ISSN 1 746-7233, England, UK
World Journal of Modelling and Simulation
Vol. 12 (2016) No. 1, pp. 48-58

Reliability evaluation of a repairable system under fuzziness

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(*Received May 27 2016, Accepted November 2, 2015*)

Abstract. Research in traditional reliability theory is based mainly on probist reliability, which uses a binary state assumption and classical reliability distributions. In the present paper the binary state assumption has been replaced by a fuzzy state assumption, thereby leading to profust reliability estimates of a repairable system, which is modeled as a four unit gracefully degradable system using Markov process. The effect of variations of system coverage factor and repair rates on the fuzzy availability is also studied.

Keywords: profust reliability, degradable system, coverage factor, Markov process

1 Introduction

A repairable system is defined as a system which can be restored to satisfactory working condition by repairing or replacing the damaged components that caused the failure to occur other than replacing the whole system[16]. Repairable systems receive maintenance actions that restore/renew system components when they fail. Ascher and Feingold[1]give a very rigorous discussion of many of the misconceptions associated with treating repairable system reliability data as if they came from a nonrepairable system. Once a system experiences a failure, different repair strategies have different influences on the system reliability, usually defined as the probability of no failures in time intervals[13]. The traditional reliability analysis of repairable system depends basically on two assumptions: Binary State Assumption and Probability Assumption[4]. But in various engineering problems, this binary state assumption is not extensively acceptable. Zadeh[18] suggested a paradigm shift from the theory of total denial and affirmation to a theory of grading to give a new concept of fuzzy set. Utkin[15] discussed the fuzzy system reliability based on the binary state assumption and possibility assumption and considered the fuzzy availability and unavailability and the fuzzy operative availability and unavailability. Since fuzzy set theory can express the gradual transition of the system from a working state to a failed state, the binary state assumption of traditional reliability theory is replaced by fuzzy state assumption. This approach of reliability theory based on the probability assumption and fuzzy-state assumption is known as Profust reliability[3].

Many researchers applied the concept of profust reliability on various systems. The collection of papers edited by Onisawa and Kacprzyk[8] gave many different approaches to fuzzy reliability. Pandey et al.[14] modeled the powerloom plant as a system with both its units gracefully degradable and obtained the profust reliability estimates of the system. Kuldeep Kumar et al.[9] calculated profust reliability and fuzzy availability of the serial processes in butter-oil processing plant using both failure and repair rates. By defining the membership functions of the system states to fuzzy success and failure, the availability of the system is analyzed using the reliability theory based on fuzzy states. Yao et al.[17] applied a statistical methodology in fuzzy system reliability analysis and got a fuzzy estimation of reliability. By applying the concept of profust reliability

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theory, Mirakbari and Ganji[11] developed a conceptual model of profust reliability for reliability analysis of a rangeland system. Atalay et al.[2] by stating the states of components as nonfuzzy and state of system as fuzzy obtained the profust reliabilities of consecutive k-out-of-n: F and G systems considering both linear and circular sequence of n components. A comparison between profust reliabilities and conventional reliabilities is also given.

The traditional reliability approaches have limitations in estimating reliability for individual systems under dynamic operating and environmental conditions as they are always based on the analysis of historical life-test data, which yield statistical results only reflecting population characteristics of the same kind of systems under typical conditions[10]. In contrast, profust reliability theory extends the traditional binary state space 0, 1 into a fuzzy state space [0, 1] and models fuzzy state transitions for a component or system representing various degrees of success and failure. This feature enables a profust reliability based approach to track real-time operational performance, and characterize the physical degradation and property evolution of a specific system. Zhao et al.[19] proposed a profust reliability based prognostic and health management approach, where the profust reliability was employed as a health indicator to evaluate the real-time system performance. The mean remaining useful life estimate is also predicted using a degraded Markov model.

In this paper we have estimated the profust reliability of a repairable system with coverage factor subjected to degradation which, in time, reduces the ability of the system to perform its intended function. A comparison between the conventional and profust reliability is also performed to see any improvements in the resultant estimates.

