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On the influence of magnetic field on darcy mixed convection from a horizontal plate in a nanofluid saturated porous medium

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Abstract. The coupled nonlinear partial differential equations modeling the transverse magnetic effect on Darcian mixed convection from a horizontal plate in a nanofluid saturated porous medium have been solved numerically by finite difference method after transforming the partial differential equations into a set of two ordinary equations in suitable similarity variables under boundary layer approximations. Results obtained by the present study are compared with those from literature and are found to be in excellent agreement. Further, the results are analyzed for the heat transfer from the plate. Also, the dimensionless velocity and temperature profile are presented in the form of xy-plots for inferring the salient features in the current mixed convection process.

Keywords: darcy, porous medium, nanofluid, horizontal flat plate, mixed convection

1 Introduction

Real life applications such as storage of radioactive nuclear waste, transpiration cooling transport process in aquifers, groundwater pollution, geothermal extraction and fibre insulation etc. draw the attention of the researchers to analyse the fluid flow and heat transfer process in porous media. A great wealth of related literature is available in the books written by Nield and Bejan[1], Vafai[2, 3], Pop and Ingham[4], Ingham and Pop[5] and Vadasz[6].

Several industrial and engineering application like an aerodynamic extrusion of plastic sheet, cooling of a metallic plate, thin film solar energy collector, electronic cooling and vehicle cooling transformer etc. are in need of enhanced fluid cooling and heating mechanisms. Nano metallic particle suspensions in fluids can provide such an enhanced heat transfer mechanism due to higher thermal conductivities of the suspended metallic particles[7–10]. Some of the important studies related to nanofluid flows are presented by Choi[11], Kleinstreuer et al.[12], Khanafer et al.[13], Tiwari and Das[14], Oztop and Abu-Nada[15], Rahman et al.[16], Das et al.[17], Kakac and Pramanjaroekij[18], Ahmed and Pop[19], Arfin et al.[20], Rosca et al.[21] etc. have considered problems related to convection of heat transfer in the nanofluid saturated porous medium based on the aptly modified models proposed for convection in fluid saturated porous media by Cheng[22], Lai and Kulacki[23], Magyari et al.[24] etc. Recently Arfin et al.[20] analyzed free and mixed convection of nanofluid flow, consisting of nano particles such as Copper, alumina and titania, past a horizontal flat plate. Garoosi et al.[25]−[29] have investigated the natural and mixed convection of nano-fluid in different configurations such as square cavity with and without heaters and coolers, differently heated cylinders etc. using either single phase or two-phase

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models. Khan and Aziz[30], Hsaio[31], Pal et al.[32], Cheng [33], Sun and Pop[34], Ahmad and Pop[35], Rashad et al.[36], Rana et al.[37], Nield and Kuznatsov[38, 39] etc. have studied different types of convection and heat transfer process in nano-fluid saturated porous media under different configurations.

Chamkha and Aly[40], Hamada et al.[41], Kameswaran et al.[42], Hsaio[43]−[45] etc. have studied MHD flows, owing to their importance in scientific and engineering applications. Aly and Vajravelu[46] recently studied MHD nano boundary layer flow over a stretching surface in porous medium. A detailed review on nano-fluid flow and heat transfer has very recently been reported by Kasaeian et al.[47]. This also includes different MHD based works related to convection in nano-fluid saturated porous media. However, the study related to the effect of the MHD field on Darcy mixed convection from a horizontal plate in nanofluid saturated porous media has not been reported in the literature. Hence this work is focused on analysing the effect of the transverse magnetic field on Darcy mixed convection process from a hot horizontal surface embedded in a nanofluid saturated porous media. Here following Hussain and Hussain[48], along the horizontal plate similarity solutions are obtained when the wall temperature varies as $x^{1/2}$, where x is the distance along the plate measured from the origin. The similarity equations are solved numerically by implicit Finite Differences Method.

