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Two-dimensional flow of a Jeffery fluid in a dilating and squeezing porous channel

Naveed Ahmed[1], Umar Khan[2], Syed Tauseef Mohyud-Din[3*]

Department of Mathematics, Faculty of Sciences, HITEC University, Taxila Cantt, Pakistan

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Abstract. In this article, the problem of a two-dimensional Jeffery fluid flow between a dilating and squeezing porous channel is considered. Using the suitable similarity transforms, the nonlinear partial differential equations governing the flow, are reduced to a highly nonlinear ordinary differential equation. The resulting equation is then solved by using Adomian's Decomposition Method (ADM). The physical effects of wall expansion ratio and other involved parameters are demonstrated graphically. For the expanding walls, increasing values of viscoelastic parameter β result in a back follow. A very important phenomenon about the permeation Reynolds number Re and the similarity transform is discussed which has been overlooked by all the previous authors who have contributed their work in these type of problems. The case of Newtonian fluid is a special case of our study (for $\lambda_1 = 0$ and $\beta = 0$[10, 21]). This study has a great deal of importance in bio-fluid mechanics and many other industrial areas.

Keywords: nonlinear Equations, two-dimensional flow, Jeffery fluid, Adomian's Decomposition Method (ADM), expanding and contracting walls

1 Introduction:

In the study of fluid transport in biological organisms, we deal with the flow between permeable walls that may expand or contract. This phenomenon really has a great importance in medical and biological sciences. Oozing through porous walls is an important phenomenon in blood flow, which contribute a lot in inter-body transportation of different substances and it may affect the entire health of the living organism.

The flows of such type have also a wide range of industrial applications. Gold miners, use the machinery in which sludge is carried away from the mine to cleansing chambers with the help of vessels which may expand/contract and have the porous walls.

The pioneer work related to the steady flow solutions in channels with porous boundaries can be traced back to Berman[9] in 1953. He introduced a method to reduce Navier Stokes equations into a single ordinary differential equation on the basis of the assumption that the suction or injection through the porous bodies is uniform. His study opened a new door for many researchers who later worked on the guidelines provided by him. In the beginning, the studies were restricted to limiting cases, or, to obtain the solution to the problem over restricted fluid domains. Such assumptions were imposed, which in fact were not appropriate to approximate real life flows. However, with the passage of time many researcher tried to overcome those hurdles and now more flexible work can be seen in literature regarding these types of flows.

The flow through dilating and squeezing permeable gaps has been a research area of many researchers[3, 4, 10, 12, 21]. Many analytical and numerical techniques [kuch references yahan b adjust ho sktay hian] have been used to determine the flow profile to simulate bio-fluid flow.

In real life, fluids inside living organisms are not Newtonian normally. Si et. al[31] studied the flow of a viscoelastic fluid through a porous channel with expanding and contracting walls. It is worth mentioning that

* Corresponding author. Tel.: *E-mail address*: syedtauseefs@hotmail.com

there is no single model available that can incorporate all the properties of every non-Newtonian fluid. Different models have been proposed for different kinds of non-Newtonian fluids[13–15, 17, 22–24, 27, 32]. To understand the transportation of materials inside the body further, we need to examine the flows of non-Newtonian fluids. This is for, here, we present this work. It takes a non-Newtonian fluid model (Jeffery Fluid model) in to consideration. A number of research works have been carried out using the said model[1, 7, 16, 25, 26] dealing with the different kinds of geometries and situations. Up to the best of our knowledge the flow of a Jeffery fluid in a porous channel with deformable walls is yet to be investigated.

In this manuscript, we have examined unsteady, laminar flow of an isothermal and incompressible Jeffery fluid inside a channel having an in-finite length. The similarity transformations, both in space and time have been used to reduce the governing equations into a highly nonlinear ordinary differential equation. A well-known analytical method, Adomian's decomposition method (ADM) has been employed to obtain the solution of the resulting equation. Many researchers[2, 5, 8, 18–20, 28–30] successfully employed ADM for solving some highly nonlinear ordinary and partial differential equations. Particularly, for flows between expanding and contracting walls, recently, Asghar et. al[6] and Chen et. al.[11] have accurately used ADM to approximate the flow of a Newtonian and viscoelastic fluid respectively. On the same grounds we have employed ADM to analyze the velocity profile and results are shown with the help of graphs.

According to our literature survey, we are the first ones investigating the flow of a Jeffery fluid in a channel with dilating and squeezing porous walls. This problem can be extended for many different situations and reader may find these directions useful for future work. Some of our recommendations include, heat and mass transfer analysis of the same problem. Effects of nonlinear thermal radiation on the flow under consideration is another open direction. The problem can also be extended for non-Newtonian nanofluids.

Organization of this paper is as follows: The paper is comprised of five section out of which the first section is dedicated to explain the brief background and history of the problem. It also highlights the contributions made by us, and, some future directions. The second section of the paper describes the problem's mathematical formulation. The third section includes the solution of the problem using ADM. The second last section of the paper is dedicated to discuss the influence of involved parameters on velocity profile. A comprehensive discussion aided with the graphical material has been provided to understand the flow behavior. The fifth and final part of the paper summarizes the discoveries made. At the end a bibliography is also included crediting the contributions made by other respectable researchers in this field of research.

