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MHD peristaltic flow of blood through porous medium with slip effect in the presence of body acceleration

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Abstract. This paper deals with the investigation of MHD non-Newtonian blood flow through a porous medium with periodic body acceleration and wall slip condition. We have considered the blood as a couple-stress fluid. In order to negate the inertia effects, low Reynolds number and long wave length approximations are assumed. Expressions for axial velocity and pressure gradient are obtained analytically. The pressure rise and frictional forces have been obtained numerically. The results obtained are explained and displayed through graphical illustrations. This study is of appreciably importance in blood flow to understand the effects in micro-circulatory system. Body acceleration parameter can be used to enhance the velocity and pumping rate of the blood flow.

Keywords: peristaltic flow, couple-stress fluid, porous medium, magnetic field, body acceleration, wall slip condition

1 Introduction

Peristaltic pumping of physiological fluids takes place because of the propagation of progressive transverse waves along the walls of the channel. It has drawn serious attention of investigators working in the area of physiological fluid dynamics. The mechanism of peristalsis is responsible for the transport of various physiological fluids such as transport of urine from the kidney to the bladder, swallowing of food through oesophagus, movement of chyme in the gastro-intestinal tract, flow of bile, transport of spermatozoa in the ducts efferents of the male reproduction tract, movement of ovum in the fallopian tube, cilia movement, circulation of blood in small blood vessels. The mechanism of peristaltic transport has found ample industrial applications like sanitary fluid transport, transport of corrosive fluids, transport of noxious fluid in the nuclear industries, heart lung machines, dialysis and blood pump machines. Mekheimer[20] investigated the effect of induced magnetic field on peristaltic flow of a couple stress fluid. Bhatti et al.[6] studied the blood flow of sisko fluid with endoscope analysis and titanium magneto-nanoparticles. Clot blood model with heat transfer analysis on peristaltically induced motion of particle fluid suspension with variable viscosity has been analyzed by Bhatti et al.[7]. Effects of heat transfer on the peristaltic transport of MHD Newtonian fluid with variable viscosity has been studied by Nadeem and Noreen[23]. Furthermore, Nadeem and Noreen[24] studied the heat transfer effects on peristatic flow of MHD fluid with partial slip. More recently, the peristaltic flow of MHD fluids has been studied Bhatti et al.[10, 11, 31]. Bhatti and Zeeshan[5] studied the heat and mass transfer analysis on peristaltic flow of particle fluid-suspension with slip effects. Bhatti et al.[8] investigated the effects of magnetohydrodynamics on entropy generation for peristaltic blood flow with casson model.

Peristaltic transport through porous medium has got considerable attention in the last few decades due to its enormous applications in biological and engineering fields. Dharmendra[29] studied the peristaltic flow

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of a Newtonian fluid through permeable medium. Elshehawey et al.[13] investigated the peristaltic transport in an asymmetric channel through porous medium. Mekheimer and Al-Arabi[21] studied non-linear peristaltic transport of MHD flow through a porous medium. Mekheimer[19] studied the effect of porous medium with inclined planer channel on non-linear peristaltic transport flow. Recently, Ellahi et al.[12] studied the blood flow of Prandtl fluid through a tapered stenosed arteries in permeable walls with magnetic field. Noreen Akbar et al.[1] described the influence of induced magnetic field and heat flux with the suspension of carbon nanotubes for the peristaltic flow in a permeable channel. Bhatti and Abbas[4] studied the simultaneous effects of slip and MHD on peristaltic blood flow of Jeffrey fluid model through a porous medium.