2 (Basic equations) Mathematical model

In the proposed problem, for the modeling purpose, let y be the coordinate normal to the heated horizontal impermeable surface along the x-axis. The physical situation of the model under consideration is presented in Fig. 1. Here the solid line refers to the thermal boundary layer of thickness δ_T and the dashed lines of thickness δ_m refers to the momentum boundary layer. U_∞ represents the uniform velocity outside the momentum boundary layer and T_∞ is the uniform ambient temperature of the nanofluid. $T_w(x) > T_\infty$ denotes the temperature of the horizontal surface. Recently Garoosi et al.[25, 26] have carried out a two-phase simulation of natural convection and mixed convection in the nanofluid saturated porous enclosure. Here they have treated nano particles of Cu, Al_2O_3, TiO_3 etc as a separate phase. Their results indicate that the thermophoretic effects of nano particles with high thermal conductivity are negligible. Further, their studies clearly indicate that the nanofluids such as Cu-water, without any loss of generality, may be treated as a signal phase homogeneous fluid flow. Hence in

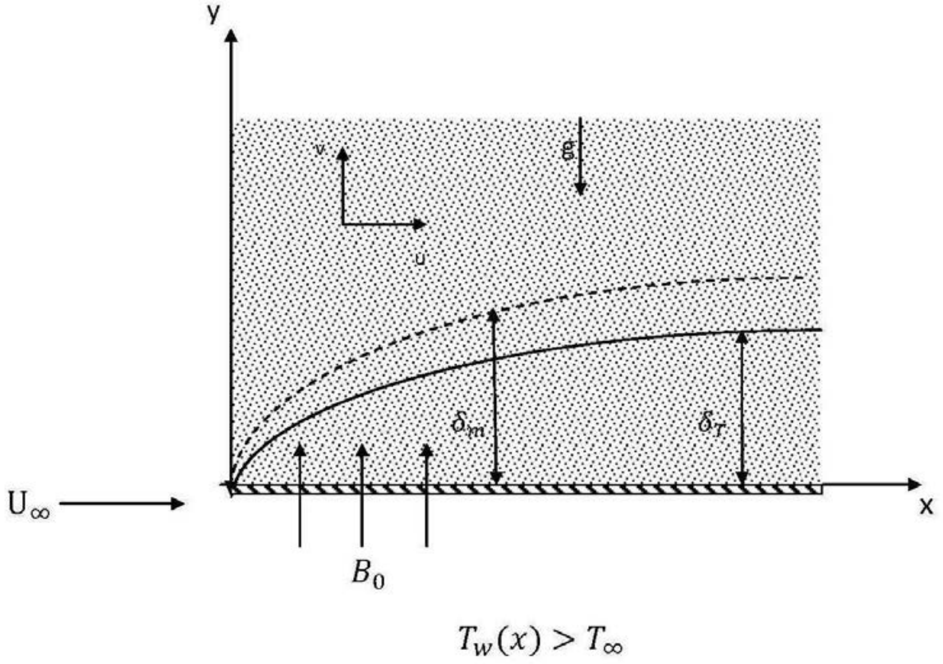

Fig. 1: Physical model and coordinate system

Table 1: Physical properties of fluid and nano-particles.

Property	Water	Cu	Al_2O_3	TiO_2
$C_p(J/kgK)$	4179	385	765	686.2
$\rho(kg/m^3)$	997.1	8933	3970	4250
k(W/m K)	0.613	400	40	8.9538
$\alpha \times 10^7(m^2/s)$	1.47	1163.1	131.7	30.7
$\beta \times 10^5(1/K)$	21	1.67	0.87	0.9

the current study, the Cu-water nanofluid is treated as a single phase homogeneous fluid. In addition, it may also be noted that the recent studies[27, 28] indicate that at low Richardson number(Ri) the effects of the drag, gravity and buoyancy effects on nano-particles distributions are negligible. Hence at low Ri Cu-water nanofluid may as well be heated as a single phase fluid. Hence in the case of Cu-water based nanofluid flow models, one can safely treat Cu-water as a single phase as considered in the current study. To obtain the volume averaged conservation equations the following assumptions are made:

- The nano-particles are considered homogeneously distributed.
- Base fluid (water) and nano-particles are in thermal equilibrium with no-slip between them.
- Nanofluid is homogeneous, Newtonian, steady and incompressible.
- The thermophysical properties of nanofluid assumed to be constant except for the density which varies according to the Boussinesq approximation.
- In the present study, the homogeneous model has been considered where the solid particles are assumed of nanometer scale size with the low concentration ($< 5\%$). In such situation, the difference between the single-phase and the two-phase model vanish. Hence, in the current model, nonoparticles are considered having 25nm diameter size with the concentration below 5% because it preserves the validity of homogeneous and single-phase models.
- Nanofluid particles are assumed sensitive to the magnetic forces.