2 Problem formulation

Consider the laminar, incompressible and isothermal flow of a Jeffrey fluid in a rectangular duct of infinite length. It contains two permeable walls, from where the fluid can enter or exit during the successive expansions or contractions. The distance between the porous walls is much smaller as compared to the width and length of the channel. It enables us to neglect the effect of lateral walls and take the rectangular cross-section as shown in Fig. 1. There is an equal permeability $-V_w$ at the both walls and they expand or contract uniformly at a time-dependent rate $\dot{a}(t) = \dfrac{da}{dt}$. We choose a coordinate system with origin at the centerline of the channel as shown in the following Fig. 1.

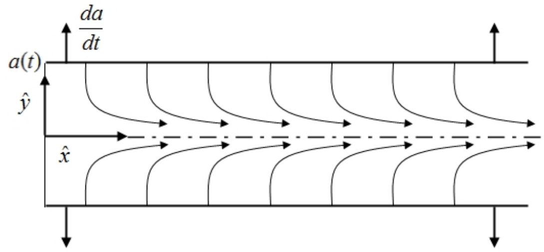


Fig. 1: Two-dimensional domain with expanding or contracting walls

The equations governing the flow are expressed as:

$$\nabla \cdot \boldsymbol{V} = 0,, \tag{1}$$

$$\rho \left[\frac{DV}{Dt} \right] = \nabla . \boldsymbol{T} \tag{2}$$

For Jeffrey fluid the constitutive equation are [1, 7, 16, 25, 26],

$$\boldsymbol{T} = -p\boldsymbol{I} + \boldsymbol{S}, \tag{3}$$

where

$$\boldsymbol{S} = \frac{\mu}{1+\lambda_1} \left(\boldsymbol{A_1} + \lambda_2 \frac{D\boldsymbol{A_1}}{Dt} \right), \tag{4}$$

in which p denotes the pressure, $\boldsymbol{I}$, $\boldsymbol{S}$ represent identity and extra stress tensor respectively, μ is dynamic viscosity, λ_1 is ratio of relaxation and retardation times, λ_2 denotes retardation time and $\frac{D}{Dt}$ is the material derivative, whereas

$$\boldsymbol{A_1} = \nabla \boldsymbol{V} + (\nabla \boldsymbol{V})^T. \tag{5}$$

In (5), ∇ and $(\cdot)^T$ are gradient and transpose operators respectively.

The boundary equations for the problem are:

$$\hat{u}(\hat{x}, a) = 0, \hat{v}(\hat{x}, a) = -V_w = -\frac{\dot{a}}{c}, \tag{6}$$

$$\left(\frac{\partial \hat{u}}{\partial \hat{y}} \right)_{\hat{x}=0} = 0, \hat{v}(\hat{x}, 0) = 0, \hat{u}(0, \hat{y}) = 0. \tag{7}$$

$c = -\frac{\dot{a}}{V_w}$ is coefficient of suction and injection i.e. the wall permeability.

The governing Eqs. (1) and (2) with help of Eqs. (3) and (4) reduce to:

$$\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} = 0, \tag{8}$$

$$\rho \left(\frac{\partial \hat{u}}{\partial \hat{t}} + \hat{u} \frac{\partial \hat{y}}{\partial \hat{x}} + \hat{v} \frac{\partial \hat{u}}{\partial \hat{y}} \right) = -\frac{\partial \hat{p}}{\partial \hat{x}} + \frac{\partial \boldsymbol{S}_{\hat{x}\hat{x}}}{\partial \hat{x}} + \frac{\partial \boldsymbol{S}_{\hat{y}\hat{x}}}{\partial \hat{y}}, \tag{9}$$

$$\rho \left(\frac{\partial \hat{v}}{\partial \hat{t}} + \hat{u} \frac{\partial \hat{y}}{\partial \hat{x}} + \hat{v} \frac{\partial \hat{v}}{\partial \hat{y}} \right) = -\frac{\partial \hat{p}}{\partial \hat{y}} + \frac{\partial \boldsymbol{S}_{\hat{x}\hat{y}}}{\partial \hat{x}} + \frac{\partial \boldsymbol{S}_{\hat{y}\hat{y}}}{\partial \hat{y}}. \tag{10}$$

