ISSN 1 746-7233, England, UK
World Journal of Modelling and Simulation
Vol. 14 (2018) No. 2, pp. 100-107

Numerical study of swimming of an organism in a viscous fluid in a channel

Ranjith Maniyeri[1] *, Sangmo Kang[2]

[1] Department of Mechanical Engineering, National Institute of Technology Karnataka (NITK), Surathkal, Mangalore-575025, India
[2] Department of Mechanical Engineering, Dong-A University, Saha Gu, Busan- 604714, Republic of Korea

(*Received November 09 2016, Accepted October 30 2017*)

Abstract. In the present work, a two-dimensional immersed boundary finite volume method based computational model is developed to capture the propelling behaviour of an organism in a channel. In this work, the organism is modelled as a one-dimensional elastic filament using discrete number of immersed boundary points and the momentum and continuity equations governing the fluid flow are solved on a staggered Cartesian grid system. First of all, the computational model is validated by comparing the numerical simulation results pertinent to the swimming of an infinite sheet with that of the existing analytical results. Later, simulations are presented to investigate the organism swimming near a channel wall and, between two walls. The swimming behaviour is analysed based on mean swimming speed of the organism under three different locations inside the channel. In the present work, higher swimming speed is observed when the organism swims near the channel wall compared to the swimming far away from the channel wall. Further, it is seen that the organism swims in a centerline fashion when it is kept at the center between two channel walls whereas the organism swims towards the channel wall when it is kept close to the channel wall. It is found that the above mentioned behavior is very similar to that of sperm motion near boundaries.

Keywords: swimming of organism, mean swimming speed, immersed boundary method, staggered grid, navier-stokes equations

1 Introduction

Study of microorganisms like sperm swimming in a viscous fluid inside a channel and near a channel wall has got significant attraction in the recent times. Generally, spermatozoa move in the reproductive tract in close proximity to boundaries. The swimming motion of the sperm is fully governed by the hydrodynamic interaction between the sperm and the surrounding fluid. To explore the propulsion behaviour and the hydrodynamic interaction fully of such motion, experimental techniques are limited. Based on the work done by Taylor[13] on infinite waving swimming sheet modelled as a small body like sperm, Katz[6] studied the swimming of infinite sheet in a channel and near a channel wall using analytical perturbation method. The above theoretical analyses were not able to extend further to general cases, since in these studies the sheet was considered as a boundary of the flow[12]. Later, Slender Body Theory (SBT) and Resistive Force Theory (RFT) were used to study sperm motion[4, 5]. Further, few more studies were reported on the modelling of sperm motion using modified versions of RFT[12]. The theoretical studies are limited to low amplitude wave motions and for large amplitude finite filament (sperm model) the problem becomes too complex to solve. This led the researchers to use efficient computational methods to capture the fluid-structure interaction problems involving complex, moving flexible structures like sperm in low Reynolds number fluid flow.

Recently, immersed boundary (IB) method has proved as a superior numerical tool in handling fluid-structure interaction problems because of the easiness in grid generation, savings in memory and CPU time.

* Corresponding author. *E-mail address*: mranji1@nitk.edu.in

The IB method was developed by Charles Peskin[1, 11] to simulate the interaction of elastic flexible structures immersed in a viscous, incompressible fluid. The elastic structure is treated as a body force term in the Navier-Stokes equation and the simulation can be performed on a regular Cartesian grid system without new mesh generation and mesh movement. Using IB method Fauci and Peskin[2] studied the swimming motion of aquatic animal in which the animal is modelled as an elastic structure immersed in a viscous fluid. Later, Fauci and McDonald[3] presented two-dimensional computational model of flagellar propulsion near walls. They present simulations of finite flagella coupled mechanically to cell bodies, within channel bounded by both rigid and elastic walls. Maniyeri[7] developed a two-dimensional computational model using IB method to study the propulsive behaviour of an organism modelled as an elastic filament in a viscous fluid domain. The main aim of the work was to observe the swimming behaviour inside a periodic domain when the filament is actuated by a driving function. The model was validated with previous researcher's results.