The nanofluid saturated Darcy porous medium under local thermal equilibrium consideration is assumed to follow the Boussinesq approximation. The basic equations governing the influence of uniform transverse magnetic field on mixed convection process as depicted in Fig. 1 are given by.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\left(1 + \frac{\sigma \mu_e^2 B_0^2 K}{\mu}\right) u = -\frac{K}{\mu_{nf}} \left(\frac{\partial p}{\partial x}\right) \tag{2}$$

$$v = -\frac{K}{\mu_{nf}} \left(\frac{\partial p}{\partial y} + \rho_{nf} g\right) \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf} \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

where

$$\rho_{nf} = \rho_{nf,\infty} \left[1 - \beta_{nf}(T - T_\infty)\right] \tag{5}$$

is the Boussinesq approximation. From Eqs. (1) and (3) one obtains

$$(1 + Ha) \left(\frac{\partial u}{\partial y}\right) - \frac{\partial v}{\partial x} = -\frac{g \rho_{nf,\infty} K \beta_{nf}}{\mu_{nf}} \left(\frac{\partial T}{\partial x}\right) \tag{6}$$

where

$$Ha = \frac{\sigma \mu_e^2 B_0^2 K}{\mu}. \tag{7}$$

Hartmann number (Ha) is the ratio of electromagnetic force to the viscous force first introduced by Hartmann.

Using the boundary layer approximation and introducing the stream function ψ defined as $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$, Eqs. (6) and (4) become:

$$(1+Ha)\left(\frac{\partial^2 \psi}{\partial y^2}\right) = -\frac{g\rho_{nf,\infty} K \beta_{nf}}{\mu_{nf}}\left(\frac{\partial T}{\partial x}\right) \tag{8}$$

$$\frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} = \alpha_{nf}\left(\frac{\partial^2 T}{\partial y^2}\right). \tag{9}$$

We assume that the boundary conditions for these equations are

$$\begin{aligned} v &= 0, \quad T_w = T_\infty + Ax^{1/2} \text{ at } y = 0 \\ u &\to U_\infty \,, \; T \to T_\infty \text{ as } y \to \infty. \end{aligned} \tag{10}$$

This is a very particular boundary condition, which was chosen to get a similarity solution. It represents a particular operating condition frequently met in common applications.

Here, in view of the mixed convection process we consider slip boundary condition. Especially, since the horizontal flat plate is at the higher temperature than its ambient surrounding and that there is a forced flow over it, slip boundary condition is both natural and appropriate in this context.

For the sake of simplicity, in the following we will denote $\rho_{nf,\infty}$ by ρ_{nf}. The physical characteristics of the nanofluid are given in the Table 1[27, 42]. Following the literature [14]−[16][48]−[52] we define the density(ρ_{nf}), thermal diffusivity (α_{nf}), effective viscosity (μ_{nf}), heat capacitance $((\rho C_p)nf)$ and thermal conductivity (k_{nf}) of the nanofluid as follows:

$$\begin{aligned} &\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, \; \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}, \; \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \\ &(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s \,, \; \frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)}. \end{aligned} \tag{11}$$

Like in Lai and Kulacki[23] on introducing the following similarities variables

$$\psi = (\alpha_f U_\infty x)^{1/2} f(\eta), \; \theta(\eta) = \frac{(T - T_\infty)}{(T_w - T_\infty)}, \; \eta = \left(\frac{U_\infty x}{\alpha_f}\right)^{1/2}\left(\frac{y}{x}\right). \tag{12}$$

The partial differential Eqs.(8) and (9) are transformed to the following ordinary (similarity) equations

$$(1+Ha)f'' = 1/2(1-\phi)^{2.5}\left[1 - \phi + \phi\left(\frac{\rho_s \beta_s}{\rho_f \beta_f}\right)\right]\lambda\left(\eta\theta' - \theta\right) \tag{13}$$

$$\left(\frac{\frac{k_{nf}}{k_f}}{1 - \phi + \phi\frac{(\rho C_p)_s}{(\rho C_p)_f}}\right)\theta'' = \frac{1}{2}\left(\theta f' - f\theta'\right) \tag{14}$$

along with the corresponding boundary conditions

$$f(0) = 0 \,, \; \theta(0) = 1 \; at \; \eta = 0 \,, \; f'(\eta) \to 1 \,, \; \theta(\eta) \to 0 \; as \; \eta \to \infty \tag{15}$$

