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Slow Steady Rotation of an axially symmetric Micropolar fluid sphere in a nonconcentric Newtonian spherical container*

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Abstract. This paper examines the slow steady rotation of a micropolar fluid sphere in an eccentric spherical container containing viscous fluid. The boundary conditions on the particle surface and cavity wall are satisfied by a collocation technique. An infinite-series solution is presented for the velocity components, microrotation component and shear stress of the flow. Numerical results for the couple acting on the micropolar fluid sphere are found with good convergence for different values of the micropolarity parameter, ratio of particle-to-container radii, viscosity ratio and distance between the centers of the particle and container. In the limit of the motion of the micropolar fluid sphere located at the center of the spherical container, the numerical values of the wall correction factor are in good agreement with the values available in the literature. It is observed that the wall correction factor increases with increasing values of the micropolarity parameter and separation parameter.

Keywords: Micropolar fluid, eccentric, sphere, couple

1 Introduction

Studies concerning rotating fluid systems have endure much attention from researchers because of its different applications both in engineering and science. Some of them are fluid gyroscopes, colloidal science and centrifuges. The couple exerted on the rotating body is useful in designing and calibration of viscometers [15]. Jeffery[14] was the first who studied the slow rotation of spheroids in an unbounded fluid. Kanwal[15] discussed the slow steady rotation of an axisymmetric body in an incompressible viscous fluid. Keh and Lu[16] investigated the translation and rotation of a porous spherical shell located at the concentric position in a spherical cavity containing Newtonian fluid. The translation and rotational motion of a porous spheroid in concentric spheroidal container was studied by Saad[25]. Srinivasacharya and Prasad[28] investigated the rotation of a porous approximate sphere located at the center of an approximate spherical container containing an incompressible Newtonian viscous fluid.

The classical Navier-Stokes theory is not suitable to explain the behavior of structured fluids like polymeric suspensions, liquid crystals, lubricating oils, body fluids, animal blood, etc. Thus, a theory is needed that can correctly describe the behavior of structured fluids. The micropolar fluid theory given by Eringen[5, 6] is one of the best theories to explain the fluids with microstructure. The micropolar fluids consist of rigid, randomly oriented particles with their own spins and microrotations, suspended in a viscous medium. These fluids show some microscopic effects coming from the micromotion of the fluid elements and local structure and they can also support couple stresses. The book by Lukaszewicz[19] and the review article by Ariman et al.[3] describe some of the theory and applications of micropolar fluids.

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In the past few years the study of non-Newtonian fluids has received special attention. Rao[22] studied the slow steady rotation of a sphere in a micropolar fluid. Ramkisson[23] analyzed the Stokes flow of an axially symmetric body rotating in a micropolar fluid. Rao and Iyengar[24] investigated the slow steady rotation of a spheroid (prolate and oblate) in an incompressible micropolar fluid. Iyengar and Srinivasacharya[13] examined the problem of slow steady rotation of an approximate sphere in an incompressible micropolar fluid. Prasad and Gurdatta[20, 21] analyzed the problem of rotational motion of a micropolar fluid sphere located at the center of a spherical container containing viscous fluid and the problem of Stokes flow of an incompressible viscous fluid past a micropolar fluid spheroid.

The boundary collocation method has been employed by many researchers to investigate viscous and micropolar fluid flows. Gluckman et al.[11] developed a truncated series boundary collocation method for studying the Stokes flow of viscous fluid past finite assemblages of particles of arbitrary shape. Leichtberg et al.[18] used this method to investigate the flow past a finite chain of rigid spheres moving slowly in a viscous fluid inside an infinitely long cylindrical tube. Ganatos et al.[9, 10] employed this method to examine the Stokes flow of a sphere between two parallel plane boundaries. Keh and Lee[17] used this method to study the axisymmetric motion of a slip spherical particle in an eccentric spherical container containing another immiscible fluid. They obtained the wall-corrected drag force acting on the particle with good convergence. Faltas and Saad[7] used a combined analytical/numerical method with the boundary collocation technique to investigate the axisymmetric Stokes flow problem of a viscous fluid contained between two eccentric spheres rotating with different angular velocities and evaluated the rotational couple coefficient acting on the particle. Faltas et al.[8] also used this technique to study the interaction between two spherical particles rotating with different angular velocities in a micropolar fluid. Ashmawy[1, 2] employed this technique to investigate the problem of steady rotation of an axisymmetric body in viscous fluid and the rotation of an axisymmetric porous particle in a viscous fluid. Saad[26] examined the translation and steady rotation of a porous sphere in a nonconcentric spherical container using a combined analytical-numerical technique. El-Sapa et al.[4] analyzed the quasi-steady translational motion of two solid spherical particles embedded in a porous medium. They assumed particles of different sizes, translating with different velocities, and allowing for the hydrodynamic slip at their surfaces. Recently, Saad[27] studied the flow of an incompressible viscous fluid past an assemblage of porous nonconcentric spherical particles using the stress jump boundary condition and obtained the numerical solutions for the drag force exerted on the porous sphere.

