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Instantaneous frequency estimation of non-stationary signals in a finitely correlated environment using a decorrelating time-varying autoregressive model

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Abstract. The performance of Time-Varying Autoregressive (TVAR) model have been shown well for Instantaneous Frequency (IF) estimation of frequency modulated (FM) components in white noise. Nevertheless the performance of the TVAR model degrades, when the model is applied to a signal containing a finitely correlated signal as well as the white noise, particularly when the correlated signal is not weak relative to the FM components. We modify the TVAR model by introducing a Decorrelation delay larger than one between the time-varying coefficients for IF estimation of non stationary signal in a finitely correlated environment. The resulting model is referred to as the Decorrelating TVAR (DTVAR) model. We showed the performance of Decorrelating TVAR (DTVAR) model based IF estimator is better than TVAR model based IF estimator in a finitely correlated environment. We also discussed the DTVAR Model order estimation using Maximum Likelihood Estimation (MLE) Algorithm. Simulation results are included to show the effectiveness of the modified method in a finitely correlated environment.

Keywords: time-varying autoregressive modeling, instantaneous frequency estimation, finitely correlated environment, decorrelating time-varying autoregressive model

1 Introduction

In the area of wireless communications, speech, biomedical, radar and sonar signal processing, IF estimation of time-varying narrow band FM components in a signal is a significant research topic[11]. The extensive applications of IF estimation arise from the underlying non-stationarity of signals encountered in these fields. Non-stationary signals have spectral characteristics that vary with time, the time variation being a result of the physical phenomena connected with its generation or methods and mediums concerned in its communication. For example, in a FM signal used in wireless communications, the frequency of the carrier is varied according to a modulating signal representing information - such as voice - that is inherently non-stationary. Consequently, the information resides in the continuously and instantaneously changing frequency (the IF) of the FM signal and its accurate estimation is of main attention at the FM receiver. In biomedical applications, IF estimation provides useful information for the analysis of electrocardiogram (ECG) and electroencephalogram (EEG) Signals.

One solution when dealing with non-stationary signals is to employ time-varying parametric models, where the associated model parameters are allowed to be time-varying (or) time-dependent. We consider a TVAR model based IF estimator. Modeling a time-varying (non-stationary) discrete time signal as an autoregressive process with time-varying coefficients was proposed by Rao in 1970. Rao introduced the idea of approximating the time-varying coefficients using a weighted linear combination of a small set of known time

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functions referred to as the basis. The method was soon used for linear estimation of time-varying signals by Liporace in 1975 and applied to time-varying linear predictive coding (LPC) of speech by Hall, Oppenheim, and Willsky in 1977. A heuristic treatment of time-dependent ARMA modeling of non-stationary stochastic processes and their application to time-varying spectral estimation was provided by Grenier in 1983. Soon, TVAR model based IF estimation of non-stationary signals was proposed by Sharman and Friedlander in 1984. In this method, the angles of the roots (poles) of the TVAR polynomial at each sample instant are used as the IF estimates of the FM components in the non-stationary signal[10].

TVAR model based IF estimation had been considered poor since being proposed in 1984 by Sharman and Friedlander. Shan and Beex[10, 11] in 1998 have shown that it is a fairly good method, and especially advantageous for those practical cases where short data records are used for modeling and/or a linear IF law cannot be assumed a priori. TVAR model performs well when applied to short data records for IF estimation of frequency modulated (FM) components in white noise[10, 11]. In wireless communications, an FM signal can appear as strong narrowband interference and we are faced with the task of separating a desired signal from the interference in the presence of white noise. This desired signal can sometimes be a finitely correlated signal - such as a Continuous Phase Modulated (CPM) signal[2]. We use terminology appropriate with wireless communications and define the ratio of the power of a correlated signal to the interfering FM signal as the signal-to-interference ratio (SIR).

When TVAR model is applied to a signal containing a finitely correlated signal of interest-such as a continuous phase modulated (CPM) signal in addition to the white noise, the TVAR model based IF estimation performance degrades particularly when the correlated signal is not weak relative to the FM components[10]. We tackle this issue by introducing a decorrelation delay (gaps) between the time-varying coefficients of the TVAR model. The decorrelation delay minimizes the correlation between the current sample of the correlated signal and the samples that are operated on by the filter in order to minimize its effect on the IF estimate of the FM component. The resulting model is referred to as the Decorrelating TVAR (DTVAR) model. The time-varying parameters of the TVAR model derived with a decorrelation delay form a generalized time-varying linear Prediction errors filter (TVLPEF) in which the delay between the predictor weights can be more than one. Furthermore, the angles of the roots of the generalized PEF weights at each sample instant can be used to obtain an improved IF estimate at moderate to high SIRs.

The paper is organized as follows. We briefly describe the DTVAR Model in section 2. In section 3 We have discussed the parameter and order selection of DTVAR Model. Wax-Kailath Algorithm is presented in section 4. TVAR based and DTVAR based IF estimators are presented in section 5. In Section 6, we show examples to illustrate the performance of the DTVAR Model. Concluding remarks are given in Section 7.

2 Decorrelating time-varying autoregressive (DTVAR) model

A p^{th} order Decorrelating Time-Varying Auto regressive (DTVAR) model representation of the non-stationary discrete-time stochastic process x_n with decorrelation delay (Ω) between the time-varying coefficients is shown below [13]

$$x_n = -\sum_{k=1}^{p} a_{k,n} x_{n-k\Omega} + v_n. \tag{1}$$

Here v_n is a stationary zero mean white noise process of variance σ_v^2 and the time-varying coefficients $a_{k,n)}$ are modeled as a linear combination of basis functions. The purpose of the basis is to allow fast and smooth time variation of the coefficients. If we denote $u_(m,n)$ as the basis function and consider set of $(q+1)$ functions for a given model, we can express the DTVAR coefficients in general as,

$$a_{k,n} = \sum_{m=0}^{q} a_{km} u_{m,n}, \tag{2}$$

where $u_{m,n}$ are the basis functions.

From (2) we examine that, we have to calculate the set of parameters a_{km} for $\{k = 1, 2, \cdots, p$; $m = 0, 1, 2, \cdots, q$; $a_{0m} = 1\}$ in order to compute the DTVAR coefficients $a_{k,n}$, and the DTVAR model is absolutely specified by this set.