Here λ is the mixed convection parameter, which is defined as

$$\lambda = \frac{Ra_x}{Pe_x^{3/2}}. \tag{16}$$

Where $Ra_x = \frac{g\rho_f K\beta_f(T_w - T_\infty)x}{(\mu_f \alpha_f)}$ is the local Rayleigh number and $Pe_x = \frac{U_\infty x}{\alpha_f}$ is the péclet number for the porous medium.
A physical quantity of practical interest is the local Nusselt number Nu_x, which is defined as

$$Nu_x = \frac{xq_w}{k_f(T_w - T_\infty)} \tag{17}$$

where q_w is the heat flux from the surface of the plate, which is given by

$$q_w = -k_{nf}\left(\frac{\partial t}{\partial y}\right)_{y=0}. \tag{18}$$

Using Eqs. (12), (17) and (18), we obtain
$Pe_x^{-1/2} Nu_x = -\frac{k_{nf}}{k_f}\theta'(0)$.

Equations (13-15) are solved by Finite Difference Method(FDM) by using three-point fd-stencil with central difference representation for both the second and first order derivatives (f", f', θ", θ') and the result matrix is solved by direct matrix inversion using Matlab.

3 Results and discussion

Different types of nano-particles (e.g. Cu, Al_2O_3, TiO_2 etc.) may be considered in the model introduce under section 2. However, in view of space and ready availability of comparison data only the case of Cu nano-particles is considered. Further here water has been considered as base fluid. The similarity equations, denoted by the ordinary differential equations (13) and (14) together with the boundary conditions (15) are solved by the finite difference scheme introduced under section 2 to an accuracy of 10^{-10} set on the relative error of results from numerical computations. To begin with, the results from the current study are compared with those from Lai and Kulacki[23] and Rosca et al.[21] in the case of regular fluid when there is no magnetic field. Next for the case of Cu nano particles in water but in the absence of magnetic field, the results are compared with those from Arifin et al. [20] and Rosca et al.[21]. These comparison results are presented in Tab. 2-3. The data from the current study is found to be in good agreement with those from the literature. The current simulation data is generated on the validated grid system. The grid validation tests have been carried out on a series of grids consisting of 50,75,100,125 and 150 etc. number of nodal points and the grid system with 100 number of points is found to be adequate based on relative error criteria to obtain the specified accuracy. The considered grid system is also tested for relative accuracy over the η range of different parameters governing the model under consideration.

The different parameters appearing in the present mathematical model are: Ha (magnetic parameter), λ(mixed convection parameter) and ϕ(nano-particle volume fraction). Detailed parametric study is carried out for $0 \leq Ha \leq 16$, $0 \leq \lambda \leq 8$, and $0 \leq \phi \leq 0.2$. The chosen ranges are in accordance with those in the literature [24, 53]. In Fig. 2-3 the dimensionless velocity $f'(\eta)$ and temperature $\theta(\eta)$ are presented for $\lambda = 1$,

Table 2: Value of the heat transfer $-\theta'(0)$ and slip velocity $f'(0)$ for $\phi = 0$ (Newtonian fluid) and several values of λ compared with the results of Arifin et al. [20] and Rosca et al. [21].

λ	Present		Rosca et al. [21]		Arifin et al. [20]		Maximum Error (%)	
	$-\theta'(0)$	$f'(0)$	$-\theta'(0)$	$f'(0)$	$-\theta'(0)$	$f'(0)$	$-\theta'(0)$	$f'(0)$
0	0.8828	0.9996	0.8862	1	0.8862	1	0.38	0.04
0.6	1.0233	1.4716	1.0281	1.4741	1.0282	1.4741	0.47	0.16
1	1.0964	1.7438	1.102	1.7474	1.102	1.7474	0.50	0.20
2	1.242	2.3413	1.2495	2.348	1.2493	2.3471	0.60	0.28
5	1.5384	3.7835	1.5503	3.7995	1.5502	3.7992	0.76	0.42
8	1.7453	4.9733	1.7609	4.9986	1.7609	4.9984	0.88	0.50
15	2.0908	7.2976	2.1137	7.3446	2.1133	7.3422	1.08	0.63