In above equations, $\boldsymbol{S}_{\hat{x}\hat{x}}$, $\boldsymbol{S}_{\hat{x}\hat{y}}$, $\boldsymbol{S}_{\hat{y}\hat{x}}$ and $\boldsymbol{S}_{\hat{y}\hat{y}}$ are the components of extra stress tensor related to Jeffery fluid. Their expression are;

$$\boldsymbol{S}_{\hat{x}\hat{x}} = \frac{\mu}{1+\lambda_1} \left[2\frac{\partial \hat{u}}{\partial \hat{x}} + \lambda_2 \left\{ 2 \left(\frac{\partial \hat{u}}{\partial \hat{x} \partial t} + \hat{u} \frac{\partial^2 \hat{u}}{\partial \hat{x}^2} + \hat{v} \frac{\partial^2 \hat{u}}{\partial \hat{x} \partial y} \right) \right\} \right], \tag{11}$$

$$\boldsymbol{S}_{\hat{x}\hat{y}} = \boldsymbol{S}_{\hat{y}\hat{x}} = \frac{\mu}{1+\lambda_1} \left[\frac{\partial \hat{u}}{\partial \hat{y}} + \frac{\partial \hat{v}}{\partial \hat{x}} + \lambda_2 \left(\frac{\partial^2 \hat{u}}{\partial \hat{y} \partial t} + \frac{\partial^2 \hat{v}}{\partial \hat{x} \partial t} + \hat{u} \frac{\partial^2 \hat{u}}{\partial \hat{y} \partial \hat{x}} + \hat{v} \frac{\partial^2 \hat{u}}{\partial \hat{y}^2} \right) \right], \tag{12}$$

$$\boldsymbol{S}_{\hat{y}\hat{y}} = \frac{\mu}{1+\lambda_1} \left[2\frac{\partial \hat{v}}{\partial \hat{y}} + \lambda_2 \left\{ 2 \left(\frac{\partial^2 \hat{v}}{\partial \hat{y} \partial t} + \hat{u} \frac{\partial^2 \hat{v}}{\partial \hat{y} \partial \hat{x}} + \hat{v} \frac{\partial^2 \hat{v}}{\partial \hat{y}^2} \right) \right\} \right] \tag{13}$$

We can now introduce the mean flow vorticity Ω as follows

$$\Omega = \left(\frac{\partial \hat{v}}{\partial \hat{x}} - \frac{\partial \hat{u}}{\partial \hat{y}} \right). \tag{14}$$

Using Eq. (14) in Eqs. (9) and (10) with Eqs. (11) - (13), we may obtain the following equation in terms of vorticity after eliminating the terms involving pressure

$$\rho\left(\Omega_t+\hat{u}\frac{\partial\Omega}{\partial\hat{x}}+\hat{v}\frac{\partial\Omega}{\partial\hat{y}}\right)=\frac{\mu}{1+\lambda_1}\left\{\frac{\partial^2\Omega}{\partial\hat{x}^2}+\frac{\partial^2\Omega}{\partial\hat{y}^2}+\lambda_2\left(\begin{array}{l}\frac{\partial^3\Omega}{\partial\hat{x}^2\partial t}+\frac{\partial^3\Omega}{\partial\hat{y}^2\partial t}+\frac{\partial^2\hat{u}}{\partial\hat{x}^2}\frac{\partial\Omega}{\partial\hat{x}}+\hat{u}\frac{\partial^3\Omega}{\partial\hat{x}^3}\\+\hat{v}\frac{\partial^3\Omega}{\partial\hat{x}^2\partial\hat{y}}+\hat{u}\frac{\partial^3\Omega}{\partial\hat{y}^2\partial\hat{x}}+\hat{v}\frac{\partial^3\Omega}{\partial\hat{y}^3}\\+\frac{\partial^2\hat{v}}{\partial\hat{x}\partial\hat{y}}\frac{\partial\Omega}{\partial\hat{x}}+2\frac{\partial\hat{v}}{\partial\hat{y}}\frac{\partial^2\Omega}{\partial\hat{y}^2}+2\frac{\partial\hat{u}}{\partial\hat{y}}\frac{\partial^2\Omega}{\partial\hat{y}\partial\hat{x}}\\+2\frac{\partial\hat{u}}{\partial\hat{x}}\frac{\partial^2\Omega}{\partial\hat{x}^2}+2\frac{\partial\hat{v}}{\partial\hat{x}}\frac{\partial^2\Omega}{\partial\hat{x}\partial\hat{y}}\end{array}\right)\right\}. \tag{15}$$

Conservation of mass enables us to establish a similarity transformation as under,

$$\Omega=\nu\hat{x}a^{-3}\hat{f}_{yy}(y,t),\quad y=\frac{\hat{y}}{a},\hat{u}=\nu\hat{x}a^{-1}\hat{f}_y(y,t),\quad \hat{v}=-\nu a^{-1}\hat{f}(y,t), \tag{16}$$

Putting Eq. (16) in Eq. (15) we have

$$\begin{array}{l}\hat{f}_{yyy}+(1+\lambda_1)\left[\alpha\left(3\hat{f}_{yy}+y\hat{f}_{yyy}\right)-\hat{f}_y\hat{f}_{yy}+\hat{f}\hat{f}_{yyy}\right]\\-\beta\left[\alpha\left(5\hat{f}_{yyyy}+y\hat{f}_{yyyy}\right)+\hat{f}_y\hat{f}_{yyyy}+\hat{f}\hat{f}_{yyyy}-2\hat{f}_{yy}\hat{f}_{yyy}\right]=0,\end{array} \tag{17}$$

where, α is wall expansion ratio defined by $\alpha=\frac{a\dot{a}}{\upsilon}$ and $\beta=\frac{\lambda_2}{\rho a^2}$ is the viscoelastic parameter. The expansion ratio is considered negative for contraction and positive for expansion. We may also obtain $\hat{f}_{yyt}=\hat{f}_{yyyyt}=0$ by setting wall expansion ratio to be a constant or a quasi-constant with respect to time [4, 10, 12, 21].