Some studies related to the couple-stress fluid are very beneficial in understanding various physiological processes as it has a potential to describe rheological complex fluids such as blood, lubricants containing a tiny amount of polymer additive and synthetic fluids[14, 32]. Stokes[28] was the first to develop the theory of couple-stress and its quite suitable to understand blood flow in micro-vessels by taking size of the erythrocytes into account. Recently Nadeem and Akbar[25] have studied the effect of induced magnetic field on the peristaltic flow of a couple-stress fluid in an asymmetric channel. Alemachy and Radhakrishnamacharya[3] studied the dispersion of a solute in peristaltic motion of a couple-stress fluid in the presence of magnetic field. Sobh[26] investigated the peristaltic flow of a couple-stress fluid in uniform and non-uniform channels with slip velocity. Sreenadh et al.[27] studied the effect of MHD on the couple-stress fluid in a porous medium. Khan et al.[2] obtained the approximation solution of couple-stress fluid with expanding or contracting porous channel.

The human body is often subjected to body accelerations in many situations of day to day life while in aeroplanes, traveling, body movements in sports activities etc. Many researchers studied the effect of body acceleration on the human body under different situations. Griffin[15] investigated the effects of body acceleration on human body. He described the diversity and complexity of human responses to vibration. Hiatt[16] gave some reports on human acceleration. Arntzenius et al. [17] described that the body acceleration applied synchronously with the heartbeat can have a substantial effect on the circulation.

In view of the above, an attempt has been made to study the effect of magnetic field and slip condition on the peristaltic flow of a couple-stress fluid under the influence of periodic body acceleration. The highly non-linear differential equations are solved by simply using the low Reynolds number and high wavelength approximation approach.

2 Mathematical formulation

We consider the peristaltic flow of blood as a couple-stress non-Newtonian fluid through a porous channel in presence of magnetic field of strength B_0. The induced magnetic field is neglected in comparison with the applied magnetic field. Assumptions of low Reynolds number and large wavelength are adopted. The geometry of peristaltic flow through porous medium in dimensionless form is as shown in Fig. 1, which is given by

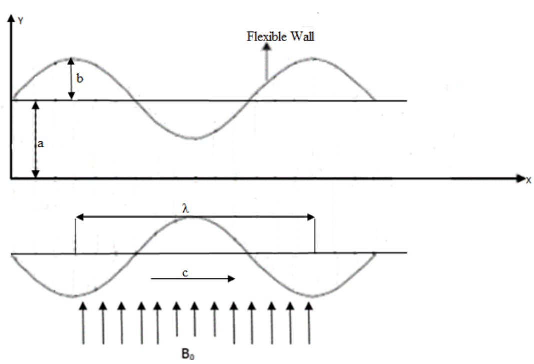

Fig. 1: The Geometry of the problem

$$h' = a + b\sin\frac{2\pi}{\lambda}(x' - ct'). \tag{1}$$

where $a, b, \lambda, x, c,$ and t are the half width of the channel, amplitude, wavelength, axial co-ordinate, wave velocity, and time respectively. The governing equations of motion for incompressible couple-stress fluid through porous medium in the presence of magnetic field with periodic body acceleration are given by

$$\frac{\partial u'}{\partial x'}+\frac{\partial v'}{\partial y'}=0, \tag{2}$$

$$\rho\left(\frac{\partial u'}{\partial t'}+u'\frac{\partial u'}{\partial x'}+v'\frac{\partial u'}{\partial y'}\right)=-\frac{\partial p'}{\partial x'}+\mu\nabla^2u'-\eta\nabla^4u'-\mu\frac{u'}{K'}-\sigma B_0^2u'+\rho g\cos\theta, \tag{3}$$

$$\rho\left(\frac{\partial v'}{\partial t'}+u'\frac{\partial v'}{\partial x'}+v'\frac{\partial v'}{\partial y'}\right)=-\frac{\partial p'}{\partial y'}+\mu\nabla^2v'-\eta\nabla^4v'-\mu\frac{v'}{K'}, \tag{4}$$

where ρ, u', v', p', μ, η, K, B_0, g, θ are the fluid density, axial velocity, transverse velocity, pressure, viscosity, material constant associated with couple-stress, permeability parameter, magnetic field, amplitude of body acceleration, phase difference respectively. Also,

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2},\ \ \nabla^4=\nabla^2(\nabla^2). \tag{5}$$