$\phi = 0.1$ and $0 \leq Ha \leq 16$. From Fig. 2 one can notice that the boundary velocities decrease with increasing values of Ha. Clearly, application of transverse magnetic field slows down the fluid flow. At all values of Ha, velocities are larger near the leading edge of the horizontal surface and they are seen to quickly taper down to the constant values within a distance of $\eta \leq 4$ from the leading edge of the horizontal surface. From Fig. 3, one can notice that the thermal boundary layer gets less prominent or thicker with increasing Ha. This is due to the fact that magnetic field is opposing to thermal boundary layer formation as it decelerates the nano-fluid flow. At all values of Ha, temperature rapidly falls down to ambient temperature with a distance of $\eta \leq 4$.

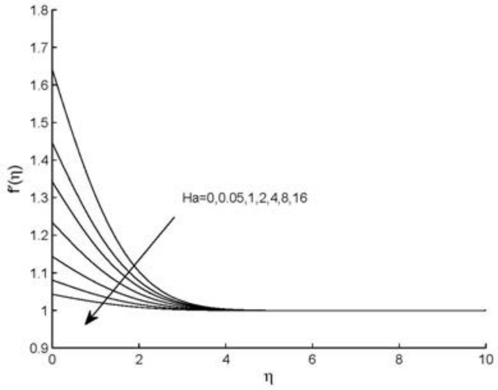

Fig. 2: Dimensionless velocity profiles $f'(\eta)$ for Ha = 0,0.05,1,2,4,8,16 when $\lambda = 1$ and $\phi = 0.1$

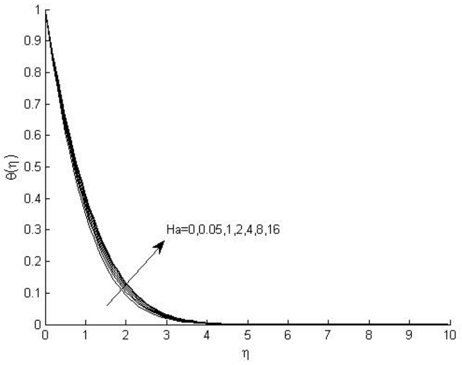

Fig. 3: Dimensionless temperature profiles $\theta(\eta)$ for Ha = 0,0.05,1,2,4,8,16 when $\lambda = 1$ and $\phi = 0.1$

Fig. 4-5 present the influence of mixed convection parameter on velocity and temperature along the horizontal surface both in the presence and absence of magnetic field for $\phi = 0.1$, Ha = 0,2 and $0 \leq \lambda \leq 8$. From Fig. 4, it is clear that irrespective of the presence of transverse magnetic field, λ is seen to favour the momentum boundary layer formation. Presence of transverse magnetic field is seen to oppose the nano fluid flow irrespective of the magnitude of mixed convection parameter. Fig. 5 presents that both in the presence and absence of transverse magnetic field thermal boundary layers get sharpen with the increasing values of mixed convection parameter. Presence of magnetic field is seen to be less favourable for the formation of sharp thermal boundary layer.

In Table 4 the data related to the influence of mixed convection parameter(λ) on local Nusselt number both in the presence and absence of the transverse magnetic field is presented. From the data, it's clear that

Table 3: Values of the heat transfer $-\theta'(0)$ and slip velocity $f'(0)$, when $\phi = 0.1$ and $\lambda = 0.1$, 0.5, 1, 2, 4, 8, 10, 20, 30,40, 50 in the case of Cu nano-particles.

λ	Present		Rosca et al. [21]		Error (%)	
	$-\theta'(0)$	$f'(0)$	$-\theta'(0)$	$f'(0)$	$-\theta'(0)$	$f'(0)$
0.1	0.7768	1.0746	0.7804	1.07	0.46	0.42
0.5	0.8472	1.3493	0.8475	1.3286	0.03	1.55
1	0.9192	1.6542	0.917	1.6175	0.23	2.26
2	1.0332	2.1868	1.0281	2.125	0.49	2.9
4	1.201	3.0829	1.1933	2.984	0.64	3.31
8	1.4336	4.5492	1.4247	4.397	0.62	3.46
10	1.5166	5.1911	1.5153	5.0178	0.08	3.45
20	1.8611	7.9317	1.8547	7.6779	0.34	3.3
30	2.1017	10.233	2.099	9.922	0.12	3.13
40	2.2944	12.2849	2.2957	11.9293	0.05	2.98
50	2.4574	14.1678	2.4628	13.7763	0.21	2.84