The DTVAR coefficients are designed as follows, we consider single realization of the process x_n. For a given realization of x_n we can analyze (1) as a time-varying linear prediction error filter and consider v_n to be the prediction error

$$v_n = x_n - \hat{x}_n, \tag{3}$$

where

$$\hat{x}_n \triangleq -\sum_{k=1}^{p} a_{k,n} x_{n-k\Omega}. \tag{4}$$

The total squared prediction error, which is as well as the error in modeling x_n, is now specified by

$$\epsilon_p = \sum_{\tau} |v_n|^2. \tag{5}$$

Substitute (2) in (4) and the prediction error v_n can be written as

$$v_n = x_n + \sum_{k=1}^{p} \sum_{m=0}^{q} a_{km} u_{m,n} x_{n-k\Omega}. \tag{6}$$

The total squared prediction error can be formulated as

$$\epsilon_p = \sum_{\tau} |x_n + \sum_{k=1}^{p} \sum_{m=0}^{q} a_{km} u_{m,n} . x_{n-k\Omega}|^2. \tag{7}$$

For modeling the non stationary stochastic process x_n,using covariance technique, we make no assumptions on the data outside $[0, N-1]$. In Eq. (7) τ is the interval over which the summation is performed and set $\tau = [p\Omega, N-1]$. By minimizing the mean squared prediction error in (7) we can estimate the time-varying parameters a_{km}[13]. We can minimize the mean squared prediction error in (7) by means of setting the gradient of ϵ_p with respect to a^*_{lg} zero

$$\frac{\partial \epsilon_p}{\partial a^*_{lg}} = \sum_{\tau} \frac{\partial v_n v^*_n}{\partial a^*_{lg}} = \sum_{\tau} \frac{\partial v^*_n}{\partial a^*_{lg}} = 0. \tag{8}$$

For $l = 1, 2, \cdots, p; g = 0, 1, \cdots, q$

$$v^*_n = x^*_n + \sum_{l=1}^{p} \sum_{g=0}^{q} a^*_{lg} u^*_{g,n} x^*_{n-l\Omega}. \tag{9}$$

And the derivative of v^*_n with respect to a^*_{lg}

$$\frac{\partial v^*_n}{\partial a^*_{lg}} = u^*_{g,n} x^*_{n-l\Omega}. \tag{10}$$

Consequently (8) becomes,

$$\sum_{\tau} v_n u^*_{g,n} x^*_{n-l\Omega} = 0. \tag{11}$$

The above mentioned condition is similar to the orthogonality principle encountered in stationary signal modeling. Substitute (6) in (11) we have

$$\sum_{\tau}\left(x_n+\sum_{k=1}^{p}\sum_{m=0}^{q}a_{km}u_{m,n}x_{n-k\Omega}\right)u_{g,n}^{*}x_{n-l\Omega}^{*}=0. \tag{12}$$

Now we define a function $c_{mg}\ (l,k)$ as shown below,

$$c_{mg}(l,k)\triangleq\sum_{\tau}u_{m,n}x_{n-k\Omega}u_{g,n}^{*}x_{n-l\Omega}^{*}. \tag{13}$$

Using the above definition in (12) we have,

$$\sum_{k=1}^{p}\sum_{m=0)}^{q}a_{km}c_{mg}(l,k)=-c_{0g}(l,0). \tag{14}$$

The above equation represents a system of $p(q+1)$ linear equations. The above system of linear equations can be efficiently represented in matrix form as follows Define a column vector a_m as follows

$$a_m=[a_{1m}\ \ a_{2m}\ \ \cdots\ \ a_{pm}]^T, \tag{15}$$

where $m=0,1,\cdots,q$.

We can use the function (13) to find the following matrix for $0\leq(m,g)\leq q$

$$C_{mg}=\begin{bmatrix} c_{mg}(1,1) & c_{mg}(1,2) & \cdots & c_{mg}(1,p)\\ c_{mg}(2,1) & c_{mg}(2,2) & \cdots & c_{mg}(2,p)\\ \vdots & \vdots & \ddots & \vdots\\ c_{mg}(p,1) & c_{mg}(p,2) & \cdots & c_{mg}(p,p)\end{bmatrix}. \tag{16}$$

The above matrix is of size pxp and all the different values for m and g resulting in $(q+1)\times(q+1)$ such matrices, by means of these matrices, we can now describe a block matrix as shown below,

$$C=\begin{bmatrix} C_{00} & C_{01} & \cdots & C_{0q}\\ C_{10} & C_{11} & \ddots & C_{1q}\\ \vdots & \vdots & \vdots & \vdots\\ C_{q0} & C_{q1} & \cdots & C_{qq}\end{bmatrix}. \tag{17}$$

The above Block matrix C has $(q+1)\times(q+1)$ elements and each element is a matrix of size $p\times p$, which implies the Block matrix C of size $p(q+1)\times p(q+1)$. Now we describe a column vector d_m as shown below

$$d_m=[(c_{0m}(1,0)\ \ c_{0m}(2,0)\ \ \cdots\ \ c_{0m}(p,0))]^T \tag{18}$$

where $m=0,1,\cdots,q$.

By using the definitions from (15)-(18) we can represent the system of linear equations in (14) in a compact matrix form as follows

$$\underbrace{\begin{bmatrix} C_{00} & \cdots & C_{0q}\\ \vdots & \ddots & \vdots\\ C_{q0} & \cdots & C_{qq}\end{bmatrix}}_{c}\underbrace{\begin{bmatrix} a_0\\ \vdots\\ a_q\end{bmatrix}}_{a}=-\underbrace{\begin{bmatrix} d_0\\ \vdots\\ d_q\end{bmatrix}}_{d} \tag{19}$$

$$Ca=-d. \tag{20}$$

The above equations reduce to the Yule-walker equations (YWE) for a stationary AR model, as soon as $q=0$. The set of TVAR parameters a_{km} are elements of a and can be computed by solving the above matrix equation using Wax-Kailath Algorithm, and is discussed in section 4.

3 Parameter selection

The DTVAR parameter selection is basically depends on three degrees of freedom, such as the DTVAR model order p, the basis function order q, and the set of basis functions $u_{m,n}$[10, 13].