Many authors have employed the boundary collocation technique by taking the particle as a solid sphere, liquid sphere and porous sphere in a nonconcentric spherical geometry and evaluated the drag or torque exerted on the particle. But only a few work have been reported in the literature concerning the problem of interaction between particles and container containing immiscible fluids. To the best of our knowledge, there is no previous work dealing with motion of a micropolar fluid sphere rotating in an eccentric spherical container containing viscous fluid. This motivated us to study the present problem. The rotation of spherical micropolar fluid droplet located at the center of a spherical cavity containing viscous fluid have already been analyzed, and closed-form expression for the torque exerted on the particle was obtained. The objective of this work is to obtain semianalytical solutions for the rotational motion of a micropolar fluid sphere in a nonconcentric spherical cavity. The creeping flow equations applicable to the systems are solved by using the boundary collocation technique. The couple acting on the particle is obtained with good convergence for various parameters considered. Numerical results for the case of motion of micropolar fluid sphere in concentric spherical container are also included.

The paper is arranged as follows: The equations of motion describing the flow are given in Section 2. The boundary conditions are discussed in Section 3. The solution procedure and the expressions for torque and wall correction factor are given in Section 4. The numerical results obtained are explained in Section 5. The concluding remarks of the study is discussed in Section 6.

2 Formulation of the problem

Consider the slow steady rotational motion of an incompressible micropolar fluid sphere of radius a located at the eccentric of a spherical container of radius b filled with Newtonian fluid. The angular velocity of the spherical container is Ω about common axis joining the center of the micropolar fluid sphere and viscous spherical container. The geometry of the spherical nonconcentric annulus considered is indicated in Fig. 1. The problem is concerned by dividing the flow in two regions: I is the region of the internal micropolar fluid sphere and II is the region of the Newtonian fluid in the spherical container. The center of the micropolar fluid sphere is located away from the cavity center at a distance d.

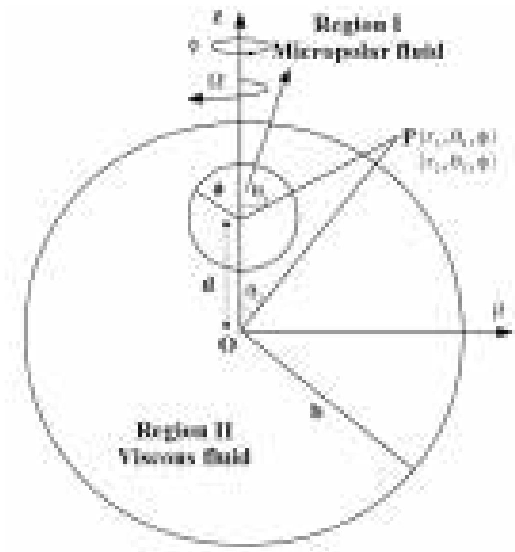


Fig. 1: Micropolar fluid sphere in an eccentric spherical container

The equations describing the motion in region I are the equations governing the Stokes flow of an incompressible micropolar fluid with the absence of body force and body couples are given by

$$\nabla \cdot \vec{q}^{(1)} = 0, \tag{1a}$$

$$\nabla p^{(1)} + (\mu_1 + \kappa)\, \nabla \times \nabla \times \vec{q}^{(1)} - \kappa\, \nabla \times \vec{\nu} = 0, \tag{1b}$$

$$\kappa\, \nabla \times \vec{q}^{(1)} - 2\,\kappa\, \vec{\nu} - \gamma_0\, \nabla \times \nabla \times \vec{\nu} + (\alpha_0 + \beta_0 + \gamma_0) \nabla\nabla \cdot \vec{\nu} = 0, \tag{1c}$$

where $\vec{q}^{(1)}$ is the velocity vector, $\vec{\nu}$ is the microrotation vector, $p^{(1)}$ is pressure, μ_1 is the coefficient of viscosity of the classical viscous fluid and $\kappa, \alpha_0, \beta_0$ and γ_0 are the new viscosity coefficients of the micropolar fluids.