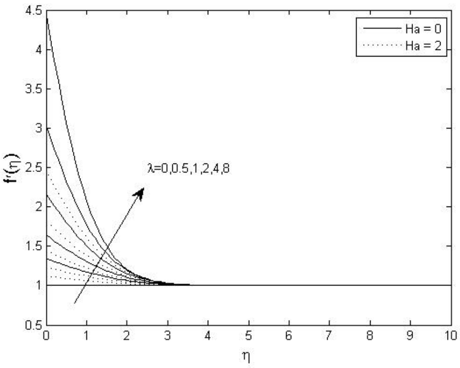

Fig. 4: Dimensionless velocity profiles $f'(\eta)$ for $\lambda = 0, 0.5, 1, 2, 4, 8$ and $\phi = 0.1$.

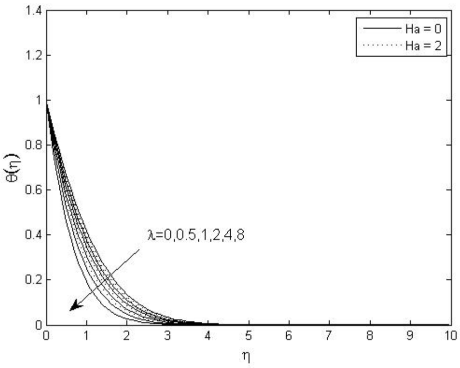

Fig. 5: Dimensionless temperature profiles $\theta(\eta)$ for $\lambda = 0, 0.5, 1, 2, 4, 8$ when $\phi = 0.1$.

irrespective of the presence of magnetic field local heat transfer is enhanced by increasing values of λ. The enhancement is relatively larger in the absence of magnetic field than in its presence. The presence of magnetic field decelerates the flow thereby the local heat and convection process.

Physically presence of nano particles enhances net thermal conductivity and the viscosity of the nano fluid. The effect of the volume fraction of nano particles (ϕ) both in the presence and absence of transverse magnetic field on velocity and temperature distribution are shown in Fig. 6-7. From Fig. 6, it is clear that the transverse magnetic field slows down the flow field irrespective of the magnitude of the volume fraction of nano-particles. Magnetic fields are opposing the mixed convection process. Close to the leading edge of horizontal surface i.e for $\eta \leq 1.5$, both in the presence and absence of magnetic field, fluid flow velocities increase with increasing values of ϕ. However, for $\eta \geq 1.5$, a reverse trend is noticed. Essentially momentum boundary layer thickness decreases near the leading edge of the plate and away from the leading edge it is seen to increase with ϕ.

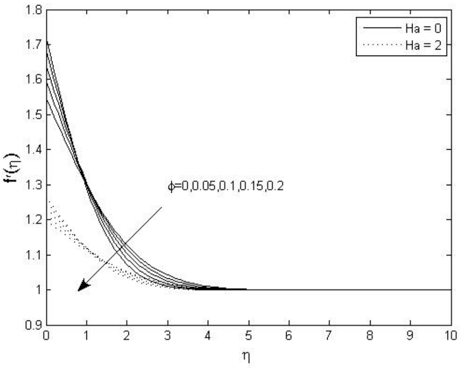

Fig. 6: Dimensionless velocity profiles $f'(\eta)$ for $\phi = 0, 0.05, 0.1, 0.15, 0.2$ when $\lambda = 1$.

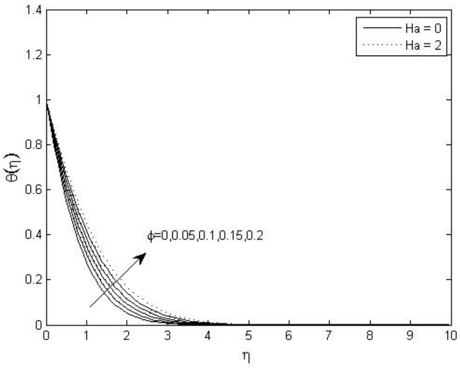

Fig. 7: Dimensionless temperature profiles $\theta(\eta)$ for $\phi = 0, 0.05, 0.1, 0.15, 0.2$ when $\lambda = 1$.