Introducing the dimensionless variables and parameters as follows:

$$\begin{aligned}&x=\frac{x'}{\lambda},y=\frac{y'}{a},u=\frac{u'}{c},t=\frac{ct'}{\lambda},v=\frac{v'}{c\delta},h=\frac{h'}{a}=1+\phi\sin(2\pi x),p=\frac{p'a^2}{\mu c\lambda},\\&K=\frac{K'}{a^2},R_e=\frac{\rho ca}{\mu},\delta=\frac{a}{\lambda},\alpha=a\sqrt{\frac{\mu}{\eta}},M^2=\frac{\sigma}{\mu}B_0^2a^2,G=\frac{\rho ga^2}{\mu c}.\end{aligned} \tag{6}$$

where $\delta, \phi, R_e, \alpha,$ and G are wave number, amplitude ratio, Reynolds number, couple-stress fluid parameter and body acceleration parameter respectively. By using Eq. (6) in Eqs. (2)–(4) and applying the low Reynolds number and high wavelength approximation approach we arrive at

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{7}$$

$$\frac{\partial p}{\partial x}=\frac{\partial^2u}{\partial y^2}-\frac{1}{\alpha^2}\frac{\partial^4u}{\partial y^4}-\frac{u}{K}-M^2u+G\cos\theta, \tag{8}$$

$$\frac{\partial p}{\partial y}=0. \tag{9}$$

The boundary conditions related to the flow are

$$\text{Slip condition: } u=-\beta\frac{\partial u}{\partial y}\ \ at\ \ y=h. \tag{10}$$

Here, β denotes the slip parameter.

$$\text{Symmetric condition: } \frac{\partial u}{\partial y}=0\ \ at\ \ y=0, \tag{11}$$

$$\text{Vanishing couple-stresses, } \frac{\partial^2u}{\partial y^2}=0\ \ at\ \ y=h,\ \frac{\partial^3u}{\partial y^3}=0\ \ at\ \ y=0. \tag{12}$$

3 Method of solution

The solution of differential Eq. (8) obtained with the help of mathematics software MATHEMATICA is given as

$$u=C_1e^{yA_1}+C_2e^{-yA_1}+C_3e^{yA_2}+C_4e^{-yA_2}+\frac{K(-\frac{\partial p}{\partial x}+G\cos\theta)}{1+KM^2} \tag{13}$$

Now to eliminate the constants C_1, C_2, C_3 and C_4, we have to apply the appropriate boundary conditions given in Eqs. (10)–(12) in Eq. (13). Hence the expression for velocity is given by

$$u = \frac{K(\frac{-\partial p}{\partial x} + G\cos\theta)}{N}\left[1 + \frac{1}{L}\left\{\frac{A_2^2\cosh(A_1 y)}{\cosh(A_1 h)} - \frac{A_1^2\cosh(A_2 y)}{\cosh(A_2 h)}\right\}\right]. \tag{14}$$

where,

$$N = 1 + KM^2, \tag{15}$$

$$L = (A_1^2 - A_2^2) + \beta A_1^2 A_2 \tanh(A_2 h) - \beta A_2^2 A_1 \tanh(A_1 h), \tag{16}$$

$$A_1 = \frac{1}{\sqrt{2}}\sqrt{\alpha^2 - \frac{\sqrt{K\alpha^2(-4 - 4KM^2 + K\alpha^2)}}{K}}, \tag{17}$$

$$A_2 = \frac{1}{\sqrt{2}}\sqrt{\alpha^2 + \frac{\sqrt{K\alpha^2(-4 - 4KM^2 + K\alpha^2)}}{K}}. \tag{18}$$

Let us consider the flow through a wave frame (X', Y') with velocity c as the steady flow and treating the flow in the laboratory frame (x', y') as unsteady flow. The relation between flow at both frames can be written as

$$x' = X' - ct', \; y' = Y', \; u' = U' - c, \text{ and } v' = V'. \tag{19}$$

where (u', v') and (U', V') are velocity components in the wave and laboratory frames of reference, respectively.