From the literatures review, it is seen that only a few works have been reported to understand the hydrodynamic interaction and the resulting propulsion behaviour of organism driven by externally driving function. The work of Fauci and Peskin[2] addressed only the swimming behaviour of filament like organism inside a periodic fluid domain where the ends of the organism coincides with the periodic boundary. Fauci and McDonald[3] used the same IB model described in[2] and studied the propulsive dynamics of sperm like finite organism inside channels of different height. Their model included a head attached to the filament like tail which eventually leads to more computational effort. To the best of author's knowledge, immersed boundary finite volume based numerical model, which is commonly used in the field of computational fluid dynamics, to simulate the effect of channel walls on organism propulsion has not been reported. Some of these inadequacies and motivation from previous studies are the main thrust of the present work. Here, we aim to develop a two-dimensional immersed boundary model to study the propulsive behavior of a sperm like organism in a viscous fluid inside a channel and near the channel walls. Here, the organism model presented in our previous work[7] is extended to explore the propulsive behavior inside the channel and near the channel walls. In the present study, a finite organism is considered inside the channel where as in the previous study[7], the organism modeled as a filament in the form of a sine wave which mimics that of sperm is considered in a periodic domain where the ends of the filament coincides with the fluid periodic boundary. Further, the present work differs from previous models in such a way that, we do not consider head of the organism in our model and the physics of propulsion in the channel is addressed by keeping the organism at three different initial positions in the same channel instead of different channel height reported in [3]. In the present work, first of all the developed numerical model is validated by comparing the numerical simulation results pertinent to the swimming of an infinite sheet with that of the existing analytical results. Then, numerical simulations are performed to investigate swimming of an organism modelled as an elastic filament, near a channel wall and, between two walls.

The paper is arranged as follows: The organism model based on IB method, the fluid model based on the Navier-Stokes and continuity equations and the numerical procedure are discussed in Section 2. The simulation results obtained using the computational model is explained in Section 3. In Section 4, the concluding remarks of the present study are presented.

2 Mathematical modelling and numerical procedure

Fig. 1 shows the schematic diagram of the physical domain which involves a finite organism modelled as one-dimensional elastic filament surrounded by a viscous fluid at a very low Reynolds number. The filament is modelled with definite number of IB points. Each IB point is connected to its neighboring points by elastic springs. These elastic springs can undergo tension, compression and bending. Between two IB points, the spring undergoes either tension or compression and between three IB points the spring can undergo bending.

Initially, a sine wave like configuration is assumed for the filament as seen in Fig. 1. Then, a driving function is applied along the length of the filament during computation. When the driving function is applied, a sine wave will travel along the length of the filament from left to right. As a result, the various elastic springs along the length of the filament activate from the first IB point (left side) to the last IB point (right side) and provide necessary elastic forces which results in the forward motion of the filament towards left direction. The forward swimming of the filament is a result of the hydrodynamic interaction with the surrounding fluid and also due to the mechanical behavior (elastic) of the filament.

The present work use IB method proposed by Peskin[1] based on an elastic energy approach to simulate the hydrodynamic interaction of the filament in a viscous fluid. For the case of incompressible, viscous, unsteady fluid flow, the Navier-Stokes and continuity equations in its dimensional form are given by

$$\rho_f\left(\frac{\partial u^*}{\partial t^*}+\mathbf{u}^*\cdot\nabla^*\mathrm{u}^*\right)=-\nabla^*\cdot p^*+\mu\nabla^{*2}\mathbf{u}^*+\mathbf{f}^* \tag{1}$$

$$\nabla^*\cdot\mathbf{u}^*=0 \tag{2}$$

where ρ_f is the fluid density, u^* the fluid velocity, p^* the fluid pressure, μ the dynamic viscosity of the fluid, f^* the Eulerian force density and t^* the time. The above sets of equations are non-dimensionalized by introducing typical filament length l_c as the characteristic length and typical speed of the filament based on wave speed as the characteristic velocity U_c to yield in the following form

$$\rho_f\left(\frac{\partial u}{\partial t}+\mathbf{u}\cdot\nabla\mathrm{u}\right)=-\nabla p+\frac{1}{Re}\nabla^2\mathbf{u}+\mathbf{f} \tag{3}$$

$$\nabla\cdot\mathbf{u}=0 \tag{4}$$

where $Re=\frac{\omega\rho f}{\mu k^2}$ is the Reynolds number with ω being the frequency of wave travel and k the wave number.