The corresponding boundary conditions become:

$$\hat{f}_{yy}(0,t)=0,\ \hat{f}(0,t)=0,\hat{f}_y(1,t)=1,\hat{f}(1,t)=\mathrm{Re}. \tag{18}$$

$\mathrm{Re}=\dfrac{\alpha V_w}{\upsilon}$ here, is the permeation Reynolds number and is taken to be positive for injection and negative for suction.

Introducing non-dimensional variables we have,

$$u=\frac{\hat{u}}{\dot{a}},\quad v=\frac{\hat{v}}{\dot{a}},\quad x=\frac{\hat{x}}{a},\quad F=\frac{\hat{f}}{\mathrm{Re}}, \tag{19}$$

or,

$$u=x\frac{F'}{c},\quad v=-\frac{F}{c},\quad c=\frac{\alpha}{\mathrm{Re}}. \tag{20}$$

Under above assumptions, Eq. (17) can be written as,

$$F^{iv}+(1+\lambda_1)[\alpha(3F''+yF''')-\mathrm{Re}(F'F''-FF''')]-\alpha\beta(5F^{iv}+yF^{v}) \tag{21}$$

$$-\beta\mathrm{Re}\left[F'F^{iv}+FF^{v}-2F''F'''\right]=0. \tag{22}$$

The auxiliary conditions are,

$$F''(0)=0,\ F(0)=0,F'(1)=0,\ F(1)=1. \tag{23}$$

One can easily observe that Eq. (21) leads to Newtonian fluid case for $\lambda_1=0$ and $\beta=0$[10, 21].

3 Solution procedure

Following the standard procedure of ADM[4] and applying it on Eq. (21) in operator form we have,

$$\begin{array}{l}LF^{iv}=-(1+\lambda_1)\left[\alpha\left(3F''+yF'''\right)-\mathrm{Re}\left(F'F''-FF'''\right)\right]-\alpha\beta\left(5F^{iv}+yF^{v}\right)\\\quad-\beta\mathrm{Re}\left[F'F^{iv}+FF^{v}-2F''F'''\right],\end{array} \tag{24}$$

where $L = \dfrac{d^4}{dy^4}$.

Applying L^{-1} on both sides of above equation we obtain,

$$F(y) = F(0) + yF'(0) + \frac{y^2}{2!}F''(0) + \frac{y^3}{3!} - L^{-1}\left\{ \begin{array}{l} (1+\lambda_1)\left[\alpha\left(3F'' + yF'''\right) - \text{Re}\left(F'F'' - FF'''\right)\right] \\ -\alpha\beta\left(5F^{iv} + yF^{v}\right) - \beta\text{Re}\left[F'F^{iv} + FF^{v} - 2F''F'''\right] \end{array} \right\} = 0, \tag{25}$$

where $L^{-1} = \int_0^y \int_0^y \int_0^y \int_0^y (.).dydydydy$. Using (23) we get,

$$F(y) = Ay + B\frac{y^3}{3!} - L^{-1}\left\{ \begin{array}{l} (1+\lambda_1)\left[\begin{array}{l} \alpha\left(3F'' + yF'''\right) \\ -\text{Re}\left(F'F'' - FF'''\right) \end{array} \right] \\ -\alpha\beta\left(5F^{iv} + yF^{v}\right) \\ -\beta\text{Re}\left[F'F^{iv} + FF^{v} - 2F''F'''\right] \end{array} \right\} = 0. \tag{26}$$

Here, $A = F(0)$ and $B = F''(0)$ which may be determined by using boundary conditions. Adomian's method assumes the solution in the form

$$F(y) = \sum_{n=0}^{\infty} F_n(y), \tag{27}$$

and

$$NF(y) = \sum_{n=0}^{\infty} A_n. \tag{28}$$

A_n are called Adomian's polynomials and can be calculated using several ways, while $NF(y)$ denotes the nonlinear terms involved in Eq. (26) and

$$\begin{aligned} NF = &(1+\lambda_1)\left[\alpha\left(yF'''\right) - \text{Re}\left(F'F'' - FF'''\right)\right] - \alpha\beta\left(yF^{v}\right) \\ &- \beta\text{Re}\left[F'F^{iv} + FF^{v} - 2F''F'''\right], \end{aligned} \tag{29}$$

rest of the linear terms are:

$$RF = (1+\lambda_1)\left[\alpha\left(3F''\right)\right] - \alpha\beta\left(5F^{iv}\right). \tag{30}$$