The volumetric flow rate in the laboratory frame is expressed as

$$Q(X', t') = \int_0^H U'(X', Y', t')dY', \tag{20}$$

where $H = h'(X', t')$, which in the wave frame can be expressed as

$$q'(x') = \int_0^H u'(x', y')dy', \tag{21}$$

where $H = h'(x')$. Using Eqs. (19) and (21) in Eq. (20), we obtain

$$Q(X', t') = q'(x') + ch'(X', t'). \tag{22}$$

The expression for time averaged volumetric flow rate with the time period T at a fixed position X' is given by

$$Q'(X') = \frac{1}{T}\int_0^T Q(X', t')dt'. \tag{23}$$

By using Eq. (22) in Eq. (23), we obtain

$$Q'(X') = q'(x') + ca. \tag{24}$$

The dimensionless mean flow rates $\overline{Q}$ (in the laboratory frame) and q (in the wave frame) are defined as

$$\overline{Q} = \frac{Q'}{ca}, \quad q = \frac{q'}{ca}. \tag{25}$$

Applying the dimensionless quantities from Eq. (25) into Eq. (24), we have

$$\overline{Q} = q + 1 = Q + 1 - h, \tag{26}$$

where

$$Q = \int_0^h u \, dy. \tag{27}$$

Using the velocity expression from Eq. (14) in Eq. (27), the volumetric flow rate Q can be written as

$$Q = \frac{\left(KG\cos\theta - K\frac{\partial p}{\partial x} \right)}{N} \left[\frac{hLA_1A_2 + A_2^3 \tanh(A_1 h) - A_1^3 \tanh(A_2 h)}{L(A_1 A_2)} \right]. \tag{28}$$

By applying Eq. (26) into Eq. (28),the pressure gradient can be written as

$$\frac{\partial p}{\partial x} = G\cos\theta - \frac{1}{K} \left(\frac{(\overline{Q} - 1 + h)(1 + KM^2)\, LA_1A_2}{h\, LA_1\, A_2 + A_2^3 \tanh(A_1 h) - A_1^3 \tanh(A_2 h)} \right). \tag{29}$$

The pressure difference (Δp) and frictional force (F), across one wave length are respectively given by

$$\Delta p = \int_0^1 \frac{\partial p}{\partial x}\, dx, \tag{30}$$

$$F = \int_0^1 h \left(-\frac{\partial p}{\partial x} \right) dx. \tag{31}$$

4 Results and discussion

The aim of this section is to discuss the numerical and computational results with the help of graphical illustrations. The influence of different emerging physical parameters have been discussed with fussy prominence. Fig. 2 illustrates the variation of axial velocity for different values of the Magnetic field parameter (M), slip parameter (β), couple-stress fluid parameter (α), body acceleration parameter (G), amplitude ratio (ϕ), and the permeability parameter (K). From Fig. 2(a), it is observed that the axial velocity decreases by increasing the value of Magnetic field parameter. The transverse magnetic field gives rise to a resistive force called as the Lorentz Force. This force slows down the fluid motion. These results are same as noted in Refs. [18, 22]. This result has an essential role in large number of industrial applications, particularly in favor to solidification processes such as casting and semiconductor single crystal growth applications. In these claims, as the liquids experience solidification, fluid flow and turbulence occur in the solidifying liquid pool and have critical conclusions on the product quality control. The practice of magnetic fields has effectively been applied to monitoring melt convection in solidification systems. From Fig. 2(b), it is observed that the axial velocity increases by increasing the values of slip parameter (β). This results holds good with the results obtained by Devakar et al.[9]. From Fig. 2(c), it is depicted that the axial velocity increases by increasing the couple stress fluid parameter. This results is in good agreement with the one obtained by Tripathi and Anwar[30]. From Fig. 2(d-f), it is observed that the behavior of body acceleration parameter, amplitude ratio and porous parameter is similar when compared with the effect of couple stress fluid parameter.