The Eulerian force density $\mathbf{f}$ acting on the fluid is given by

$$\mathbf{f}(x,t)=\int\mathbf{F}(\mathbf{s},\mathbf{t})\delta(\mathbf{x}-\mathbf{X}(\mathbf{s},\mathbf{t}))ds \tag{5}$$

where $\mathbf{f}$ is the Lagrangian force density acting on the organism and $\delta(x-X(s,t))$ is the two-dimensional Dirac delta function.

The IB method used in the present study employs an elatic energy function $E[\Box]$ to compute the Lagrnagian force density. The elastic energy function is time dependent which also depends upon the configuration of consequtive triples of points along the boundary.

Let X_k^n represents the position of an IB point on the filament model at the n the time level. Then the Lagrangian force acting on the IB point X_k^n is computed as below

$$\mathbf{F}(\mathbf{s},\mathbf{t})=-\frac{\partial\mathbf{E}}{\partial\mathbf{x_k^n}} \tag{6}$$

The details on the determination of elastic energy function can be found in our previous paper[7]. In the present simulation, the following initial configuration for the filament is assumed.

$$y=asin(ks-\omega t) \tag{7}$$

where a is the amplitude of the wave, k is the wave number and ω is the frequency of wave travel.

At the beginning of the simulation, the elastic force between the two IB points is zero. When a driving function is applied (more details can be found in [7]) on each IB point along the length of the filament, the elastic springs connecting the IB points become activated due to change in their lengths, which induces necessary elastic force at the IB point. There will be both elastic forces due to tension/compression between the two IB points and due to bending between three consecutive triples of IB points.

A fractional step based finite volume method is employed to solve the Eqs. (3) and (4) to obtain the fluid velocity $u^{(n+1)}(x)$ and the fluid pressure $p^{(n+1)}(x)$ at the next time leve $(n+1)$. More details on fractional step based finite volume method can be found in our previous papers[8–10].

The fluid velocity $u^{(n+1)}(x)$ are then interpolated to the IB points to obtain the Lagrangian velocity of the IB point as below

$$\mathbf{U}(\mathbf{s},\mathbf{t})=\int\mathbf{u}(\mathbf{x},\mathbf{t}),\mathbf{t}\delta(\mathbf{x}-\mathbf{X}(\mathbf{s},\mathbf{t}))\mathbf{dx} \tag{8}$$

where the two-dimensional Dirac delta function is defined as,

$$\delta(\mathbf{x}) = \frac{\mathbf{1}}{\mathbf{h^2}}\emptyset(\frac{\mathbf{x}}{\mathbf{h}})\emptyset(\frac{\mathbf{y}}{\mathbf{x}}) \tag{9}$$

where (x, y) are Eulerian grid points and h is the Eulerian grid size (Refer to [7] for more details). The IB point X_k^n is then moved to its new position $X_k^{(n+1)}$ at this Lagrangian velocity according to the following equation

$$\mathbf{X_k^{n+1}} = \mathbf{X_k^n} + \mathbf{\Delta t} \cdot \mathbf{U(s, t)} \tag{10}$$

where Δt is the time step-size.