2 Profust reliability

Profust reliability theory is based on probability assumption and fuzzy state assumption that is, the system failure behavior is characterized in the context of probability and the system success and failure are characterized by fuzzy states. Suppose that the system has n topological (non-fuzzy) states $S_1, \cdots, S_n$. Let $U = S_1, \cdots, S_n$ be the universe of discourse. On this universe we can define a fuzzy success state S as:

$$S = \{S_i, u_S(S_i)\}; i = 1, 2, \cdots, n. \tag{1}$$

And a fuzzy failure state F as:

$$F = \{S_i, u_F(S_i)\}; i = 1, 2, \cdots, n. \tag{2}$$

Where $u_S(S_i)$ and $u_F(S_i)$ are the corresponding membership functions respectively.

A fuzzy state is just a fuzzy set and is defined to represent the system level of performance. When fuzziness of interest is discarded, the fuzzy success state and the fuzzy failure state become a conventional success state and failure state respectively.

In the conventional reliability theory, one is interested in the event of transition from system success state to system failure state and hence reliability of the system from time t_0 to $(t_0 + t)$ is defined as $R(t_0, t_0 + t) = Prob$ (no transition from success state to failure state occurs in time interval $[t_0, t_0 + t]$). Whereas, in case of fuzzy reliability, T_{SF} which denotes the transition from the fuzzy success state to the fuzzy failure state, is the event of interest.

Assume that the behavior of n system states is completely stochastically characterized in the time domain, profust reliability is defined as:

$$R(t_0, t_0 + t) = Pr\left\{T_{SF} \text{ does not occur in time int erval } (t_0, t_0 + t)\right\}. \tag{3}$$

Since both S and F are fuzzy states, the transitions between them are certainly fuzzy, and thus T_{SF} can be viewed as a fuzzy event[18]). Apparently, T_{SF} may occur to some extent only when some state transition occurs among the n non-fuzzy system states $S_1, S_2, \cdots, S_n$. So, T_{SF} can be defined in the universe $U_T = u_{ij}, i, j = 1, 2, \cdots, n$ as:

$$T_{SF} = \{u_{ij}, \mu_{T_{SF}}(u_{ij}), i, j = 1, 2, \cdots, n\}, \tag{4}$$

where u_{ij} represents the transition from state S_i to state S_j, with membership function

$$\{\mu_{T_{SF}}(u_{ij}), i, j = 1, 2, \cdots, n\}. \tag{5}$$

The transition possibility of working to failed state of the system (membership function) is determined as:

$$\mu_{T_{SF}}(u_{ij}) = \begin{cases} \delta_{F/S}(S_j) - \delta_{F/S}(S_i) & \delta_{F/S}(S_j) > \delta_{F/S}(S_i) \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

Let $\delta_{F/S}(S_i) = \dfrac{\mu_F(S_i)}{\mu_F(S_i) + \mu_S(S_i)}, i = 1, 2, \cdots, n.$

$\delta_{F/S}(S_i)$ can be interpreted as the grade of membership of S_i, with respect to S to F. Then the fuzzy interval reliability is defined as:

$$R_{PF}(t_0, t_0 + t) = 1 - \sum_{i=1}^{n}\sum_{j=1}^{n} \mu_{T_{FS}}(u_{ij}) \times \Pr\left\{ \begin{array}{l} \text{system is in state} \\ S_i \text{ at time}(t_0 + t) \end{array} \right\}, \tag{7}$$

where u_{ij} is the transition from state S_i to S_j without passing via any intermediate state. Cai and Chuan–Yuan[4] showed that can be represented as:

$$R_{PF}(t_0, t_0 + t) = \sum_{i=1}^{n}\sum_{j=1}^{n} \bar{\mu}_{T_{FS}}(u_{ij}) \times \Pr\left\{ \begin{array}{l} \text{system is in state} \\ S_i \text{ at time } (t_0 + t) \end{array} \right\}, \tag{8}$$