The variation of pressure gradient for different values of physical parameters has been illustrated in Fig. 3. From Fig. 3(a), it is observed that by increasing the magnetic field parameter, the pressure gradient increases in the narrow part of the channel $x \in (0.6, 0.9)$ while an opposite is observed in the wider part $x \in (0, 0.6) \cup (0.9, 1)$ of the channel. In Fig. 3(b)-(d), it is illustrated that the effect of porous parameter, couple-stress fluid parameter and slip parameter on the pressure gradient is totally opposite when compared with the magnetic field parameter. In Fig. 3(e), it is observed that by increasing the body acceleration parameter, the pressure gradient increases in the the narrow part as well as in the wider part of the channel.

The Variation of Pressure rise (Δp) over one wavelength, friction forces (F) across one wavelength against the average volume flux (Q) has been illustrated in Figs. 4 and 5 for different values of M, K, β, G and α. It is determined that the behavior of pressure rise and volume flow rate is quite opposite . In these figures the region is divided into four parts: peristaltic pumping region $(\Delta p > 0, Q > 0)$, retrograde pumping region $(\Delta p > 0, Q < 0)$, augmented region $(\Delta p < 0, Q > 0)$ and free pumping region $(\Delta p = 0)$. The region in which

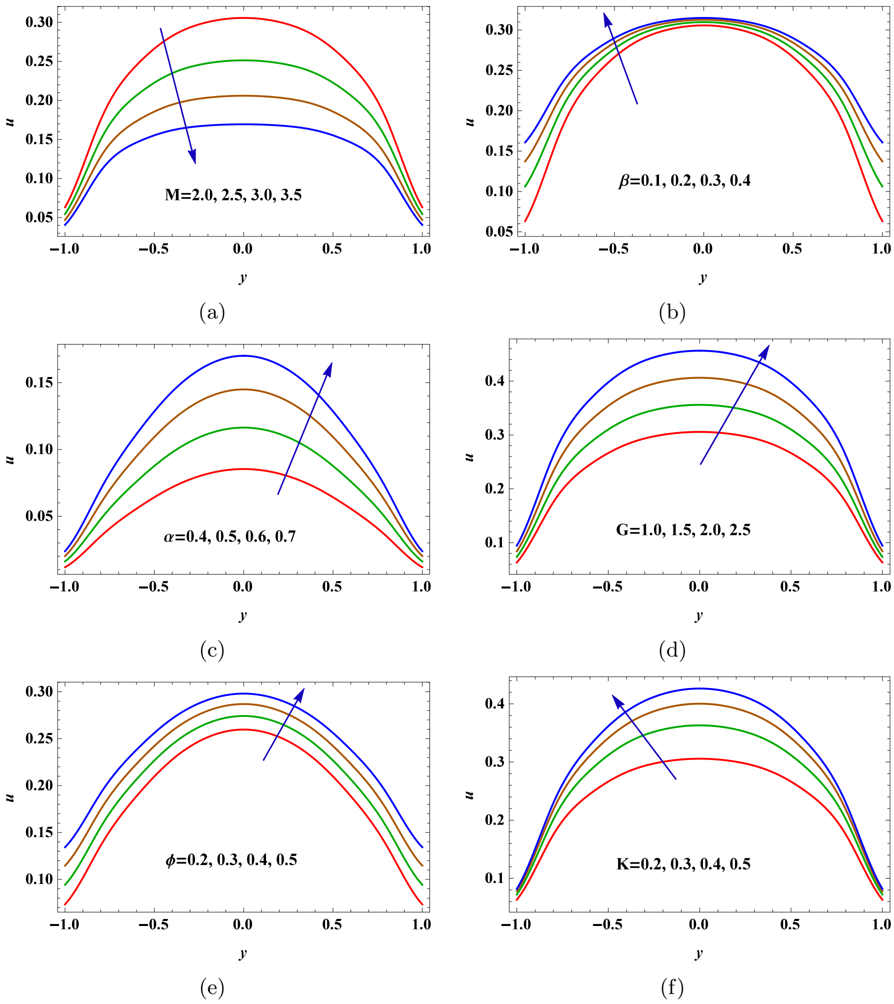

Fig. 2: Variation of M, K, α, β, G over the axial velocity (u) with respect to y with $\theta = 0$, $\frac{dp}{dx} = -2, \phi = 0.2, x = 1$