3.1 Choice of the basis functions

The basis functions $u_{m,n}$ must be independent and non-zero for $n = 0, 1, \cdots, N-1$, and $u_{m,n} = 1$, if $n = 0$. If a priori information about the signal variation is known, the basis functions should be chosen such that the trends in parameter change is retained. In case, when a priori information is unavailable selection of basis is trial and error[10, 11].

According to Eq. 2, no particular constraint is imposed on the basis $u_{m,n}$, consequently one will be able to track only variations which are approximable by this set of functions. Numerous solutions have been projected in the literature such as time basis functions, Legendre polynomial, Chebyshev polynomial, Discrete Prolate Spheroidal (DPSS) sequence, Fourier basis, Discrete cosine basis, Walsh basis, Multi wavelet basis, none of these solutions seems to be perfect, since the selection of $u_{m,n}$ desires some priori information upon the time variations present in x_n[10].

Then again, basis such as prolate spheroidal functions are extremely tough to generate. We propose here to apply traditional polynomial functions (namely time polynomial, Legendre polynomials, and Chebyshev polynomials) since their realization is easy, and they can fairly accurate a broad range of variations. In this article, we employ basis functions that are powers of the time variable n (time basis function) as given below,

$$u_{m,n} = \left(\frac{n - p\Omega}{N}\right)^m, \tag{21}$$

$$m = 0, 1, \cdots, q; n = p\Omega, p\Omega + 1, p\Omega + 2, \cdots, N - 1. \tag{22}$$

3.2 Order selection

Maximum Likelihood Estimation (MLE)

The DTVAR Model for the non stationary discrete-time stochastic process x_n is

$$x_n = -\sum_{k=1}^{p}\sum_{m=0}^{q} a_{km} u_{m,n} x_{n-k\Omega} + v_n.$$

The above can be represented in compact form as

$$x_n = -Z^T[n]a + v_n, \tag{23}$$

where $Z[n]$ is

$$Z[n] = \Phi[n]u[n]. \tag{24}$$

Here, $\bigotimes$ denote Kronecker multiplication.

$$\Phi[n] = [x_{n-\Omega},\ x_{n-2\Omega},\ \cdots,\ x_{n-p\Omega}]^T, \tag{25}$$

$$u[n] = [u_{0n}, u_{1n}, \cdots, u_{qn}]^T. \tag{26}$$

Moreover

$$a = [a_1^T,\ a_2^T,\ \cdots,\ a_p^T]. \tag{27}$$

Here

$$a_k^T = [a_k 0,\ a_k 1,\ \cdots,\ a_k q]. \tag{28}$$

Step 1. compute

$$Z[n] = \Phi[n]u[n]. \tag{29}$$

Step 2. calculate

$$C = -\left(\sum_{n=p\Omega}^{N-1} Z[n]Z^T[n]\right)^{-1}\left(\sum_{n=p\Omega}^{N-1} Z[n]x[n]\right). \tag{30}$$

Step 3. Estimate

$$\hat{\beta} = \frac{1}{N}\sum_{n=0}^{N}[x_n - C^T Z[n]]^2. \tag{31}$$

Step 4. Obtain the cost function

$$J(p,q) = \frac{p(q+1)+2-N}{2}\log(2\pi\beta) - \frac{1}{2}\log|\sum_{n=0}^{N} Z[n]Z^T[n]|. \tag{32}$$

Step 5. Maximize the above cost function to select the expansion dimension $q = q_opt$ and the model order $p = p_opt$, where $p_opt \in \{1, 2, 3, 4, \cdots, p_{\max}\}$ and $q_opt \in \{1, 2, 3, 4, \cdots, q_{\max}\}$.

4 Wax-Kailath algorithm

Computationally efficient inversion of block-Toeplitz matrix with Hermitian-Toeplitz blocks is an area of noteworthy research attention since it is used in a diversity of applications such as IF estimation based on TVAR model, co channel interference mitigation in cellular communication systems, equalization of time-varying multipath fading of wireless channels and multichannel filtering. The Wax-Kailath algorithm[8] can be used to competently invert a block-Toeplitz matrix with Hermitian-Toeplitz blocks.

The calculation of the TVAR parameters a_{km} by solving $Ca = -d$ is one of the difficult aspects in TVAR based IF estimation. The direct (or) iterative techniques are available for solving the matrix. In the direct technique by means of Gaussian elimination which require $O(p^3(q+1)^3)$ Calculations, and is not an competent technique to solve the linear system. Hence, for realistic execution of IF estimation the recursive and competent inversion of C is necessary. When the covariance method by means of time polynomial basis functions is used for TVAR modeling, interestingly the covariance matrix C is found to have Hermitian block-Hankel arrangement. We can translate this block-Hankel arrangement to a block-Toeplitz arrangement by rewriting the system of equations. By adding an appropriate error matrix to this block-Toeplitz matrix we can obtain block-Toeplitz matrix with Hermitian-Toeplitz blocks. The Wax-Kailath algorithm can be used to competently invert this matrix, which involves only $O(p^3(q+1)^2)$ calculations

The Covariance matrix C in Eq. (19) has a Hermitian block-Hankel arrangement. To convert C to a block-Toeplitz arrangement, we revise Eq. (19) as shown below

$$\underbrace{\begin{bmatrix} C_{0q} & C_{0q-1} & \cdots & C_{00} \\ C_{1q} & C_{1q-1} & \ddots & C_{10} \\ \vdots & \vdots & \vdots & \vdots \\ C_{qq} & C_{qq-1} & \cdots & C_{q0} \end{bmatrix}}_{R_{q+1}} \underbrace{\begin{bmatrix} a_q \\ a_{q-1} \\ \vdots \\ a_0 \end{bmatrix}}_{\check{a}} = - \underbrace{\begin{bmatrix} d_0 \\ d_1 \\ \vdots \\ d_q \end{bmatrix}}_{d}.$$

The matrix R_{q+1} in the above equation has block-Toeplitz structure with the individual blocks that are Hermitian as well as being close to Toeplitz. Consequently, we add a suitable error matrix E to R_{q+1} to get a block-Toeplitz matrix $T_{p,q+1}$ with Hermitian-Toeplitz blocks, which are then inverted using Wax-Kailath Algorithm.