where $\bar{\mu}_{T_{SF}}(u_{ij}) = 1 - \mu_{T_{SF}}(u_{ij})$. When $t_0 = 0$, we have $R(t_0, t_0 + t) = R(t)$. $R(t)$ is referred to as the fuzzy reliability of the system at time t. Let us now define another reliability measure called availability. The system fuzzy availability at time t, denoted by $A_{PF}(t)$, is defined as

$$\begin{aligned} A_{PF}(t) &= \Pr(\text{the system is in fuzzy success state at time } t) \\ &= \sum_{j=1}^{n} \mu_S(S_i) \times \Pr(\text{system is in state } S_i \text{ at time } t). \end{aligned} \tag{9}$$

For the computation of fuzzy reliability measures of the degradable system, we consider the following two cases of degradation:

Case 1. When units degrade linearly

Assuming that the units degrade linearly as shown in Fig. 2, we define fuzzy success state S and fuzzy failure state F with their membership functions as follows:

$$\mu_S(S_i) = \frac{i}{n}, \qquad i = 1, 2, \cdots, n,$$

and

$$\mu_F(S_i) = \frac{n - i}{n}, \qquad i = 1, 2, \cdots, n.$$

Where the ratio in $\mu_S(S_i)$ implies that i out of N components are working in a unit. S_N and S_0 represent the fully working and fully failed sates respectively. Then the membership grades of the transition T_{SF} from state S_i to S_j is given at time t are

$$\mu_{T_{SF}}(u_{ij}) = \begin{cases} \frac{i-j}{n}, & \text{if } i > j, \\ 0, & \text{if } i \le j. \end{cases}$$

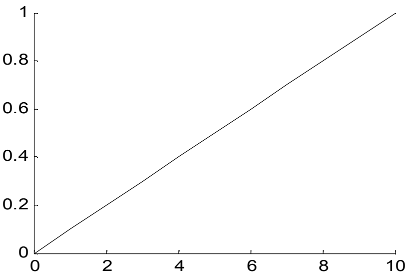

Fig. 1: Linear degradation

Case 2. When units degrades quadratically

If the degradation in units is considered to be quadratic, as shown in Fig. 3, the membership function for fuzzy success state S is defined as:

$$\mu_S(S_i) = \begin{cases} \frac{2i^2}{n^2}, & 0 \leq i \leq \frac{n}{2}, \\ 1 - \frac{2(n-i)^2}{n^2}, & \frac{n}{2} < i \leq n. \end{cases}$$

Then the corresponding membership grades of the transition are

$$\mu_{T_{SF}}(u_{ij}) = \begin{cases} 1 - \frac{2i^2}{n^2}, & 0 \leq i \leq \frac{n}{2}, \\ \frac{2(n-1)^2}{n^2}, & \frac{n}{2} < i \leq n. \end{cases}$$

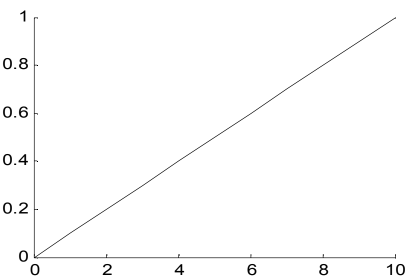

Fig. 2: Quadratic degradation

3 System modeling and reliability analysis

For the purpose of modeling and analysis, most of the classical reliability assessments are based on the probabilistic Markov models. Markov models are frequently used in reliability & maintainability analysis where events, such as the failure or repair of a module, can occur at any point in time. A Markov model breaks the system configuration into a number of states and develops the probability of an item being in a given state, as a function of the sequence through which the item has traveled. The Markov process can thus easily describe degraded states of operation, where the item has either partially failed or is in a degraded state where some functions are performed while others are not[6]. Also, when the coverage factor is considered, the Markov model becomes a more suitable modeling approach, since the covered and uncovered failures of components are supposed to be mutually exclusive events.