The equations for the stress tensor $t_{i\,j}$ and the couple stress tensor $m_{i\,j}$ are

$$t_{i\,j} = -p\,\delta_{i\,j} + \mu_1\,(q_{i,j} + q_{j,i}) + \kappa\,(q_{j,i} - \epsilon_{i\,j\,m}\,\nu_m), \tag{2}$$

$$m_{i\,j} = \alpha_0\,\nu_{m,m}\,\delta_{i\,j} + \beta_0\,\nu_{i,j} + \gamma_0\,\nu_{j,i}, \tag{3}$$

where the comma denotes the partial differentiation, $\delta_{i\,j}$ and $\epsilon_{i\,j\,m}$ are the Kronecker delta and the alternating tensor.

The equations describing the motion in region II are

$$\nabla \cdot \vec{q}^{(2)} = 0, \tag{4a}$$

$$\nabla p^{(2)} + \mu_2\, \nabla \times \nabla \times \vec{q}^{(2)} = 0, \tag{4b}$$

where $\vec{q}^{(2)}$is the velocity vector, $p^{(2)}$ is the pressure and μ_2 is the coefficient of viscosity.

Let (r_1, θ_1, ϕ) be the spherical coordinates based on the center of the micropolar fluid sphere, where (ρ, ϕ, z) and (r_2, θ_2, ϕ) be the circular cylindrical and spherical coordinates based on the center of the spherical container. The relation between r_1 and r_2 is given by $r_1^2 = r_2^2 + d^2 - 2\, r_2\, d\, \cos\theta_2$ or $r_2^2 = r_1^2 + d^2 + 2\, r_1\, d\, \cos\theta_1$. Since the rotation is assumed to be slow, the velocity $\vec{q}$ has its only component and the microrotation vector $\vec{\nu}$ lies in the meridian plane. All the flow quantities are independent of ϕ. Thus, we choose the velocity and microrotation vectors in cylindrical coordinates as

$$\vec{q}^{(i)} = q_\phi^{(i)}(r,\theta)\, \vec{e}_\phi, \;\; i = 1, 2, \tag{5}$$

$$\vec{\nu} = \nu_r(r,\theta)\, \vec{e}_r + \nu_\theta(r,\theta)\, \vec{e}_\theta. \tag{6}$$

Substituting Eqs. 5 in 1b and 4b, 6 in 1c and then introducing the nondimensional variables $r = a\tilde{r}$, $q_\phi^{(i)} = \Omega\, \tilde{q}_\phi^{(i)}$, $\nu_r = \Omega\, \tilde{\nu}_r$, and $\nu_\theta = \Omega\, \tilde{\nu}_\theta$ in the resulting equations and dropping the tildes, we get the following dimensionless equations: In the micropolar fluid region $r \leq a$,

$$\frac{\partial p^{(1)}}{\partial \rho} = 0, \;\; \frac{\partial p^{(1)}}{\partial z} = 0, \tag{7}$$

$$L(L - m^2) q_\phi^{(1)} = 0, \tag{8}$$

and for the viscous region $a \leq r \leq b$

$$\frac{\partial p^{(2)}}{\partial \rho} = 0, \;\; \frac{\partial p^{(2)}}{\partial z} = 0, \tag{9}$$

$$L\, q_\phi^{(2)} = 0, \tag{10}$$

where

$$L = \nabla^2 - \frac{1}{\rho^2}, \;\; \nabla^2 = \frac{\partial^2}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial}{\partial \rho} + \frac{\partial^2}{\partial z^2} - \frac{1}{\rho^2}.$$