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$\Delta p > 0, Q > 0$ is known as the peristaltic pumping region. In this region the peristaltic wave overwhelms the pressure rise and propagates the fluid in the direction of its propagation. The region in which $\Delta p > 0$ and $Q < 0$ is called a retrograde pumping region. In this locale, the stream is inverse to the course of the peristaltic movement. The region in which $\Delta p < 0, Q > 0$ is known as augmented pumping region or co-pumping region. In this region, the negative pressure rise enhances the flow due to the peristalsis of the walls. In the free pumping region, where $\Delta p = 0$, the flow is totally prompted by the peristalsis of the walls.

In Fig. 4(a), it is clear that with an increase in magnetic field parameter the pumping rate increases up to a critical value of (Q) and decreases after a critical value of (Q) in the retrograde pumping region while it decreases in the peristaltic, free and co-pumping regions. Fig. 4(b)-(c) illustrates that the permeability parameter and slip parameter have opposite behavior as compared to the magnetic field parameter (M), that is, the pumping rate decreases in the retrograde pumping region, peristaltic and free pumping region while it increases in the augmented region. This result is in good agreement with the results obtained by Tripathi and Anwar[30]. From Fig. 4(e), it is observed that the pumping rate shows negligible change by changing the values of the couple-stress fluid parameter (α). Fig. 5(a)-(e) describes the variation of frictional forces against the volume flow rate for diverse rising parameters. The frictional forces precisely have an inverse conduct when contrasted with the pressure rise.