By using m and g to index the blocks, the error matrix E can be formed as follows,

$$E_{mg} = T_{mg} - C_{mg} 0 \leq (m, g) \leq q. \tag{33}$$

Here C_{mg} indicate the blocks in $R_{q+1)}$ and T_{mg} is the equivalent Toeplitz block. The Hermitian-Toeplitz blocks are formed as shown below,

$$T_{mg} = \text{toeplitz}(\text{mean}(diag(C_{mg}, i))), \tag{34}$$

where $i = 0, 1, \cdots, p-1$

Here, the function diag extract the ith diagonal of the matrix C_{mg}, the function mean, calculates the mean of the ith diagonal, at last, the function toeplitz forms a Hermitian-Toeplitz matrix whose foremost row elements equivalent to the mean of the diagonals. Using the above equation, we get $T_{p,q+1}$ and calculate E_{mg} to form the error matrix E.

$$T_{p,q+1} = R_{q+1} + E. \tag{35}$$

The R_{q+1} can be computed as

$$R_{q+1} = T_{p,q+1} - E, \tag{36}$$

where $T_{p,q+1}$ is a block-Toeplitz matrix with Hermitian-Toeplitz blocks of size $p(q+1) \times p(q+1)$ as shown below

$$T_{p,q+1} = \begin{bmatrix} T_0 & T_1 & T_2 & \cdots & T_q \\ T_{-1} & T_0 & T_1 & \cdots & T_{q-1} \\ T_{-2} & T_{-1} & T_0 & \cdots & T_{q-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ T_{-q} & T_{-q+1} & T_{-q+2} & \cdots & T_0 \end{bmatrix}. \tag{37}$$

Here T_i and T_{-i} for $i = 0, 1, \cdots, q$ are $p \times p$ matrices having Hermitian-Toeplitz arrangement as shown below

$$T_i = \begin{bmatrix} t_{0,i} & t^*_{1,i} & t^*_{2,i} & \cdots & t^*_{p-1,i} \\ t_{1,i} & t_{0,i} & t^*_{1,i} & \cdots & t^*_{p-2,i} \\ t_{2,i} & t_{1,i} & t_{0,i} & \cdots & t^*_{p-3,i} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ t_{p-1,i} & t_{p-2,i} & t_{p-3,i} & \cdots & t_{0,i} \end{bmatrix}.$$

Now we describe a $p(q+1) \times p(q+1)$ exchange matrix $J_{p,q+1}$ as shown below,

$$J_{p,q+1} = \begin{bmatrix} 0 & \cdots & 0 & J_p \\ 0 & \cdots & J_p & 0 \\ \vdots & \cdots & \ddots & \vdots \\ J_p & \cdots & 0 & 0 \end{bmatrix}. \tag{38}$$

Here, J_p is a $p \times p$ matrix obtained by reversing the identity matrix, columns

$$J_p = \begin{bmatrix} 0 & \cdots & 0 & 1 \\ 0 & \cdots & 1 & 0 \\ \vdots & \cdots & \ddots & \vdots \\ 1 & \cdots & 0 & 0 \end{bmatrix}. \tag{39}$$

Here, 0 is the $p \times p$ zero matrix.

4.1 Wax-kailath recursions

The matrices $T_{p,i}$ for $i = 1$ to $q + 1$ contain nested structure and we can characterize these as

$$T_{p,i+1} = \begin{bmatrix} T_{p,i} & \gamma_i \\ \gamma_{-i}^T & T_0 \end{bmatrix}. \tag{40}$$

Here

$$\gamma_i \triangleq \begin{bmatrix} T_i \\ T_{i-1} \\ T_1 \end{bmatrix} = \begin{bmatrix} T_i \\ \gamma_{i-1} \end{bmatrix} \tag{41}$$

and

$$\gamma_{-i}^T = [T_{-i} \;\; T_{-i+1} \;\; \cdots \;\; T_{-1}]. \tag{42}$$

To invert a partitioned block matrix, consider the following formula

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} = \begin{bmatrix} A^{-1} + A^{-1}B(D - CA^{-1}B)^{-1}CA^{-1} & -A^{-1}B(D - CA^{-1}B)^{-1} \\ -(D - CA^{-1}B)^{-1}CA^{-1} & D - CA^{-1}B \end{bmatrix}.$$

Here the matrices A and D have to be invertible. Using the above formula we have,

$$T_{p,i+1}^{-1} = \begin{bmatrix} T_{p,i}^{-1} + T_{p,i}^{-1}\gamma_i \Omega_i^{-1}\gamma_{-i}^T T_{p,i}^{-1} & -T_{p,i}^{-1}\gamma_i \Omega_i^{-1} \\ -\Omega_i^{-1}\gamma_{-i}^T T_{p,i}^{-1} & \Omega_i^{-1} \end{bmatrix}. \tag{43}$$

Here

$$\Omega_i = T_0 - \gamma_{-i}^T T_{p,i}^{-1}\gamma_i, \tag{44}$$

$$W_i = -T_{p,i}^{-1}\gamma_i, \tag{45}$$

$$V_i^T = -\gamma_{-i}^T . T_{p,i}^{-1} \tag{46}$$

Consequently,

$$T_{p,i+1}^{-1} = \begin{bmatrix} T_{p,i}^{-1} + W_i \Omega_i^{-1} V_i^T & W_i \Omega_i^{-1} \\ \Omega_i^{-1} V_i^T & \Omega_i^{-1} \end{bmatrix}. \tag{47}$$

To calculate the above inverse recursively, we require to build up recursive relations for W_i, V_i^T, and Ω_i. To get the recursive relations; we first initiate the following notations,

$$\breve{\gamma}_i^T \triangleq J_{p,i}\gamma_i = \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_i \end{bmatrix}, \tag{48}$$

where $\breve{T}_i = J_p T_i$.

$$\gamma_{-i}^T \triangleq \gamma_{-i}^T J_{p,i} = [\breve{T}_{-1} \;\; \breve{T}_{-2} \;\; \cdots \;\; \breve{T}_{-i}], \tag{49}$$

where $\breve{T}_{-i} = T_{-i}J_p$

The Wax-Kailath Algorithm is out lined below

1.Initialization $(i = 0)$

Set $T_{p,i+1}^{-1} = T_0^{-1}$. Calculate the following,

- $W_{i+1} = -T_0^{-1} T_1$,
- $V_{i+1} = -T_0^{-T} T_{-1}^{-T}$,
- $\Omega_{i+1} = T_0 - T_{-1} T_0^{-1} T_1$,
- $\breve{W}_{i+1} = J_p W_{i+1}$ and $\breve{V}_{i+1} = J_p V_{i+1}$,
- $\gamma_{i+1} = T_1$,
- $\gamma_{-i-1}^T = T_{-1}$.