Assume that $\mathrm{div}\vec{\nu} = \varsigma(r,\theta)$, $\mathrm{curl}\vec{\nu} = \upsilon(r,\theta)\, \vec{e}_\phi = -N^{-1}\, L q_\phi^{(1)}\, \vec{e}_\phi$, we have

$$\left(\nabla^2 - c^2\right) \varsigma = 0, \tag{11}$$

where

$$c^2 = \frac{2\,\kappa\, a^2}{\alpha_0 + \beta_0 + \gamma_0}, \;\; m^2 = \frac{a^2\, \kappa(2+\chi)}{\gamma_0(1+\chi)}, \;\; \chi = \frac{\kappa}{\mu_1}, \;\; N = \frac{\chi}{1+\chi},$$

$$\nu_r = \frac{1}{c^2}\frac{\partial \varsigma}{\partial r} - \frac{\gamma_0}{2\,\kappa}\frac{1}{r}\left(\frac{\partial \upsilon}{\partial \theta} + \upsilon \cot\theta\right) + \frac{1}{2\,r}\left(\frac{\partial q_\phi^{(1)}}{\partial \theta} + q_\phi^{(1)} \cot\theta\right), \tag{12}$$

$$\nu_\theta = \frac{1}{c^2}\frac{1}{r}\frac{\partial \varsigma}{\partial \theta} + \frac{\gamma_0}{2\,\kappa}\left(\frac{\partial \upsilon}{\partial r} + \frac{\upsilon}{r}\right) - \frac{1}{2}\left(\frac{\partial q_\phi^{(1)}}{\partial r} + \frac{q_\phi^{(1)}}{r}\right). \tag{13}$$

3 Boundary conditions

The boundary conditions on the micropolar fluid sphere and the spherical container to be satisfied by the collocation technique are as follows:

At a surface of the micropolar fluid sphere $r_1 = a$, continuity of velocity component along with the tangential stress and no-spin condition

$$q_\phi^{(1)} = q_\phi^{(2)}, \tag{14}$$

$$t^{(1)}_{r_1\phi} = t^{(2)}_{r_1\phi}, \tag{15}$$

$$\nu_r = 0, \tag{16}$$

$$\nu_\theta = 0. \tag{17}$$

On the container surface $r_2 = b$, it is assumed that the tangential velocity at the container surface is continuous

$$q^{(2)}_\phi = -\, r_2 \, \sin\theta_2. \tag{18}$$

4 Solution of the problem

The solutions of Eqs. 8, 10 and 11 are ([7, 8, 12]):

$$q^{(1)}_\phi = \sum_{n=1}^{\infty} \left[c_n\, r_1^n + f_n\, r_1^{-1/2}\, I_{n+1/2}(m\, r_1) \right] P^1_n(\cos\theta_1), \tag{19}$$

$$q^{(2)}_\phi = \sum_{n=1}^{\infty} \left[a_n\, r_1^{-n-1}\, P^1_n(\cos\theta_1) + b_n\, r_2^n\, P^1_n(\cos\theta_2) \right], \tag{20}$$

$$\varsigma(r,\theta) = \sum_{n=1}^{\infty} r_1^{-1/2}\, h_n\, I_{n+1/2}(c\, r_1)\, P^1_n(\cos\theta_1), \tag{21}$$

$$\upsilon(r,\theta) = -\,\frac{m^2}{N} \sum_{n=1}^{\infty} r_1^{-1/2}\, f_n\, I_{n+1/2}(m\, r_1)\, P^1_n(\cos\theta_1), \tag{22}$$

Where I denotes the modified Bessel function of the first kind and P^1_n is the associated Legendre function of order n. The coefficients a_n, b_n, c_n, f_n and h_n are unknown constants that will be determined using the boundary conditions.

Now using the expressions for υ, ς and $q^{(1)}_\phi$ in the Eqs. 12 and 13, the expressions for ν_r and ν_θ are obtained as:

$$\begin{aligned} \nu_r = \sum_{n=1}^{\infty} \Bigg[& \frac{n(n+1)}{2} c_n\, r_1^{n-1} + \frac{n(n+1)}{N} f_n\, r_1^{-3/2}\, I_{n+1/2}(m\, r_1) \\ & - \frac{1}{c^2} h_n\, r_1^{-3/2} \left((n+1)\, I_{n+1/2}(c\, r_1) - c\, r_1\, I_{n-1/2}(c\, r_1) \right) \Bigg] P_n(\cos\theta_1), \end{aligned} \tag{23}$$

$$\begin{aligned} \nu_\theta = \sum_{n=1}^{\infty} \Bigg[& -\frac{(n+1)}{2} c_n\, r_1^{n-1} + \frac{1}{N} f_n\, r_1^{-3/2} \left(n\, I_{n+1/2}(m\, r_1) - m\, r_1\, I_{n-1/2}(m\, r_1) \right) \\ & - \frac{1}{c^2} h_n\, r_1^{-3/2}\, I_{n+1/2}(c\, r_1) \Bigg] P^1_n(\cos\theta_1). \end{aligned} \tag{24}$$