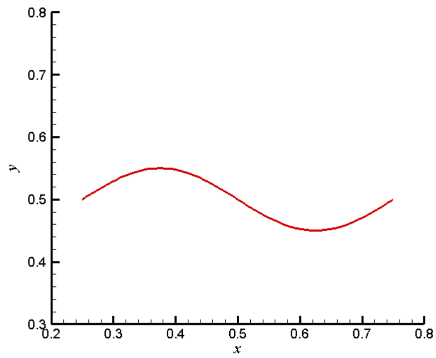

Fig. 1: Schematic diagram of the physical domain involving a finite organism modelled as one-dimensional elastic filament surrounded by viscous fluid

3 Results and discussions

The governing equations depicted in the previous section are solved in a two-dimensional square dimensionless domain of 1×1 with periodic boundary condition in x direction and no-slip boundary condition in y direction. We use 64×64 grid points with uniform Eulerian grid size. Here, the numerical simulations are performed for a filament with length, $\lambda = 0.5$, amplitude, $a = 0.05$, and wave number, $k = 4\pi$ and frequency of wave travel as $\omega = 8\pi^2$. We use 64 immersed boundary points along the axial length of the filament. The fluid is at rest at the beginning of the simulation.A numerical model based on an immersed boundary finite-volume method is developed to solve the set of equations depicted in Section 2. The dimensionless time-step size for the simulation is set as $\Delta t = 0.0002/(\pi)$. Numerical simulations are performed for eight periods for a total dimensionless time of $t = 0.6369$. The numerical results are presented in a dimensionless form and the conversion to the dimensional form can be easily obtained using the viscosity of water, $\mu = 0.01 g/cms$, characteristic length, $l_c = 0.2cm$, and characteristic velocity, $U_c = \frac{0.4}{\pi} cm/s$[2, 7]. The Reynolds number for the present study is 1.3. In this study, the propulsion behavior of the organism inside the channel bounded by rigid walls and near one of the channel walls is mainly investigated based on the mean swimming speed of the organism (U_m). The mean swimming speed of the organism is computed based on the time and space average of the displacement of all the IB points constituting the entire filament model.

The developed code is validated by studying the swimming of an infinite sheet in an unbounded fluid domain[2, 3, 7]. Then, the numerical results are compared with the asymptotic analysis done for small amplitude motion by Taylor[2] and Tuck[3]. Fig. 2 shows the validation of the developed code. The ratio of swimming velocity to wave velocity is plotted for different wave amplitudes. A close agreement of our results with that of other two researchers can be seen in the graph which proves the validation. More details can be found in our previous work[7].

We first present simulations designed to investigate the organism swimming inside a channel between two channel walls. In this case, the initial position of the organism is at the centre of the channel ($x_c = 0.25$,