Hence Eq. (26) takes the form,

$$F(y) = Ay + B\frac{y^3}{3!} - L^{-1}\left\{RF_n + \sum_{n=0}^{\infty} A_n\right\}. \tag{31}$$

Some of the Adomian's polynomials are given by:

$$\begin{aligned} A_0 = &(1+\lambda_1)\left[\alpha\left(yF_0{'''}\right) - \text{Re}\left(F_0{'}F_0{''} - F_0F_0{'''}\right)\right] - \alpha\beta\left(yF_0{}^{v}\right) \\ &- \beta\text{Re}\left[F_0{'}F_0{}^{iv} + F_0F_0{}^{v} - 2F_0{''}F_0{'''}\right], \end{aligned} \tag{32}$$

$$\begin{aligned} A_1 = &(1+\lambda_1)\left[\alpha\left(yF_1{'''}\right) - \text{Re}\left(F_0{'}F_1{''} + F_1{'}F_0{''} - F_0F_1{'''} - F_1F_0{'''}\right)\right] - \alpha\beta\left(yF_1{}^{v}\right) \\ &- \beta\text{Re}\left[F_0{'}F_1{}^{iv} + F_1{'}F_0{}^{iv} + F_0F_1{}^{v} + F_1F_0{}^{v} - 2\left(F_0{''}F_1{'''} + F_1{''}F_0{'''}\right)\right], \end{aligned} \tag{33}$$

$$\begin{aligned} A_2 = &(1+\lambda_1)\left[\alpha\left(yF_2{'''}\right) - \text{Re}\left(F_0{'}F_2{''} + F_1{'}F_1{''} + F_2{'}F_0{''} - F_0F_2{'''} - F_1F_1{'''} - F_2F_0{'''}\right)\right] \\ &- \alpha\beta\left(yF_2{}^{v}\right) - \beta\text{Re}\left[\begin{array}{l} F_0{'}F_2{}^{iv} + F_1{'}F_1{}^{iv} + F_2{'}F_0{}^{iv} + F_0F_2{}^{v} + F_1F_1{}^{v} \\ +F_2F_0{}^{v} - 2\left(F_0{''}F_2{'''} + F_1{''}F_1{'''} + F_2{''}F_0{'''}\right) \end{array} \right]. \end{aligned} \tag{34}$$

The remaining linear terms are given by:

$$RF_0 = (1+\lambda_1)\left[\alpha\left(3F_0{''}\right)\right] - \alpha\beta\left(5F_0{}^{iv}\right),$$
$$RF_1 = (1+\lambda_1)\left[\alpha\left(3F_1{''}\right)\right] - \alpha\beta\left(5F_1{}^{iv}\right), \tag{35}$$
$$RF_2 = (1+\lambda_1)\left[\alpha\left(3F_2{''}\right)\right] - \alpha\beta\left(5F_2{}^{iv}\right),$$
$$\vdots \tag{36}$$

First few terms of the solution are,

$$\begin{aligned} F_0(y) =& Ay + B\frac{y^3}{3!}, \\ F_1(y) =& \frac{1}{2520}\mathrm{Re}(1+\lambda_1)B^2y^7 - \frac{1}{5}\left(\frac{1}{6}\alpha(1+\lambda_1)B + \frac{1}{12}\beta\mathrm{Re}B^2\right)y^5; \\ F_2(y) =& \frac{1}{2494800}\mathrm{Re}^2(1+\lambda_1)^2B^3y^{11} \\ & - \frac{1}{9}\left(\frac{1}{2520}\alpha(1+\lambda_1)^2\mathrm{Re}B^2 + \frac{1}{336}\mathrm{Re}(1+\lambda_1)B\left(-\frac{1}{30}\alpha(1+\lambda_1)B\right)\right)y^9 + \cdots \end{aligned} \tag{37}$$

The series solution can now be obtained as under:

$$\begin{aligned} F(y) =& F_0(y) + F_1(y) + F_2(y) + \cdots \\ F(y) =& \frac{1}{2520}\mathrm{Re}(1+\lambda_1)B^2y^7 \\ & - \frac{1}{5}\left(\frac{1}{6}\alpha(1+\lambda_1)B + \frac{1}{12}\beta\mathrm{Re}B^2\right)y^5 - \frac{1}{2494800}\mathrm{Re}^2(1+\lambda_1)^2B^3y^{11} \\ & - \frac{1}{9}\left(\frac{1}{2520}\alpha(1+\lambda_1)^2\mathrm{Re}B^2 + \frac{1}{336}\mathrm{Re}(1+\lambda_1)B\left(-\frac{1}{30}\alpha(1+\lambda_1)B\right)\right)y^9 + \cdots \end{aligned} \tag{38}$$

The constants A and B can be determined by using (23) later on.