The tangential stresses $t^{(1)}_{r\phi}$ and $t^{(2)}_{r\phi}$ are given by

$$\begin{aligned} t^{(1)}_{r\phi} = \sum_{n=1}^{\infty} \Bigg[& \frac{(2\mu_1 + \kappa)}{2} (n-1)\, c_n\, r_1^{n-1} - (2\,\mu_1 + \kappa) f_n\, r_1^{-3/2} I_{n+1/2}(m\, r_1) - \\ & \frac{1}{c^2} \kappa\, h_n\, r_1^{-3/2}\, I_{n+1/2}(c\, r_1) \Bigg] P^1_n(\cos\theta_1), \end{aligned} \tag{25}$$

$$t_{r\phi}^{(2)} = -\mu_2 \sum_{n=1}^{\infty} \left[(n+2)\, a_n\, r_1^{-n-2} P_n^1(\cos\theta_1) - (n-1)\, b_n\, r_2^{n-1} P_n^1(\cos\theta_2) \right]. \tag{26}$$

To determine the unknown constants $a_n,\ b_n,\ c_n,\ f_n$ and h_n, we apply the boundary conditions Eqs. 14-18 to the Eqs. 19, 20, 23, 24, 25 and 26 as:

$$\sum_{n=1}^{\infty} [a_n\, a_{1\,n}(a,\theta_1) + [b_n\, b_{1\,n}(r_2,\theta_2)]_{r_1=a} + c_n\, c_{1\,n}(a,\theta_1) + f_n\, f_{1\,n}(a,\theta_1)] = 0, \tag{27}$$

$$\begin{aligned}&\sum_{n=1}^{\infty} [a_n\, a_{2\,n}(a,\theta_1) + [b_n\, b_{2\,n}(r_2,\theta_2)]_{r_1=a} + c_n\, c_{2\,n}(a,\theta_1)\\ &+ f_n\, f_{2\,n}(a,\theta_1) + h_n\, h_{2\,n}(a,\theta_1)] = 0,\end{aligned} \tag{28}$$

$$\sum_{n=1}^{\infty} [c_n\, c_{3\,n}(a,\theta_1) + f_n\, f_{3\,n}(a,\theta_1) + h_n\, h_{3\,n}(a,\theta_1)] = 0, \tag{29}$$

$$\sum_{n=1}^{\infty} [c_n\, c_{4\,n}(a,\theta_1) + f_n\, f_{4\,n}(a,\theta_1) + h_n\, h_{4\,n}(a,\theta_1)] = 0, \tag{30}$$

$$\sum_{n=1}^{\infty} [[a_n\, a_{5\,n}(r_1,\theta_1)]_{r_2=b} + b_n\, b_{5\,n}(b,\theta_2)] = -1, \tag{31}$$

where the expressions $a_{in},\ b_{in},\ c_{in},\ f_{in}$ and h_{in}, with $i = 1, 2, 3, 4, 5$ are given in Appendix A.

To determine the velocity and microrotation vectors, the boundary conditions Eqs. 14-18 should be satisfied exactly along the surface of micropolar fluid sphere and container surface. This leads to infinite linear algebraic equations, which are impossible to solve as it has infinite number of unknown coefficients. But, by using multipole collocation technique this difficulty can be avoided and solution can be obtained. The collocation method truncates the infinite series into finite ones by enforcing the boundary conditions at a finite number of points.