2. Recursion
For $i = 1$ to $q - 1$

- $\breve{\gamma}_i = J_{p,i} \gamma_i$;
- $T_{i+1} = J_p T_{i+1}$;
- Utilize the equation $\Gamma_i = W_i^T \breve{\gamma}_i + \breve{T}_{i+1}$ to calculate Γ_i;
- Utilize the equation $\breve{W}_{i+1} = \begin{bmatrix} W_i \\ 0_{pxp} \end{bmatrix} - \begin{bmatrix} V_i \\ I_{pxp} \end{bmatrix} \Omega_i^{-T} \Gamma_i$ to calculate $\breve{W}_{i+1}$;
- $\breve{\gamma}_{-i} = (\gamma_{-i}^T J_{p,i})^T$;
- $\breve{T}_{-i-1} = T_{-i-1} J_p$;
- Use equation $\Delta_i = V_i^T \breve{\gamma}_{-i} + \breve{T}_{-i-1}$ to compute Δ_i;
- Utilize equation $\breve{V}_{i+1} = \begin{bmatrix} V_i \\ 0_{p\times p}; \end{bmatrix} - \begin{bmatrix} W_i \\ I_{p\times p}; \end{bmatrix} \breve{\Omega}_i^{-T}$ to calculate $\breve{V}_{i+1}$;
- Compute $\Omega_{i+1} = \Omega_i - \Delta_i \Omega_i^{-T} \Gamma_i$;
- Compute $W_{i+1} = J_{p,i+1} \breve{W}_{i+1}$ and $V_{i+1} = J_{p,i+1} \breve{V}_{i+1}$ for the subsequent iteration;
- Update $\gamma_{i+1} = \begin{bmatrix} T_{i+1} \\ \gamma_i \end{bmatrix}$ and $\gamma_{-i-1}^T = \begin{bmatrix} T_{-i-1} & \gamma_{-i}^T \end{bmatrix}$;

end

3. After calculating the recursions, we have the following: Ω_q, $\breve{W}_q$ $\breve{V}_q$, W_q, and V_q. Using these, we form $T_{p,q+1}^{-1}$ as follows

- Initialize $T_{p,q+1}^{-1} = \begin{bmatrix} J_p \Omega_q^{-T} J_p & J_p \Omega_q^{-T} \breve{W}_q^T \\ \breve{V}_q \Omega_q^{-T} J_p & 0_{pq\times pq} \end{bmatrix}$
- Calculate $B = V_q \Omega_q^{-T} W_q^T - W_q \Omega_q^{-1} V_q^T$
- Using m and g to directory the blocks, we use equation

$$(T_{p,q+1}^{-1})_{m+1,g+1} = (T_{p,q+1}^{-1})_{m,g} + (\breve{V}_q \Omega_q^{-T} \breve{W}_q^T - W_q \Omega_q^{-1} V_q^T)_{m,g}$$

To calculate the remaining blocks of the inverse matrix as follows,

for $m = 1$ to q
    for $g = 1$ to q
    $(T_{p,q+1}^{-1})_{m+1,g+1} = (T_{p,q+1}^{-1})_{m,g} + (B)_{m,g}$
    end
end

The inverse of the block-Toeplitz matrix $T_{p,q+1}$ with Hermitian-Toeplitz blocks is obtained, by means of Wax-Kailath algorithm; nevertheless, our interest is in computing the inverse of the block-Toeplitz covariance matrix R_{q+1} with Hermitian blocks. From Eq. (36) we have $R_{q+1} = T_{p,q+1} - E$, the inverse of the R_{q+1} is $R_{q+1}^{-1} = (T_{p,q+1} - E)^{-1}$, can be computed by means of Neumann series

4.2 Neumann series

Using Neumann series R_{q+1}^{-1} can be written as

$$R_{q+1}^{-1} = (\sum_{k=0}^{\infty}[T_{p,q+1}^{-1}E]^k)T_{p,q+1}^{-1}. \tag{50}$$

If $\rho([T_{p,q+1}^{-1}E]) < 1$,then the series is converged , where $\rho([T_{p,q+1}^{-1}E])$ is the maximum absolute eigenvalue of the matrix $T_{p,q+1}^{-1}E$. which also imply,

$$\lim_{k\to\infty}[T_{p.q+1}^{-1}E]^k = 0. \tag{51}$$

If the series converges, we are able to approximate R_{q+1}^{-1} by means of a finite number of iterations and we can calculate the solution vector estimate. $\hat{\hat{a}} = -\hat{R}_{q+1}^{-1}d$. Using this estimate we can calculate an estimate of the solution of the linear system of equations $Ca = -d$ as follows

$$\hat{a}_{WK} = F_{q+1}\hat{\hat{a}}, \tag{52}$$

where

$$F_{q+1} = \begin{bmatrix} 0 & \cdots & 0 & I_p \\ 0 & \cdots & I_p & 0 \\ \vdots & \cdots & \ddots & \vdots \\ I_p & \cdots & 0 & 0 \end{bmatrix}.$$

Here, I_p is the $p \times p$ identity matrix and 0 is the $p \times p$ zero matrix. The elements of $\hat{a}_{WK}$ are the a_{km} elements in Eq. (2), subsequently calculate the TVAR parameters $a_{k,n}$ as follows

$$a_{k,n} = \sum_{m=0}^{q} a_{km}u_{m,n}.$$

The DTVAR model is a generalization of the time varying autoregressive (TVAR) model, which corresponds to DTVAR with $\Omega = 1$,