4 Results and discussions

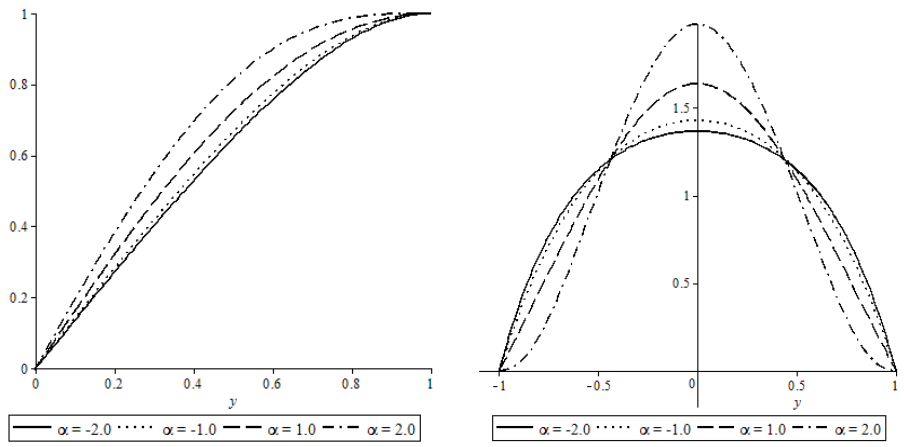
First of all it is important to observer that the similarity transforms we used earlier (19), (20) become undefined when the permeation Reynolds number (Re) is set equal to 0, which physically means that the flow is only due to suction or injection. When Re is taken to be 0, the similarity transforms loses its sense and we cannot obtain the resulting ordinary differential equation.

The following graphs show the analytical simulation of the flow behavior for different ranges of emerging parameters. The effect of Reynolds number Re viscoelastic parameter β, λ_1 and the wall expansion ration α on $F(y)$ and $F'(y)$ is discussed and demonstrated graphically. Figs. 2 and 3 show the effects of wall expansion ratio α on $F(y)$ and $F'(y)$ respectively. Because of the equal permeability at both the walls, maximum velocity is observed at the center of the channel. Also, the flow is symmetric and the maximum of velocity becomes larger for contracting walls and smaller for the expanding walls.

The effect of varying permeation Reynolds number Re on the flow is shown in Figs. 4 and 5. It shows that the flow is symmetric and again the maximum velocity is seen near the center of channel. Increasing Re results in an increased velocity for contracting walls, while, it is quite opposite for the expanding walls.

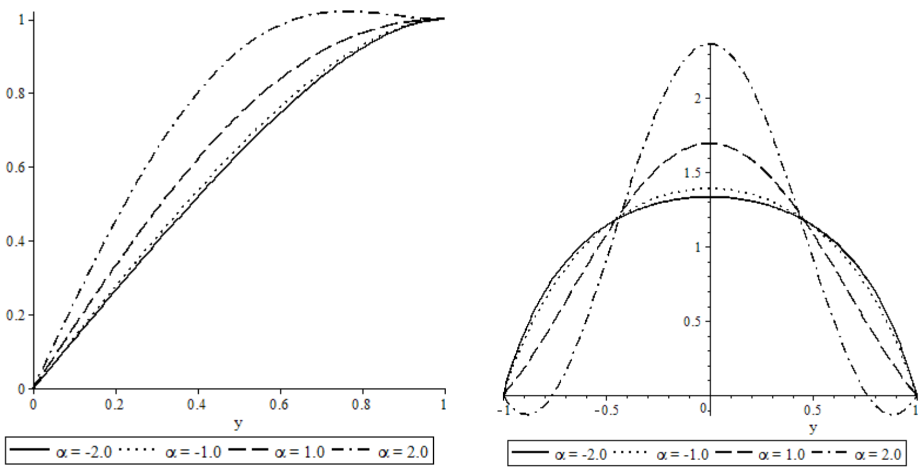
The effect of λ_1 on the velocity is illustrated in Figs. 6 and 7. For contracting walls, the velocity is an increasing function of λ_1, on the other hand, it is decreases for the expanding walls.

The influence of viscoelastic parameter β on the flow velocity is shown in Figs. 8 and 9. For increasing β, an accelerated flow is observed. It is also interesting to see that the effect of viscoelastic parameter becomes critical for some value of β and a backflow emerges in case of expanding walls with rising β. The velocity also increases when the walls contract.