Presence of transverse magnetic field, as seen in Fig. 7, is seen to favour thermal boundary layer formulation. With the increase in nano particle volume fraction thermal boundary layer is seen to get relatively less prominent or thick irrespective of the presence of the transverse magnetic field. This is due to the increase in the net thermal conductivity of nano-fluid with increasing values of nano particle volume fraction. It is also to be noted that in all case temperatures are seen to get ambient with a distance of $\eta \leq 4$ from leading edge of the horizontal surface.

Finally, to explore the effect of different nano-particles on flow and temperature fields, simulations have been carried out with Cu, Al_2O_3 and TiO_2 nano-particles for $\lambda = 4$, $\phi = 0.1$ and $Ha = 0, 2$. The velocity and temperature variations along the horizontal surface have been presented in Figs. 8-9. Both in presence and absence of magnetic fields with varying nano-particles, while a small variation is noticed in velocities,

Table 4: Values of local Nusselt number, when $\phi = 0.1$ and $\lambda = 0.1, 0.5, 1, 2, 4, 8, 10, 20, 30,40, 50$ in the case of Cu nano-particles.

λ	$Nu_x/Pe_x^{1/2}(Ha = 0)$	$Nu_x/Pe_x^{1/2}(Ha = 4)$
0.1	1.0344	1.0068
0.5	1.1281	1.0279
1	1.2240	1.0528
2	1.3758	1.0987
4	1.5992	1.1786
8	1.9091	1.3079
10	2.0297	1.3626
20	2.4783	1.5806
30	2.7987	1.7458
40	3.0552	1.8816
50	3.2723	1.9984

no variations in temperatures and thermal boundary layer is noticed. Next to investigate the effect of varying heat transfer coefficient (k_{nf}) , thermal gradients in the form of $\frac{k_{nf}}{k_f}\theta'(\eta)$ are plotted for $\lambda = 4$, $\phi = 0.1$ and $Ha = 0, 2$ in Fig. 10. Here one can note that decreasing k_{nf} leads to a small fall in thermal gradients along the horizontal wall. This fact gets amply clear from the comparison of local Nusselt number profiles corresponding to different nano-particles when $\phi = 0.1$, $Ha = 4$ and $\lambda = 4$ as presented in Fig. 11.

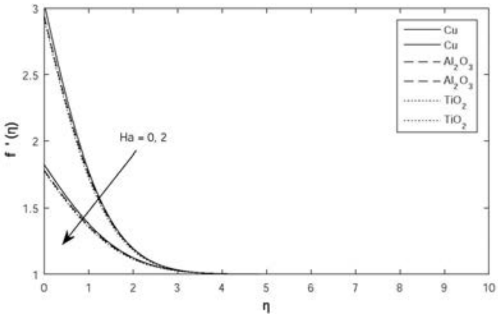

Fig. 8: Comparison of Dimensionless velocity profiles $f'(\eta)$ for Cu, Al_2O_3, TiO_2 nano-particles when $\lambda = 4$ and $\phi = 0.1$.

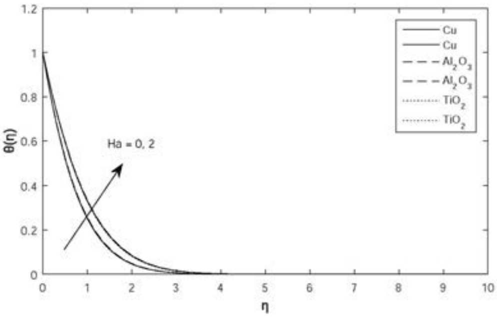

Fig. 9: Dimensionless temperature profiles $\theta(\eta)$ for Cu, Al_2O_3, TiO_2 nano-particles when $\lambda = 4$ and $\phi = 0.1$.

4 Conclusions

Under the boundary layer assumptions, using similarity transformation, the simplified coupled non-linear differential equations modelling the phenomenon of mixed convection process from a horizontal plate in a nano-fluid saturated porous medium under transverse magnetic field lead to following conclusions:

(a) Studies with Cu, Al_2O_3 and TiO_2 nano-particles indicates that while transverse magnetic field decelerates the flow near the horizontal surface the increasing thermal buoyancy forces or decreasing thermal diffusivity of nano-fluid are found to accelerate the flow. The presence of magnetic forces is not favourable for local heat transfer. But both the increasing thermal buoyancy forces and decreasing thermal diffusivity of nano-fluid lead to thermal boundary layer formation and favour local heat transfer.