5 Instantaneous frequency (IF) estimation

5.1 Tvar based if estimation procedure

Step 1. Compute TVAR model order p and q using MLE algorithm, choose the basis function $u_{m,n}$, $m = 1, 2, \cdots, q$, $n = 1, 2, \cdots, N$, $\Omega = 1$.
Step 2. For covariance technique of signal modeling set $\tau = [p, N-1]$ and with $\Omega = 1$ compute $c_{mg}(l,k)$ by means of Eq. (13) to find the matrix C_{mg} in (16), subsequently, set up the matrix C in (17), as well, use $c_{mg}(l,k)$ to calculate $d_{\min}$ (18).
Step 3. Calculate the TVAR parameters a_km by solving $Ca = -d$ in (20) and form the coefficients $a_{k,n}$ using (2).
Step 4. Solve the roots of the time-varying auto regressive polynomial formed by TVAR linear prediction filter $A(z;n) = 1 + \sum_{k=1}^{p} a_{k,n}z^{-k}$ at each instant n to find the time-varying poles: $P_{i,n}$, $i = 1, 2, \cdots, p$.
Step 5. The instantaneous frequency of the non stationary signal for each sample instant n can be estimated from the instantaneous angles of the poles using the formula $f_{i,n} = \frac{\arg[P_{i,n}])}{2\pi}$ for $|P_{i,n}| \simeq 1$.

5.2 DTVAR based IF estimation procedure

In this section, we demonstrate how the IF estimates can be derived from a DTVAR model parameters. The time-varying filter $H(z;n)$ corresponding to the DTVAR model can be expressed as,

$$H(z;n) = \frac{1}{(A(z;n)} = \frac{1}{1 + \sum_{k=1}^{p} a_{k,n}z^{-k\Omega}}. \tag{53}$$

The weights of the corresponding Time-Varying Prediction Error Filter (TVLPEF) at a given sample instant are given by,

$$a_n = \left[1 \underbrace{0\ 0\ \cdots\ 0}_{\Omega-1}, a_{1,n}, \underbrace{(0\ 0\ \cdots\ 0)}_{\Omega-1}, a_{2,n}, \cdots, a_{p,n} \right]. \quad (54)$$

If there are p FM components in the modeled signal, then by rooting the TVLPEF coefficients a_n in (54) and taking the angles of the poles at every sample instant n, we can achieve the $p\Omega$ notch frequencies. Out of these, p notch frequencies are estimates of the IF of the p FM components. Therefore, we are now faced with the non-trivial problem of identifying the proper p IF estimates from a set of $p\Omega$ frequencies and classification algorithms are required to obtain the correct IF explicitly.

In this paper, we suggest a two step approach to get the correct IF.

Step 1. In the first step, we obtain p rough IF estimates $\widehat{f_{i,n}} (i = 1, 2, \cdots, p)$, using TVAR based IF estimation procedure. In a finitely correlated environment, these estimates are generally poor but still relatively close to the true IF $f_{i,n}$.

Step 2. Next, we use the DTVAR model with decorrelation delay $\Omega > 1$ to obtain $p\Omega$ frequency estimates of which p are expected to be better IF estimates than $\widehat{f_{i,n}}$. From the set of $p\Omega$ frequencies, the ones that are nearby to $\widehat{f_{i,n}}$ for $i = 1, 2, \cdots, p$ are selected as the better IF estimates $\widehat{f_{i,n}}$.

6 Simulation results

To demonstrate the performance of the DTVAR based IF estimator, we compare it with a conventional TVAR based IF estimator. We consider a received signal r_n consisting of a Continuous Phase Modulated (CPM) signal s_n having finite correlation, white noise v_n, and FM interference in as shown below[13].

$$r_n = s_n + v_n + i_n.$$

6.1 Continuous phase modulated (CPM) signal

CPM refers to a class of modulation schemes for which the instantaneous phase is constrained to be continuous. Phase continuity is maintained in CPM signals by altering the phase for the duration of each symbol gradually and by the knowledge of the phase of previous symbols[12].

A CPM signal at complex baseband is defined as

$$S_n = exp(j\varphi(n; I). \quad (55)$$

The instantaneous phase $\varphi(n; I)$ of the CPM signal during the $(m + 1)$ interval is given by,

$$\varphi(n; I) = 2\pi \sum_{k=0}^{m} I_k h G_{n-kv}, (mv + 1) \leq n \leq (m + 1)v. \quad (56)$$

Here, I_k denotes the k^{th} CPM symbol effective over v samples for each symbol and the symbols are selected from M-ary CPM alphabets ± 1, ± 3, ± 5, $\cdots$, $\pm(M - 1)$ with M is constrained to be a power of 2. The constant h denotes the modulation index and determines the amount of phase change that can happen with symbol duration of v samples. The function G_n is called the phase smoothing response and it can be some function whose initial value is greater than zero and final value is 0.5 after a given number of samples L_v .Integer L determines how many symbols are reflected in G_n .

The function G_n is expressed as the integral of some pulse shape g_n with regularly used pulse shaping functions being a Raised Cosine (LRC) and Rectangular (LREC) functions. The class of CPM signals that we consider in this paper uses the 1REC pulse shape with $M = 4$ and is referred to as rectangular quaternary full response CPM. Since the pulse shape is rectangular, the function has the form of a ramp.

From (56), this implies that the instantaneous frequency obtained by differentiating the phase is piecewise constant with each piece representing a symbol time.