Masdi, et al.[12] presents a development of model based on Markov process for a degraded multi-state system to evaluate the system performance. Markov process was chosen to model the system due to its versatility which can be used for finite number of states and different assumptions of repair. Fuzzy Markov models are proposed to incorporate the uncertainties associated with transition rates or probabilities. Asgarpoor and Ge[7] developed a fuzzy Markov model which incorporates uncertainties of transition rate and applied it in modeling aging equipment and substations. The transition rates/probabilities with uncertainty are represented by fuzzy membership functions and the extension principle was used for calculating the reliability indices.

To begin with the model, we consider a redundant degradable computing system with four identical and independent modules operating in parallel. Each module has only two states: working or failed. The time to failure for each module follows an exponential distribution with parameter λ. If a module fails, a reconfiguration operation immediately detects and removes the failed module from the system. All the other fault free modules will continue to do their work if the reconfiguration operation is performed successfully. The probability of successful reconfiguration operation is defined as coverage factor and denoted by c. Also we assume that one repairman is available to repair one faulty module at any time. The repair time is exponentially distributed with parameter μ.

3.1 Design and characteristics of the model

Let S_k represents the system state that k operational (working) modules are available. Initially there are 4 active modules in the system. The system may then have five states: S_0, S_1, S_2, S_3, and S_4.

Table 1: The status of system

States	Number of Modules	
	Working	Failed
S_0	0	4
S_1	1	3
S_2	2	2
S_3	3	1
S_4	4	0

This system shown in Fig. 3 represents the Markovian transition among the system states, where c represents the system coverage factor. The differential equations describing the system are:

$$\frac{dP_4}{dt} = -4\lambda P_4 + \mu P_3,$$
$$\frac{dP_3}{dt} = -3\lambda P_3 + 4c\lambda P_4 + \mu P_2,$$
$$\frac{dP_2}{dt} = -2\lambda P_2 + 3c\lambda P_3 + \mu P_1,$$
$$\frac{dP_1}{dt} = -\lambda P_1 + 2\lambda c P_2 + \mu P_0,$$
$$\frac{dP_0}{dt} = -\mu P_0 + \lambda P_1 + 2\lambda(1-c)P_2 + 3\lambda(1-c)P_3 + 4\lambda(1-c)P_4,$$

where $P_i(t)$ is the probability of the system being in ith state at time t. On solving these equations together with the initial conditions,

$$P_k(0) = \begin{cases} 1, & k = 1, \\ 0, & \text{otherwise}. \end{cases}$$

The reliability of the system is computed as $R_i(t) = \sum_{i=1}^{4} P_i(t)$.

3.2 Steady-state availability

The steady state probabilities of the system are obtained from the condition when $t \to \infty$, $\frac{d}{dt} \to 0$. Thus, the steady-state equations of the system are obtained as:

$$\mu P_0 = \lambda P_1 + 2\lambda(1-c)P_2 + 3\lambda(1-c)P_3 + 4\lambda(1-c)P_4,$$
$$(\lambda+\mu)P_1 = \mu P_0 + 2\lambda c P_2,$$
$$(2\lambda+\mu)P_2 = \mu P_1 + 3\lambda c P_3,$$
$$4\lambda P_4 = \mu P_3,$$

where P_k is the probability of the system being in kth state as $t \to \infty$.

These equations are solved using the normalized condition $\sum_{k=0}^{4} P_k = 1$ to obtain the steady-state probablities. Then the system steady-state availability is obtained as:

$$A_(\infty) = \sum_{i=1}^{4} P_i = P_1 + P_2 + P_3 + P_4.$$

For the system given in Fig. 3, let the universe of discourse be $U = \{S_0, S_1, S_2, S_3, S_4\}$, where S_i represents that i operational units are available, i = 0, 1, 2, 3, 4.

