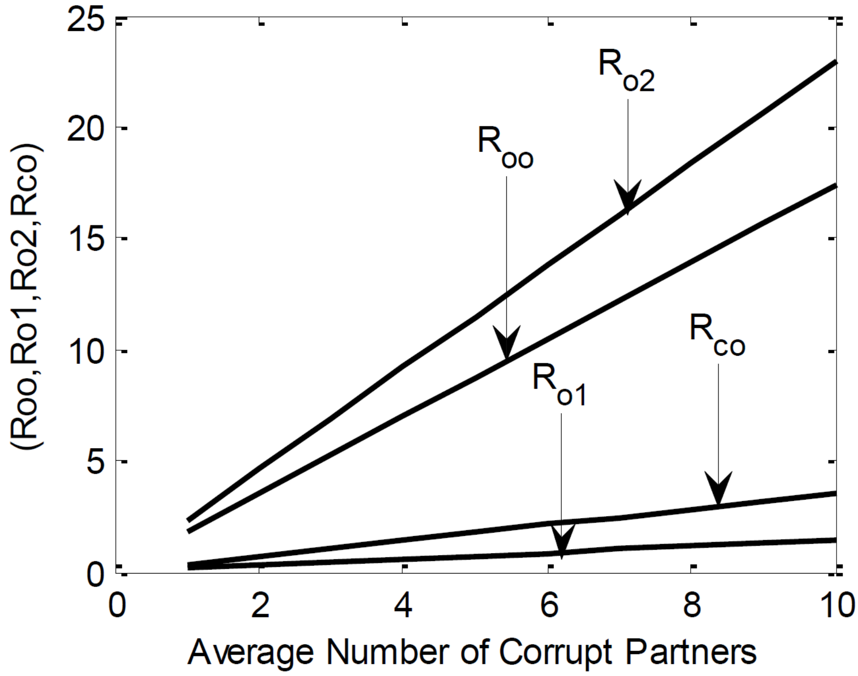

Fig. 4: Simulation results showing the behavioural relationship of the reproduction ratios $R_{oo}, R_{o1}, R_{o2} and R_{co}$ for varying number of corruption partners. Corruption partners are varied from 1 to 10 per unit time. Parameters used are in Table. 2 with $u = 0.4, \alpha_1 = 0.0001041, \alpha_2 = 0.35, \alpha_3 = 0.05 and \kappa = 2$.

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