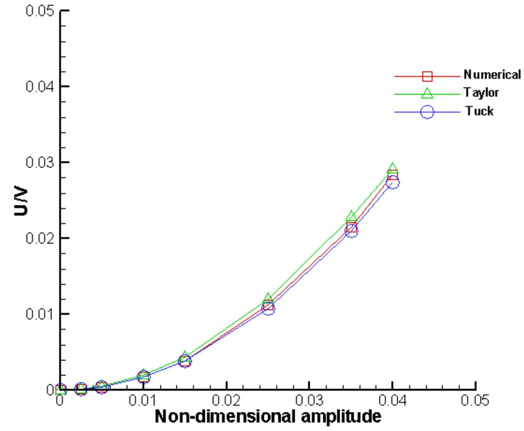

Fig. 2: Plot of swimming velocity (U)/ Wave velocity (V) for various amplitudes

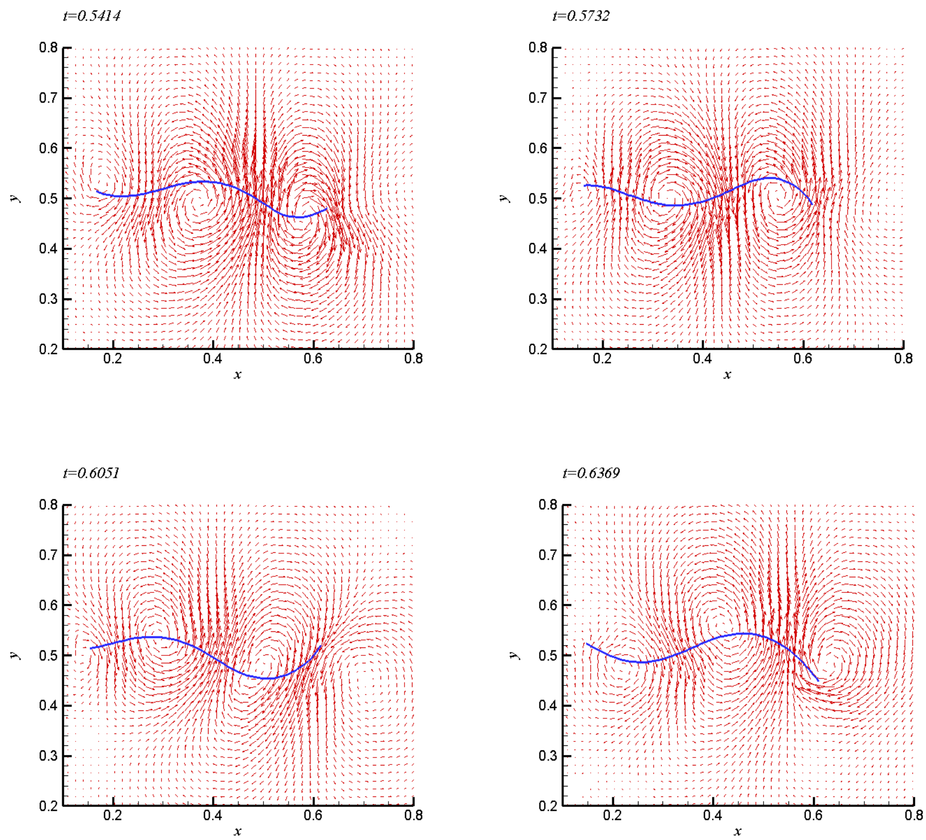

Fig. 3: Instantaneous shape and position of the organism together with flow field at four different time instants in a period

$y_c = 0.5$). Fig. 3 shows instantaneous shape and position of the the organism, modelled as the elastic filament, and the surrounding flow field at four different time instants in the last period of swimming motion. The wave goes over the filament from left to right, and the resulting swimming motion is towards the left. In this case, the dimensionless mean swimming speed (U_m) of the organism at the end of the simulation is found to be 0.1783. Fig. 4 shows the trajectory of the one of the IB point(mid point of the filament) during the simulation for eight periods. In the next case, the organism is kept at another location near to one of the channel wall (top wall) with position ($x_c = 0.25$, $y_c = 0.7$). Fig. 5 shows the final shape and position of the organism together with the flow field. In this case, it is found that the mean swimming speed of the organism is 0.1902. Hence, it can be seen that there is 6 % increase in the mean swimming speed when the organism is kept near to one of the channel walls. Fig. 6 shows the trajectory of the midpoint of the filament during simulation for eight period of time. From the figure, it can be inferred that the midpoint is moving towards the channel wall which indicates that the organism is attracted slightly towards the channel wall.

Finally, numerical simulation is performed for the case in which the organism is kept at very close to the top channel wall with the position set as ($x_c = 0.25$, $y_c = 0.85$). In this position, the organism is very close to the top wall in the beginning of the simulation. Fig. 7 shows the final shape and position of the organism together with the flow field at the end of the simulation time. Through simulation, it is computed that the mean swimming speed at the end of the simulation is 0.2534. Hence, it is interesting to note that there is 30 % increase in the mean swimming speed of the organism when it swims near to one of the channel walls (top wall, $y_c = 0.85$), compared with those when swimming between the walls (center of the channel, $y_c = 0.5$). Also, it can be seen that there is a 25% increase in the mean swimming speed of the organism when it swims near the top wall, compared with those when swimming between the walls at another initial location (between the center of the channel and the top wall, $y_c = 0.7$). Fig. 8 shows the trajectory of the midpoint of the filament during the simulation. From the figure it can be seen that the organism moves towards the channel wall during the forward motion. The motion of sperm like organism near the boundary has studied experimentally and numerically by previous researchers[3, 12]. In all the cases, it is found that there exists an attraction between the boundary and the sperm which results in the motion of the sperm towards the boundary. In the numerical studies of Fauci and McDonald[3] and Quin et al. [12] sperm with head is considered. With a simplified two-dimensional immersed boundary finite volume method based model, without considering the head of the organism, we have well captured such propelling behavior of the sperm like organism inside a channel and very close to one of the channel walls. These observations captured numerically are well in agreement with the real swimming behavior of spermatozoa near the boundaries of the reproductive tract. This proves the reliability of our developed numerical model.