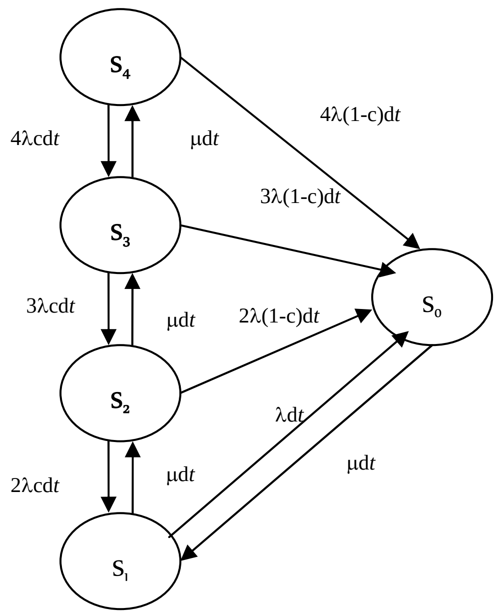

Fig. 3: Markov model of a 4-unit degradable system

Then the fuzzy availability of the system in steady-state is defined as:

$$A_{PF}(\infty) = \Pr\{\text{ that the system is in fuzzy success state }\}.$$

That is,

$$A_{PF}(\infty) = \sum_{i=1}^{4} \mu_S(S_i) \times \Pr(\text{system is in state } S_i).$$

Thus the profust availabilitiy of the system in steady-state for both linear and quadratic degradations are obtained as

For Case 1:

$$A_{PF1}(\infty) = \frac{1}{4}P_1 + \frac{1}{2}P_2 + \frac{3}{4}P_3 + P_4,$$

and For Case 2:

$$A_{PF2}(\infty) = \sum_{i=1}^{4} \mu_S(S_i).P_i = \frac{1}{8}P_1 + \frac{1}{2}P_2 + \frac{7}{8}P_3 + P_4.$$

4 Result and discussion

Numerical results obtained to study the effect of various parameters on fuzzy availability for the steady sate of the system given in Fig. 3. If the parameters like failure rate, repair rate and coverage factor are altered, the fuzzy availability is affected. The numerical values of fuzzy availability of the system in steady-state are tabulated in Tab. 2 and Tab. 3 respectively for linear and quadratic degradations.

Table 2: Effect of failure, repair and coverage factor on fuzzy availability under linear degradation

$\mu \downarrow$	$\lambda \rightarrow$			
	0.001	0.002	0.003	0.004
$c = 0.1$				
0.1	0.9175	0.8492	0.7915	0.7423
0.5	0.9822	0.9650	0.9486	0.9328
1.0	0.9910	0.9822	0.9735	0.9650
$c = 0.2$				
0.1	0.9249	0.8615	0.8072	0.7601
0.5	0.9839	0.9684	0.9534	0.9389
1.0	0.9919	0.9839	0.9761	0.9717
$c = 0.3$				
0.1	0.9324	0.8743	0.8236	0.7791
0.5	0.9856	0.9717	0.9582	0.9451
1.0	0.9928	0.9856	0.9786	0.9717
$c = 0.4$				
0.1	0.9401	0.8875	0.8409	0.7994
0.5	0.9874	0.9751	0.9631	0.9515
1.0	0.9936	0.9874	0.9812	0.9751
$c = 0.5$				
0.1	0.9480	0.9013	0.8592	0.8211
0.5	0.9891	0.9785	0.9681	0.9579
1.0	0.9945	0.9891	0.9839	0.9785
$c = 0.6$				
0.1	0.9560	0.9156	0.8785	0.8443
0.5	0.9909	0.9819	0.9731	0.9645
1.0	0.9954	0.9909	0.9864	0.9819
$c = 0.7$				
0.1	0.9642	0.9306	0.8990	0.8693
0.5	0.9926	0.9854	0.9782	0.9712
1.0	0.9963	0.9926	0.9890	0.9854
$c = 0.8$				
0.1	0.9725	0.9461	0.9207	0.8962
0.5	0.9954	0.9889	0.9834	0.9779
1.0	0.9972	0.9944	0.9916	0.9889
$c = 0.9$				
0.1	0.9811	0.9623	0.9437	0.9254
0.5	0.9962	0.9924	0.9886	0.9848
1.0	0.9981	0.9962	0.9943	0.9924
$c = 1.0$				
0.1	0.9898	0.9792	0.9683	0.9570
0.5	0.9980	0.9960	0.9939	0.9919
1.0	0.9990	0.9980	0.9970	0.9960