To demonstrate the above concepts we show in Fig. 1, and Fig. 2. the real and imaginary parts of a CPM signal for the symbol sequence $[-3, 3, -3, 1, 1, -1, 3, -3, 1, -1]$. The instantaneous phase of CPM Signal is shown in Fig. 3. and instantaneous frequency (IF) of CPM Signal is shown in Fig. 4. The IF (instantaneous frequency) is shown in units of fractional frequency, obtained by dividing the analog frequency in Hertz by the sampling frequency. The following parameters were used in the generation of this CPM signal[12]:

Symbol Rate(R_s) $= 4800$ symbols/sec

Number of samples per symbol $v = 8$

Modulation index $h = 0.25$

Sampling Frequency $(F_s) = vR_s = 38400$ samples/sec

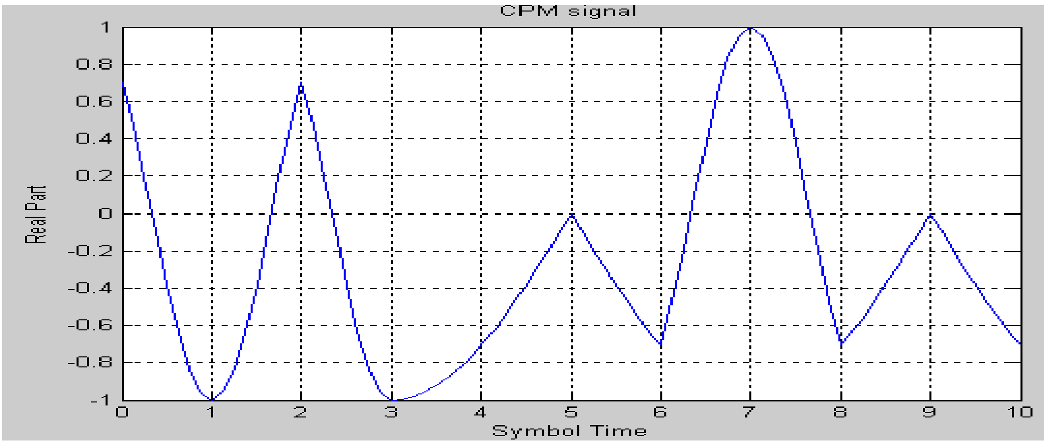

Fig. 1: Real part of CPM Signal

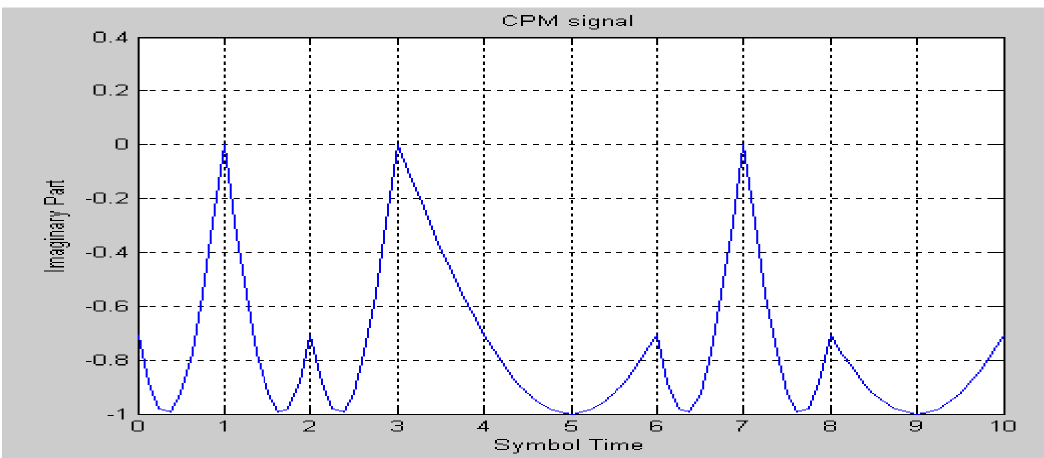

Fig. 2: Imaginary part of CPM Signal

We observe in Fig. 4 that-for each symbol-the IF takes on one of 4 discrete levels, which correspond to the CPM symbols -1, 1, -3 and 3. Denoting this discrete IF level as

$$\beta_k = I_k(\frac{1}{2v})h. \tag{57}$$

The modulation index h is-actually-conveyed to the receiver using a CPM preamble. The number of samples per symbol (v) can be determined from the sampling frequency used at the receiver and the symbol

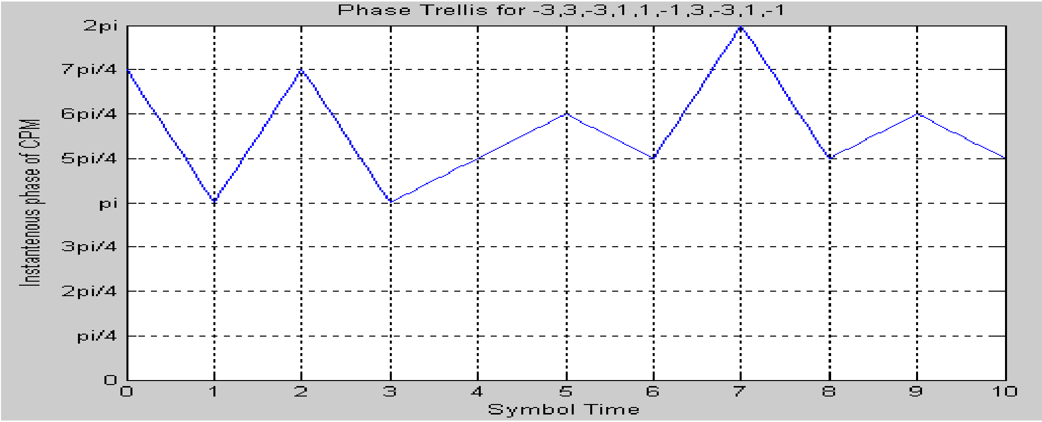

Fig. 3: Instantaneuos phase of CPM Signal

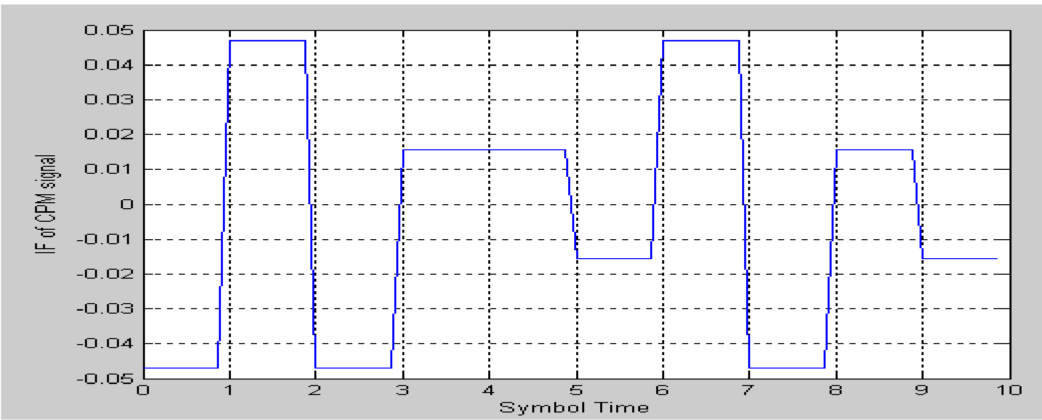

Fig. 4: Instantaneuos Frequency of CPM Signal

rate which is determined from the preamble tone locations. For a given sequence of CPM symbols, (57) can thus be used to obtain $\varphi(n; I)$ in (56). Theoretically, (57) can also be used to estimate CPM symbols given estimates of the CPM IF levels β_k