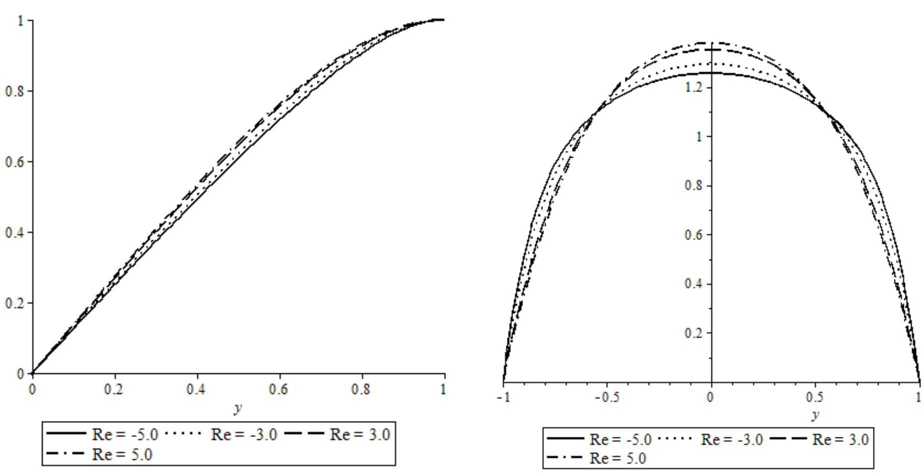
(a) $F(y)$ (b) $F'(y)$

Fig. 2: Characteristics of $F(y)$ and $F'(y)$ for varying α: $Re = 2.5$, $\beta = 0.02$ and $\lambda_1 = 0.2$



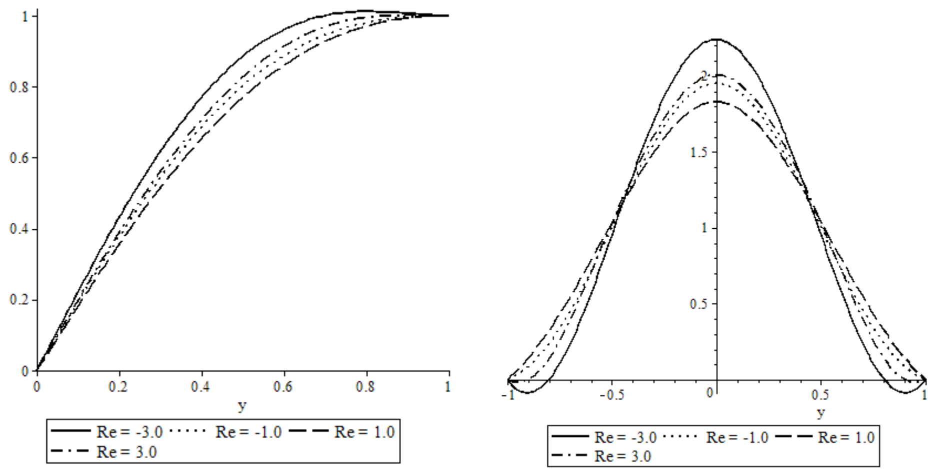
(a) $F(y)$ (b) $F'(y)$

Fig. 3: Characteristics of $F(y)$ and $F'(y)$ for varying α: $Re = -2.5$, $\beta = 0.02$ and $\lambda_1 = 0.2$



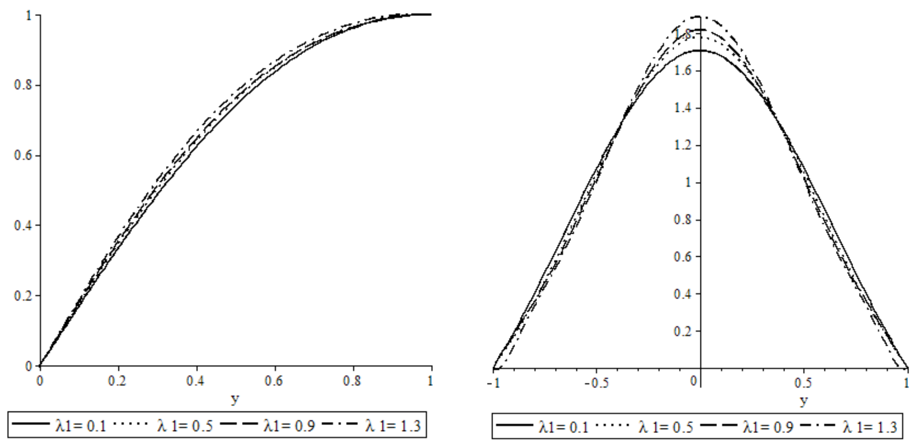
(a) $F(y)$ (b) $F'(y)$

Fig. 4: Characteristics of $F(y)$ and $F'(y)$ for varying Re: $\alpha = -2$, $\beta = 0.02$ and $\lambda_1 = 0.5$



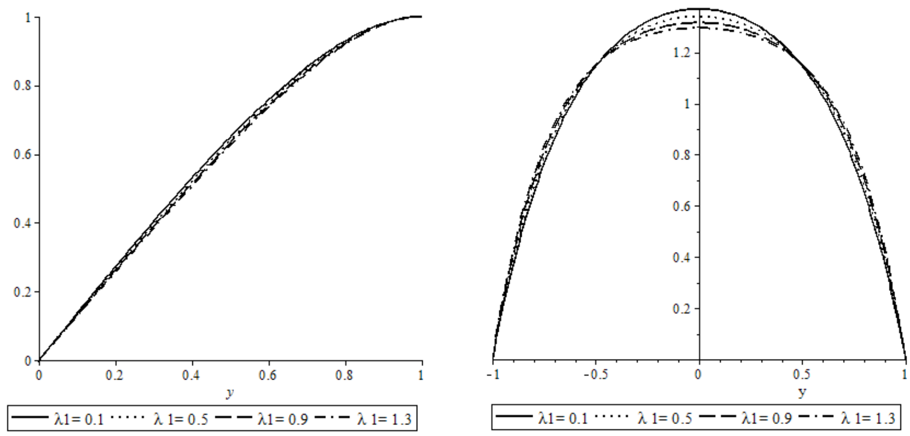
(a) $F(y)$ (b) $F'(y)$

Fig. 5: Characteristics of $F(y)$ and $F'(y)$ for varying Re: $\alpha = 2$, $\beta = 0.02$ and $\lambda_1 = 0.2$