4.1 Effect of coverage factor

To explain the effect of coverage factor on system availability, the numerical results for system availability in steady-state for various values of coverage factor are tabulated in Tab. 4 for $\lambda = 0.002$ and $\mu = 0.9$. The

Table 3: Effect of failure, repair and coverage factor on fuzzy availability under quadratic degradation

$\mu \downarrow$	$\lambda \rightarrow$			
	0.001	0.002	0.003	0.004
$c = 0.1$				
0.1	0.9178	0.8492	0.7911	0.7412
0.5	0.9823	0.9652	0.9488	0.9330
1.0	0.9910	0.9823	0.9736	0.9652
$c = 0.2$				
0.1	0.9256	0.8624	0.8074	0.7605
0.5	0.9841	0.9687	0.9539	0.9395
1.0	0.9920	0.9841	0.9763	
$c = 0.3$				
0.1	0.9336	0.8761	0.8256	0.7810
0.5	0.9859	0.9723	0.9590	0.9461
1.0	0.9929	0.9859	0.9790	0.9723
$c = 0.4$				
0.1	0.9418	0.8903	0.8443	0.8030
0.5	0.9878	0.9758	0.9642	0.9529
1.0	0.9938	0.9878	0.9818	0.9758
$c = 0.5$				
0.1	0.9501	0.9050	0.8639	0.8264
0.5	0.9896	0.9794	0.9695	0.9597
1.0	0.9948	0.9896	0.9845	0.9794
$c = 0.6$				
0.1	0.9586	0.9203	0.8848	0.8516
0.5	0.9915	0.9831	0.9748	0.9667
1.0	0.9957	0.9955	0.9845	0.9831
$c = 0.7$				
0.1	0.9673	0.9363	0.9068	0.8787
0.5	0.9933	0.9867	0.9802	0.9737
1.0	0.9967	0.9933	0.9900	0.9867
$c = 0.8$				
0.1	0.9762	0.9530	0.9302	0.9079
0.5	0.9952	0.9904	0.9857	0.9810
1.0	0.9976	0.9952	0.9928	0.9904
$c = 0.9$				
0.1	0.9853	0.9703	0.9550	0.9395
0.5	0.9971	0.9942	0.9912	0.9883
1.0	0.9985	0.9971	0.9956	0.9942
$c = 1.0$				
0.1	0.9946	0.9884	0.9815	0.9737
0.5	0.9990	0.9979	0.9969	0.9957
1.0	0.9995	0.9990	0.9985	0.9979

graph in Fig. 4 shows the relationship between system steady-state availability and coverage factor. The availability of the system in steady state decreases with the decrease in coverage factor. This decrease in availability is more gradual in fuzzy estimates, which is consistent with the degradability of the system.

4.2 Effect of maintenance

Maintainability is defined as the probability that a failed component or system will be restored or repaired to a specific condition within a period of time when maintenance is performed in accordance with the prescribed procedures[5]. In order to study the effect of maintenance on system steady-state availability of the degradable system the numerical values of fuzzy reliability in steady-state for various repair-rates taking $\lambda = 0.002$ and $c = 0.9$ are tabulated in Tab. 5. The corresponding graph is given in Fig. 4.