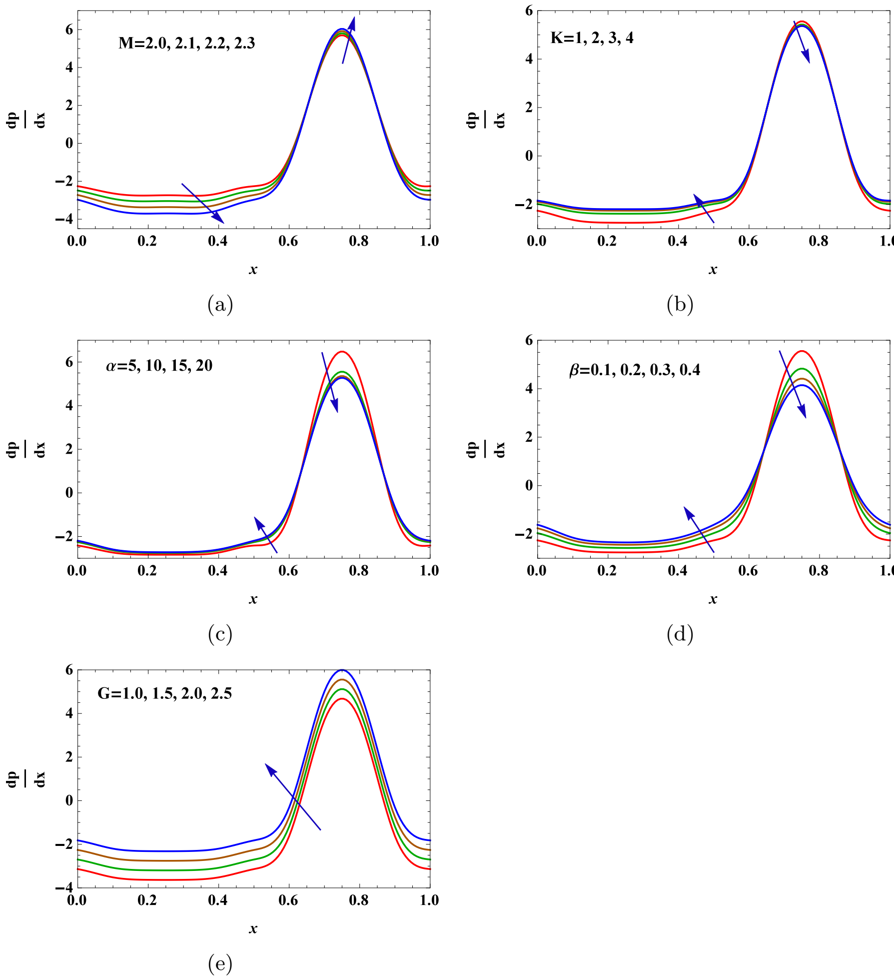

Fig. 3: Variation of M, β, m, Θ, D_a and N on the pressure gradient (g) with respect to x with $\theta = 0.5, Q = 0.5, \phi = 0.6$

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5 Conclusion

In this article, we have studied the effect of slip parameter and body acceleration on the peristaltic flow of a couple-stress fluid in presence of the magnetic field. This study can be extended by including the effects of thermal radiation, heat source/sink and chemical reaction. The key findings are summarized below:

- By increasing the magnetic field parameter, the axial velocity decreases.
- The behavior of magnetic field and body acceleration on the axial velocity is totally opposite.
- Couple-stress fluid parameter and permeability parameter enhances the velocity.
- The influence of couple-stress fluid parameter, slip parameter and permeability parameter on the pressure gradient is quite opposite when compared with the magnetic field.
- pressure rise and volumetric flow rate are reverse to each other.
- By increasing the body acceleration parameter, the pumping rate enhances.
- The behavior of volumetric flow rate on pressure rise is totally opposite when compared with the frictional forces.

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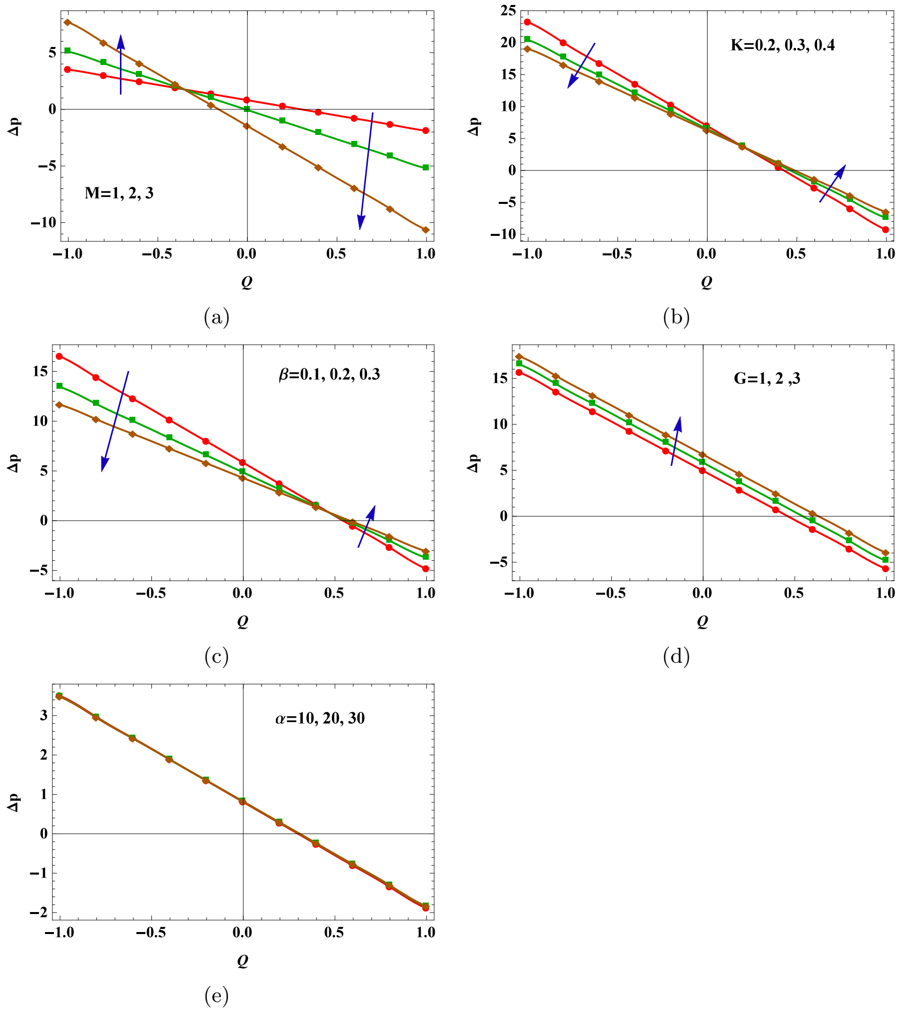