(b) Increasing the volume fraction of nano-particles is seen to decelerate the flow near the leading edge of the plate but accelerate it a little away from the leading edge. Thermal convection is dominating near leading horizontal plate whereas thermal conduction, which apparently enhances fluid viscosity, is dominating away

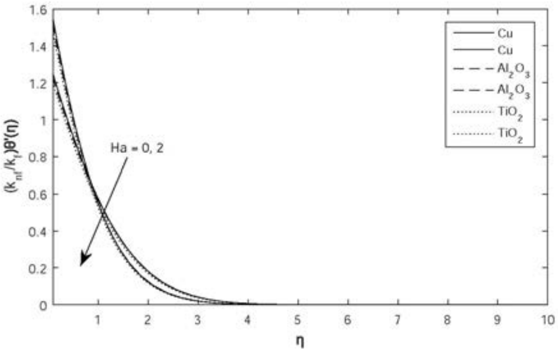

Fig. 10: Dimensionless temperature profiles $\theta(\eta)$ for Cu, Al_2O_3, TiO_2 nano-particles when $\lambda = 4$ and 0.1, $\phi = 0.1$.

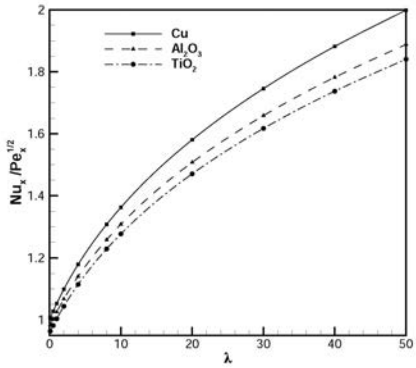

Fig. 11: The local Nusselt number profiles, when $\phi = 0.1, Ha = 4.0$ and $\lambda = 0.1, 0.5, 1, 2, 4, 8, 10, 20, 30,40,$ 50 in the case of Cu, Al_2O_3, TiO_2 nano-particles.

from the leading edge. Increasing nano-particle number leads to a loss in the thermal boundary layer and hence the local heat transfer.

(c) Nano-particles with larger heat transfer coefficients favour the local heat transfer process. In particular, fluid with Cu-nano-particles have higher local Nusselt numbers than those with Al_2O_3 or TiO_2.

Nomenclature

A	A positive constant i.e. $A > 0$;
B_0	specific heat at a constant temperature, $Jkg^{-1}K^{-1}$;
C_p	strength of the magnetic field, $Nm^{-1}A^{-1}$;
$(\rho C_p)_{nf}$	heat capacitance of the nanofluid;
g	acceleration due to gravity, ms^{-2};
k	thermal conductivity, $Wm^{-1}K^{-1}$;
K	permeability, m^2;
k_{nf}	thermal conductivity of the nanofluid;
Nu_x	local nusselt number for the porous medium;
p	pressure, Nm^2;
Pe_x	local Péclet number for the porous medium, $(= \nu_f/\alpha_f)$;
Ra_x	local Rayleigh number for the porous medium, $(g\beta_f(T_h - T_c)H^3/\alpha_f\nu_f)$;
q_w	heat flux from the surface of the plate;
T	fluid temperature, K;
T_w	surface temperature, T;
T_∞	ambient fluid temperature;
u, v	velocity components along the x and y directions, ms^{-1};
U_∞	free stream velocity, ms^{-1};
x, y	Cartesian coordinates along the plate and normal to it, respectively, m;
Ha	Hartmann number.

Greek letters

α_{nf}	thermal diffusivity of the nanofluid, m^2s;
β_{nf}	thermal expansion coefficient of the nanofluid, K^{-1};
ϕ	nano-particle volume fraction (volume of nano-particles / volume of whole mixture);
λ	constant mixed convection parameter;
η	similarity variable;
μ	dynamic viscosity, $kg\, m^{-1}s^{-1}$;

μ_e magnetic permeability, NA^{-2};
ν kinematic viscosity, $m^2 s^{-2}$;
θ dimensionless temperature;
ρ density, $kg\, m^{-3}$;
ρ_{nf} density of the nanofluid, $kg\, m^{-3}$;
ψ stream function.

$Subscripts$
f fluid fraction;
s solid fraction;
nf nanofluid.

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