Table 4: Steady-state availability for $\lambda = 0.002$ and $\mu = 0.9$

Coverage Factor c	Fuzzy Availability		Availability (Classical)
	Linear	Quadratic	
0.0	0.9783	0.9783	0.9913
0.1	0.9802	0.9803	0.9922
0.2	0.9821	0.9823	0.9930
0.3	0.9841	0.9844	0.9939
0.4	0.9860	0.9864	0.9947
0.5	0.9879	0.9885	0.9956
0.6	0.9899	0.9905	0.9965
0.7	0.9918	0.9926	0.9974
0.8	0.9938	0.9947	0.9982
0.9	0.9958	0.9968	0.9991
1.0	0.9978	0.9989	1.000

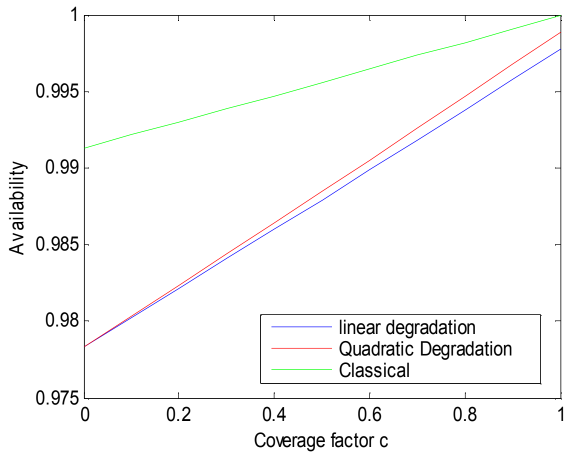

Fig. 4: Profust and classical availability for various coverage factors ($\lambda = 0.002$ and $\mu = 0.9$)

From the numerical results given in Tab. 5 and graph in Fig. 5, it is observed that the minimal improvement in maintenance facility shows substantial increase in fuzzy availability of the system in steady-state. It is also known that if mean time to repair (MTTR) follows an exponential distribution, in that case, MTTR $= \frac{1}{\mu}$. Thus, the above discussion suggests that the MTTR should be maintained as close to 10 hours to 3 hours to improve the profust availability of the system. However, from Tab. 5, it is apparent that the decrement below 3 hours in mean time to repair is not very significant.

Table 5: Fuzzy availability for various repair-rates

Repair-rate μ	Fuzzy Availability		Availability (Classical)
	Linear	Quadratic	
0.1	0.9623	0.9703	0.9922
0.2	0.9811	0.9853	0.9960
0.3	0.9874	0.9903	0.9974
0.4	0.9905	0.9927	0.9980
0.5	0.9924	0.9942	0.9984
0.6	0.9937	0.9951	0.9987
0.7	0.9946	0.9958	0.9989
0.8	0.9953	0.9964	0.9990
0.9	0.9958	0.9968	0.9991
1.0	0.9962	0.9971	0.9992

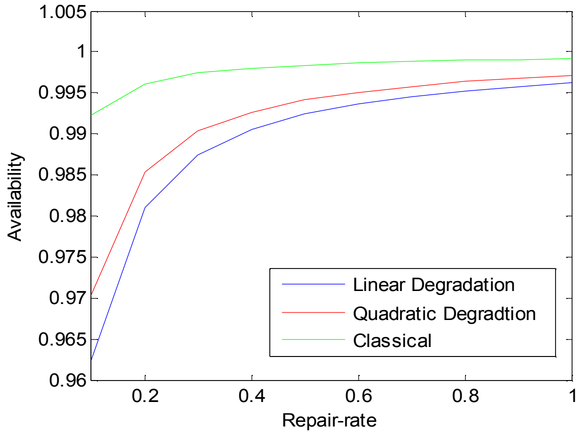

Fig. 5: Profust and classical Availabilities for various repair-rates ($\lambda = 0.002$ and $c = 0.9$)

4.3 Conclusion

A gracefully degradable system shows performance deterioration and makes it difficult to define the failure/success state. Since fuzzy set theory can handle all the possible states between working to completely failed, the membership functions of the system states to fuzzy success and failure are defined. The concept of profust reliability theory is applied to obtain the reliability measures such as steady-state availability of a repairable system subjected to degradation. Numerical results are obtained in order to show the effects of coverage factor and maintenance on the profust reliability of the system in long run. A comparison between the conventional and profust reliabilities is also presented which shows that profust reliability provides more realistic estimates than the conventional ones.

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