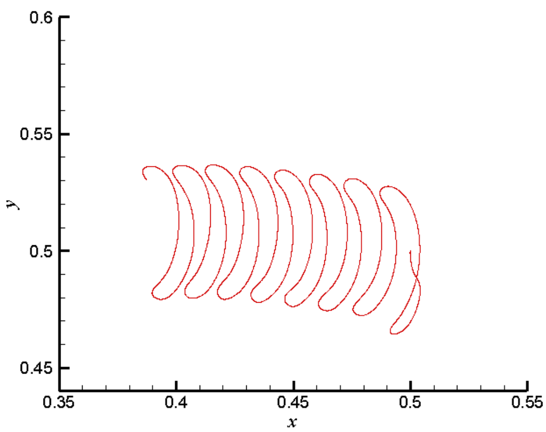

Fig. 4: Trajectory of the one of the IB point(mid point of the filament) from $t = 0.0$ to $t = 0.6369$

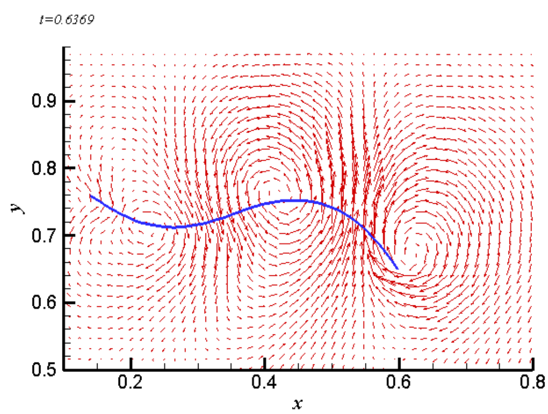

Fig. 5: Final shape and position of the organism together with the flow field for the case of y_c=0.7) from $t = 0.0$ to $t = 0.6369$

4 Conclusions

This paper presents a twoCdimensional computational model based on an immersed boundary finite volume method to study the swimming behavior of an organism modelled as an elastic filament in a viscous fluid inside a channel between two channel walls and close to one of the channel walls. Accordingly, an immersed boundary model of the filament is presented and the governing equations describing the fluid flow are solved on a staggered Cartesian grid system using fractional step based numerical scheme. The swimming behavior of the organism is analyzed based on mean swimming speed.

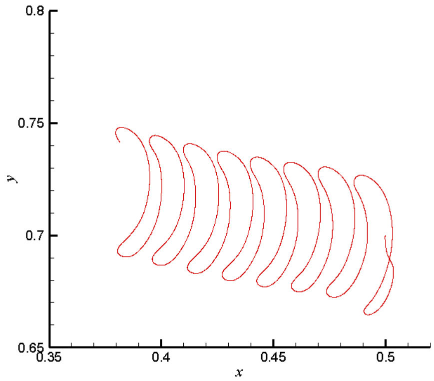

Fig. 6: Trajectory of the one of the IB point(mid point of the filament) from $t = 0.0$ to $t = 0.6369$ for the case of $y_c = 0.7$

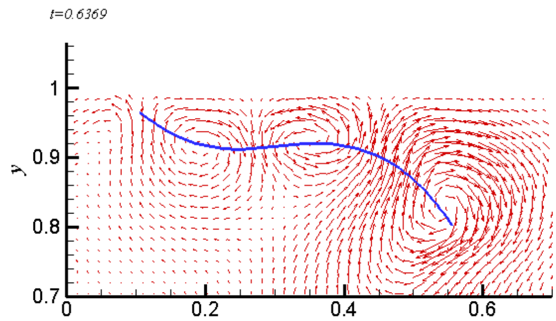

Fig. 7: Final shape and position of the organism together with the flow field for the case of $y_c = 0.85$