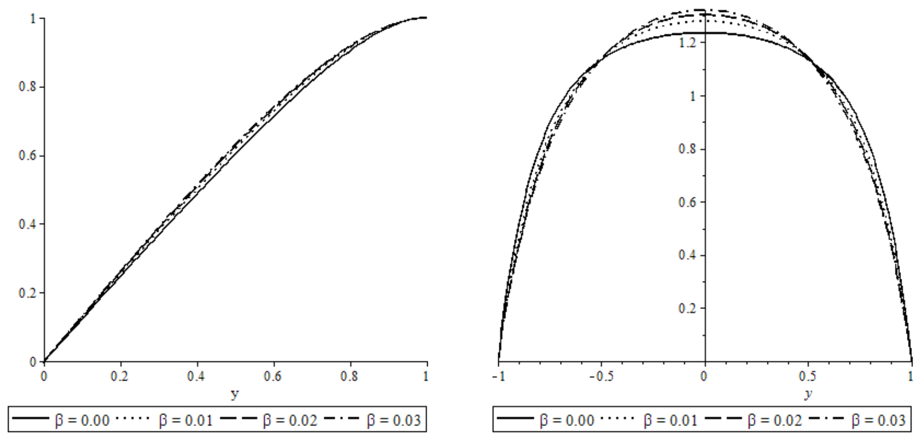
(a) $F(y)$ (b) $F'(y)$

Fig. 6: Characteristics of $F(y)$ and $F'(y)$ for varying λ_1: $\alpha = 1.5$, $\beta = 0.02$ and $Re = 2$



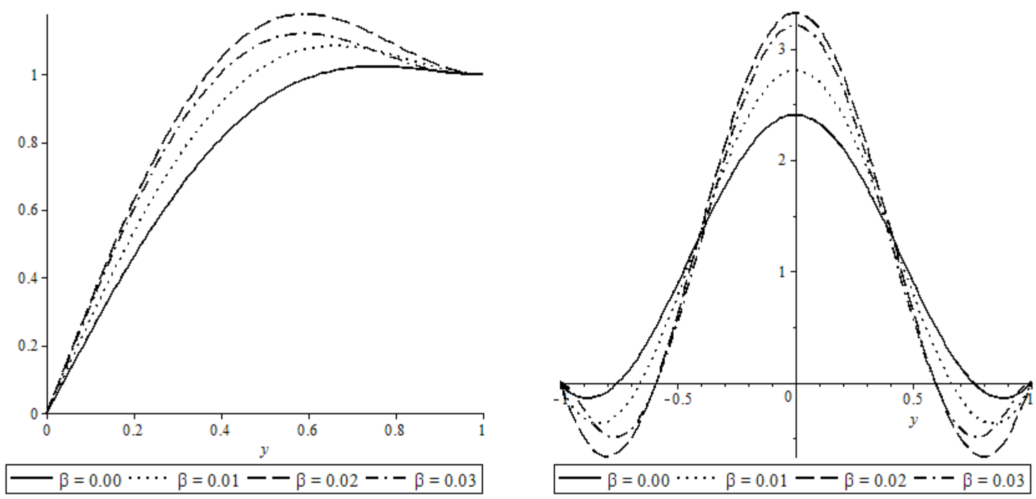
(a) $F(y)$ (b) $F'(y)$

Fig. 7: Characteristics of $F(y)$ and $F'(y)$ for varying λ_1: $\alpha = -2$, $\beta = 0.02$ and $Re = 2$



(a) $F(y)$ (b) $F'(y)$

Fig. 8: Characteristics of $F(y)$ and $F'(y)$ for varying β: $\alpha = -2$, $\lambda_1 = 0.5$ and $Re = -2$



(a) $F(y)$ (b) $F'(y)$

Fig. 9: Characteristics of $F(y)$ and $F'(y)$ for varying β: $\alpha = 2$, $\lambda_1 = 0.5$ and $Re = -2$

5 Conclusion

The flow of a Jeffery fluid in a porous channel with expanding and contracting walls is presented analytically by applying ADM. The effects of different flow parameters are discussed and presented graphically. It is observed for the case when Re=0, the similarity transform loses its physical significance because the flow is due to suction or injection. For vanishing Reynolds number, there will be no flow possible physically. This important point was overlooked by the researchers[6, 10–12, 21]. Backflow is observed in the case of expanding walls by increasing the viscoelastic parameter which may cause separation that has a significant importance in many physical applications.

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