The torque exerted on the micropolar fluid sphere is

$$T = \int_S \vec{r} \times (\vec{n} \cdot t) \cdot \vec{k}\, dS, \tag{32}$$

where $\vec{r} = a\,\vec{e_r}$, $\vec{n} = \vec{e_r}$, and $\vec{k}$ is the unit vector in the direction of the axis of rotation and taking the integral over the surface of the boundary, we get

$$T = 2\pi\, a^3 \int_0^{\pi} r^3\, t_{r\phi}^{(2)}|_{r=1} \sin^2\theta\, d\theta = -8\,\pi\,\mu_2\,\Omega\, a^3\, a_1. \tag{33}$$

The torque exerted on the micropolar fluid sphere in an unbounded medium is

$$T_\infty = -8\,\pi\,\mu_2\,\Omega\, a^3\,\chi\,\Delta_1\,\Delta_2^{-1}, \tag{34}$$

where

$$\Delta_1 = -2\left(c\,(2+\chi)\, I_{3/2}(m)\, I_{1/2}(c) + I_{3/2}(c)\left(m\,(1+\chi)\, I_{1/2}(m) - 3\,(3+2\,\chi)\, I_{3/2}(m)\right)\right),$$

$$\begin{aligned}\Delta_2 =& c\,(2+\chi)\, I_{1/2}(c)(-3\,m\,\lambda\,(1+\chi)\, I_{1/2}(m) + I_{3/2}(m)(3\,\lambda + 6\,\lambda\,\chi - 2\,\chi))\\ &+ I_{3/2}(c)(3\,\chi\, I_{3/2}(m)(4\,\chi + 6 - 3\,(2+\chi)\,\lambda) + 2\,m\,(1+\chi)\, I_{1/2}(m)\\ &\times (3\,(2+\chi)\,\lambda - \chi)),\end{aligned}$$

$\lambda = \dfrac{2\,\sigma}{2+\chi}$ and $\sigma = \dfrac{\mu_2}{\mu_1}$.

The wall correction factor W_c is defined as the ratio of the actual couple experienced by the sphere in the container wall to the couple experienced by a sphere in an infinite expanse of fluid. Using Eqs. (33) and (34) this becomes

$$W_c = \frac{T}{T_\infty}. \tag{35}$$

5 Results and discussion

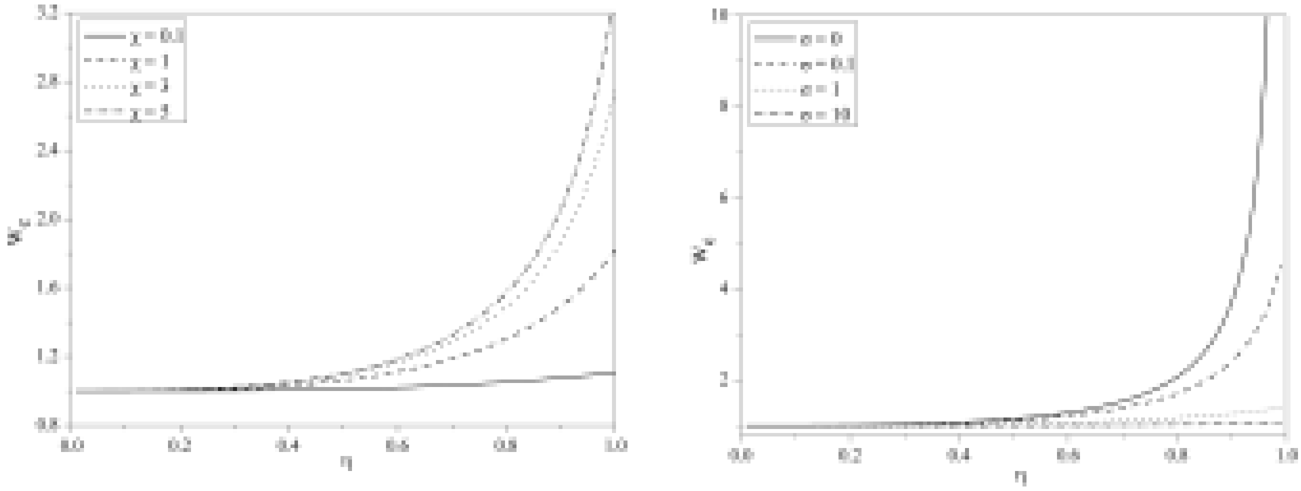
To specify the points along the semi-circular generating arc of the micropolar fluid sphere and the container surfaces where the boundary conditions are exactly satisfied, we should choose the points $\theta = 0$ and π, as these points control the gap between the particle and the container surfaces. Along with this, the point $\theta = \pi/2$ is also important. But, the system of linear algebraic Eqs 27-31 shows that if these points are used, then the coefficient matrix would become singular. In order to avoid this, we use the method given in literature[9–11, 18]. The four basic points that should be taken on the half unit circle are $\theta = \epsilon,\ \pi/2 - \epsilon,\ \pi/2 + \epsilon,\ \pi - \epsilon$, where ϵ is so chosen that the the singularities at $\theta = 0,\ \pi/2$ and π can be avoided. Remaining points are selected as mirror-image pairs about $\theta = \pi/2$ in order to divide the two quarter-circular arcs into equal segments. Then, the couple acting on the micropolar fluid sphere is calculated by solving the coefficient matrix using Gauss elimination method.