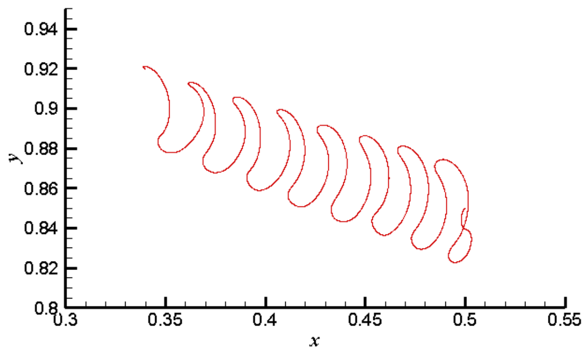

Fig. 8: Trajectory of the one of the IB point(mid point of the filament) from $t = 0.0$ to $t = 0.6369$ for the case of $y_c = 0.85$

We performed numerical simulations using the developed model to explore the swimming behavior at three different locations inside a channel. It is found that when the organism swims near the channel wall, it possess higher swimming speed compared to the swimming far away from the channel walls. Also, it is further noticed that, when the organism is kept to close to one of the channel walls, during swimming, it tends to swim with very high speed towards the channel wall rather than swimming in a center line fashion. It is worth to mention that with the help of a simplified two-dimensional computational model, we are able to capture fluid-structure interaction of sperm like organism in a viscous fluid effectively. The developed numerical model can be further extended to explore the hydrodynamic interaction between multiple organisms in a viscous fluid inside the channel and mutual interaction between the organisms in the fluid.

References

[1] C. S. Peskin. The immersed boundary method. *Acta Numerica*, 2002, **11**: 479–517.

[2] L. J. Fauci, C. S. Peskin A computational model of aquatic animal locomotion. *Journal of Computational Physics*, 1988, **77**(1): 85–108.

[3] L. J. Fauci, Mc. Mcdonald. Sperm motility in the presence of boundaries. *Bulletin of Mathematical Biology*, 1995, **57**(5): 679–699.

[4] J. Gray, G. J. Hancock. The propulsion of sea-urchin spermatozoa. *Journal of Experimental Biology*, 1955, **32**(4): 620C622.

[5] G. J. Hancock. The self-propulsion of microscopic organisms through liquids. *Proceedings of the Royal Society A*, 1953, **217**(1128): 96–121.

[6] D. F. Katz. On the propulsion of micro-organisms near solid boundaries. *Journal of Fluid Mechanics*, 1974, **64**(1): 33–49.

[7] R. Maniyeri. Simulation of propulsive dynamics of an organism in a viscous fluid using an immersed boundary finite volume method. *Applied Mechanics & Materials*, 2014, **592-594**(2): 1945–1949.
[8] R. Maniyeri, S. Kang. Numerical study on the rotation of an elastic rod in a viscous fluid using an immersed boundary method. *Journal of Mechanical Science & Technology*, 2012, **26**(5): 1515–1522.
[9] R.Maniyeri, S. Kang et al. Numerical study on bacterial flagellar bundling and tumbling in a viscous fluid using an immersed boundary method. *Applied Mathematical Modelling*, 2014, **38**(14): 3567–3590.
[10] R. Maniyeri, Y. K. Suh et al. Numerical study on the propulsion of a bacterial flagellum in a viscous fluid using an immersed boundary method. *Computers & Fluids*, 2012, **62**(12): 13–24.
[11] C. S. Peskin. Numerical analysis of blood flow in the heart. *Journal of Computational Physics*, 1977, **25**(3): 220–252.
[12] F. H. Quin, W. X. Huang, H. J. Sung Simulation of small swimmer motions driven by tail/flagellum beating. *Computers & Fluids*, 2012, **55**(4): 109–117.
[13] G. I. Taylor. Analysis of the swimming of microscopic organisms. *Proceedings of the Royal Society A Mathematical Physical & Engineering Sciences*, 1951, **209**(209): 447–461.