Fig. 4: Variation of M, K, β, G and α on the Pressure rise Δp with respect to Q with $\theta = 0.5, \phi = 0.6$

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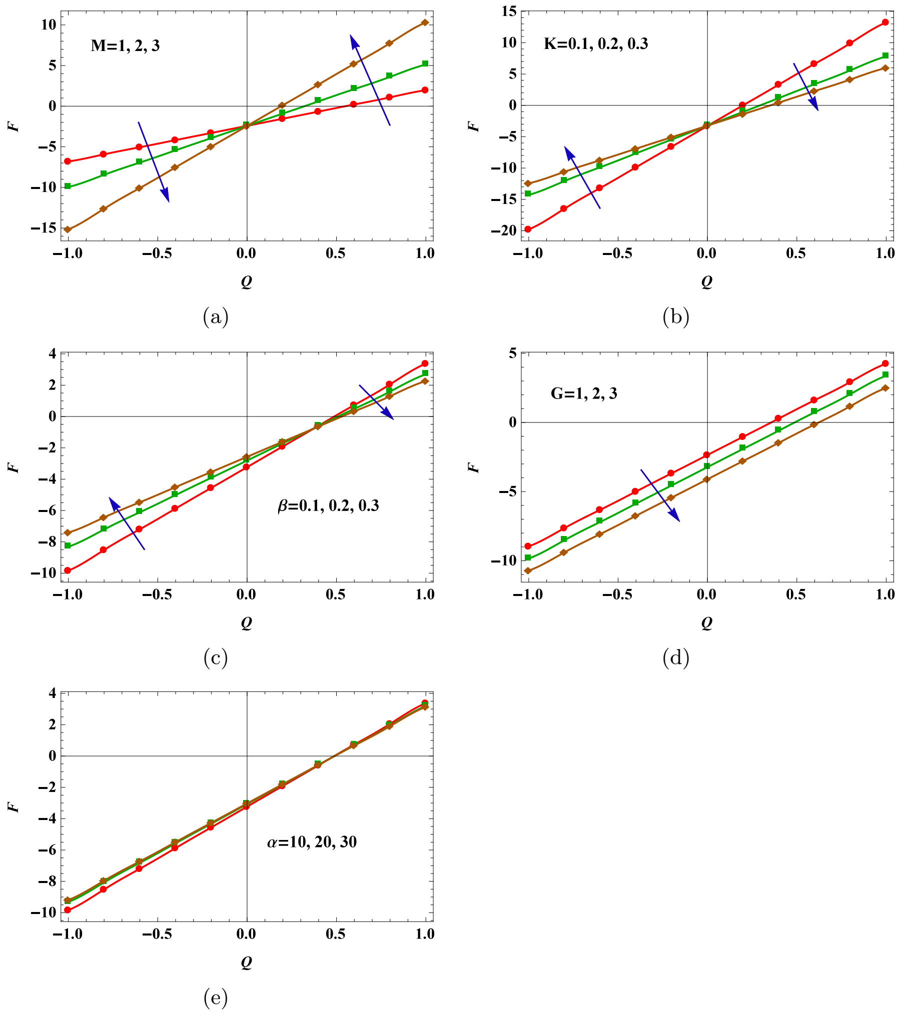

Fig. 5: Variation of M, K, β, G and α on the Frictional Force (F) with respect to Q with $\theta = 0.5, \phi = 0.6$

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