The collocation solutions of the wall correction factor exerted on the micropolar fluid sphere in an eccentric spherical container containing viscous fluid for different values of the distance between the centers of the spherical particle and container $\delta = d/(b - a)$, micropolarity parameter χ, separation parameter $\eta = a/b$ and viscosity ratio σ are presented in the Figs. 2-3 and Table 1. In numerical calculation we assumed the values of $\dfrac{\gamma_0}{\mu_1\, a^2} = 0.3$ and $\dfrac{(\alpha_0 + \beta_0 + \gamma_0)}{\mu_1\, a^2} = 0.4$. All results obtained under this collocation method converge to at least five decimal places.

Figure 2a illustrates the variation of the wall correction factor with separation parameter η for various values of micropolarity parameter χ with $\delta = 0.25$ for the case of $\sigma = 0.3$. It is observed that the wall correction factor increases with increasing micropolarity parameter for fixed values of σ and δ. For small values of χ, the wall correction factor varies slowly with η compared with larger values of χ.
Figure 2b illustrates the graphical representation of wall correction factor with η for different values of viscosity ratio σ for the case of $\chi = 2$. The case $\sigma = 0$ corresponds to the steady rotation of a solid sphere in an eccentric spherical cavity. Table 1 and Fig. 2b depicts that the wall correction factor decreases with increasing viscosity ratio. It can be perceived from the figure that the wall correction factor of solid sphere is greater than that of fluid sphere. The variation of wall correction factor with δ for different values of η is shown in Fig. 3 for the case of $\sigma = 0.3$. It shows that the wall correction factor increases with increasing values of separation parameter η, and has a large value when the surface of spherical particle touch the container surface. However, Fig. 3 and Table 1 indicate that the wall correction factor is an increasing or a decreasing function of δ depending on the value of σ. For values of σ less than or equal to 1, it is an increasing function of δ. While for values of σ greater than 1 wall correction factor is a decreasing function of δ. Also, our collocation results of the wall correction factor in the concentric limit $\delta \to 0$ agree excellently with the analytical solution [20].

6 Conclusion

In this paper, we presented the combined analytical-numerical solution for the rotational motion of a micropolar fluid sphere at an eccentric position in a spherical container containing viscous fluid. Numerical evaluations of the wall correction factor are performed and the results are illustrated graphically for various values of the considered parameters. The results show that the wall correction factor not only changes with the viscosity ratio but also with the micropolarity parameter and separation parameter. The distance between the centers of particle and container plays a significant role in evaluating the values of the couples. It has been found that the wall correction factor is an increasing or a decreasing function of δ depending on value of σ.



(a) For different χ with $\delta = 0.25$ and $\sigma = 0.3$

(b) For different σ with $\delta = 0.25$ and $\chi = 2$

Fig. 2: Variation of the wall correction factor W_c versus separation parameter η

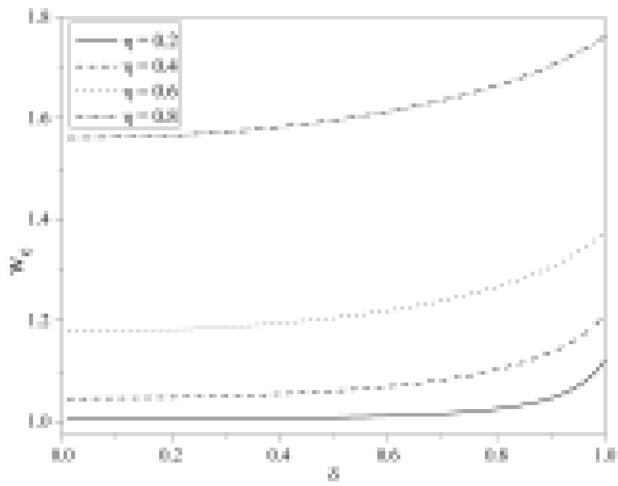


Fig. 3: Variation of the wall correction factor W_c versus δ for different η with $\chi = 5$ and $\sigma = 0.3$

A Appendix

The functions appearing in Eqs. 27-31 are

Table 1: Wall correction factor W_c at different values of σ, η and δ when $\chi = 2$

	W_c				
δ	η	$\sigma = 0$	$\sigma = 1$	$\sigma = 10$	$\sigma \to \infty$
0.00001	0.1	1.00100	1.00029	1.00004	1.00000
	0.3	1.02775	1.00784	1.00105	1.00000
	0.5	1.14286	1.03735	1.00488	1.00000
	0.7	1.52207	1.10962	1.01352	1.00000
	0.9	3.69004	1.26577	1.02917	1.00000
	0.99	33.6689	1.38787	1.03921	1.00000
0.25	0.1	1.00117	1.00032	1.26606	0.99998
	0.3	1.03055	1.00810	1.00048	0.99930
	0.5	1.15110	1.03798	1.00354	0.99838
	0.7	1.53870	1.11037	1.01223	0.99850
	0.9	3.73676	1.26606	1.02888	0.99968
	0.99	34.1014	1.38787	1.03920	0.99999
0.5	0.1	1.00198	1.00049	0.99997	0.99989
	0.3	1.04170	1.00916	0.99827	0.99660
	0.5	1.18071	1.03989	0.99870	0.99262
	0.7	1.59581	1.11262	1.00811	0.99374
	0.9	3.89483	1.26690	1.02799	0.99873
	0.99	35.5624	1.38789	1.03919	0.99998

$$
\begin{aligned}
&a_{1\,n}(r,\theta) = r^{-n-1}\, P_n^1(\cos\theta), \quad b_{1\,n}(r,\theta) = r^n\, P_n^1(\cos\theta),\\
&c_{1\,n}(r,\theta) = -r^n\, P_n^1(\cos\theta),\\
&f_{1\,n}(r,\theta) = -r^{-1/2}\, I_{n+1/2}(m\,r)\, P_n^1(\cos\theta),\\
&a_{2\,n}(r,\theta) = (n+2)\lambda\, r^{-n-2}\, P_n^1(\cos\theta),\\
&b_{2\,n}(r,\theta) = -(n-1)\lambda\, r^{n-1}\, P_n^1(\cos\theta),\\
&c_{2\,n}(r,\theta) = (n-1) r^{n-1}\, P_n^1(\cos\theta),\\
&f_{2\,n}(r,\theta) = -2\, r^{-3/2}\, I_{n+1/2}(m\,r)\, P_n^1(\cos\theta),\\
&h_{2\,n}(r,\theta) = -\left(\tfrac{2\chi}{2+\chi}\right)\, c^{-2}\, r^{-3/2}\, I_{n+1/2}(c\,r)\, P_n^1(\cos\theta),\\
&c_{3\,n}(r,\theta) = \tfrac{n(n+1)}{2} r^{n-1}\, P_n(\cos\theta),\\
&f_{3\,n}(r,\theta) = n(n+1)\, N^{-1}\, r^{-3/2}\, I_{n+1/2}(m\,r)\, P_n(\cos\theta),\\
&h_{3\,n}(r,\theta) = -c^{-2}\, r^{-3/2}\, \big((n+1)\, I_{n+1/2}(c\,r) - c\,r\, I_{n-1/2}(c\,r)\big)\, P_n(\cos\theta),\\
&c_{4\,n}(r,\theta) = -\tfrac{(n+1)}{2} r^{n-1}\, P_n^1(\cos\theta),\\
&f_{4\,n}(r,\theta) = N^{-1}\, r^{-3/2}\, \big(n\, I_{n+1/2}(m\,r) - m\,r\, I_{n-1/2}(m\,r)\big)\, P_n^1(\cos\theta),\\
&h_{4\,n}(r,\theta) = -c^{-2}\, r^{-3/2}\, I_{n+1/2}(c\,r)\, P_n^1(\cos\theta),\\
&a_{5\,n}(r,\theta) = \tfrac{1}{r_2\,\sin\theta_2} a_{1\,n}(r,\theta), \quad b_{5\,n}(r,\theta) = \tfrac{1}{r_2\,\sin\theta_2} b_{1\,n}(r,\theta).
\end{aligned}
$$

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