Simulations are performed at moderate to high SIRs and signal-to-noise ratio (SNR) of 20 dB. IF estimation of a single component linear FM chirp interference and a non-linear FM interference are considered at complex baseband. We have used 6400 symbols corresponding to a CPM burst of length $D = 51200$ to execute the simulations. The reciprocal of the mean squared error (MSE) in the IF estimates as defined below is used as the performance measure

$$MSEi = \frac{1}{D}\sum_{n=0}^{D-1}(|f_{in} - \widehat{f_{i,n}}|^2). \tag{58}$$

6.2 Linear FM interference

The interference signal i_n we use here is a single component linear FM that chirps in frequency from 4800 Hz to 4800 Hz over the entire length of the CPM signal burst. The linear IF law is given by [13],

$$f_n = f_0 + 2\alpha n = f_0 + 2\frac{f_e - f_0}{2(D-1)}n. \tag{59}$$

The FM signal generated is given below,

$$i_n = e^{j2\pi[(f_0+\alpha n)n+\varphi]}. \tag{60}$$

Here, φ is a random initial phase, f_0 is the initial frequency, f_e is the end frequency, and α is the rate of change of frequency. In our example, $f_0 = -4800$Hz, $f_e = 4800$Hz. $f_0 = 0.125$ fractional and $f_e = 0.125$ fractional is shown in units of fractional frequency, obtained by dividing the analog frequency in Hertz by the sampling frequency, and $\alpha = |f_0|/(D-1) = 0.0938$Hz/sample. In our example we have chosen $v(=8)$ samples per each CPM Symbol, hence we can choose the decorrelation delay $\Omega = 8$ in DTVAR Modeling.

The DTVAR model is applied to blocks of the received signal (r_n).In this simulation we have chosen block size $N = 128$ and in order to avoid the end effects in modeling we overlap the blocks by 50% and use the coefficient estimates for the middle of a block. The order of the DTVAR model are computed using MLE Algorithm, with $\Omega = 8$, from the Algorithm we have $p = 1$ and $q = 3$. Also, we use the covariance method of signal modeling and therefore $n = [p\Omega, \cdots, N-1]$.

The DTVAR parameters a_{kn} are derived as shown in Section 2 and the IF estimates are obtained using the two step procedure given in Section 5. In Fig. 5, we show the true and estimated IF with and without a decorrelation delay at a SIR of 5 dB. From Fig. 5, we see that the TVAR based IF estimates deviate significantly from the true IF estimates whereas the DTVAR based IF estimates follow it much more closely. The mean square error (MSE) among the true IF and estimated IF is calculated to -74.365dB for TVAR Model and -138.143dB for DTVAR Model. In Fig. 6, we show the reciprocal of the MSE plotted against SIR, for IF estimation with and without using a decorrelation delay. The decorrelation delay we used here is $\Omega = 8$. For each SIR, the MSE is calculated using 51200 IF estimates.

From Fig. 6, we observe that the DTVAR based IF estimation yields noteworthy performance gains for SIR between -35 dB and -5 dB. When the SIR is more than 0 dB, the power of the correlated signal dominates and IF estimation results for either of the approaches deteriorates. When the SIR falls below -35 dB, the 1/MSE for either of the approaches is high, with the TVAR based IF estimator providing a somewhat better result.

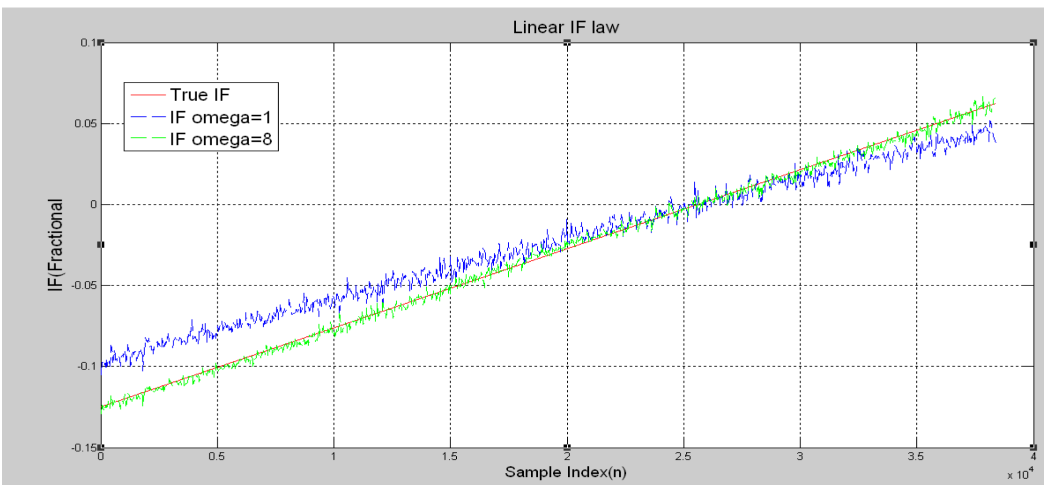

Fig. 5: Linear IF law

6.3 Non-linear FM interference

Now we demonstrate simulation results for a non-linear IF law. We consider a signal having a single FM component with a sinusoidally varying frequency. The order of the DTVAR model are computed using MLE Algorithm, with $\Omega = 8$ from the Algorithm we have $p = 1$ and $q = 3$. The non-linear IF law used in this simulation is given by,

$$f_n = -0.01 + 0.004(0.0009n\cos(0.0009n) + \sin(0.0009n). \tag{61}$$

The FM signal generated is given below,

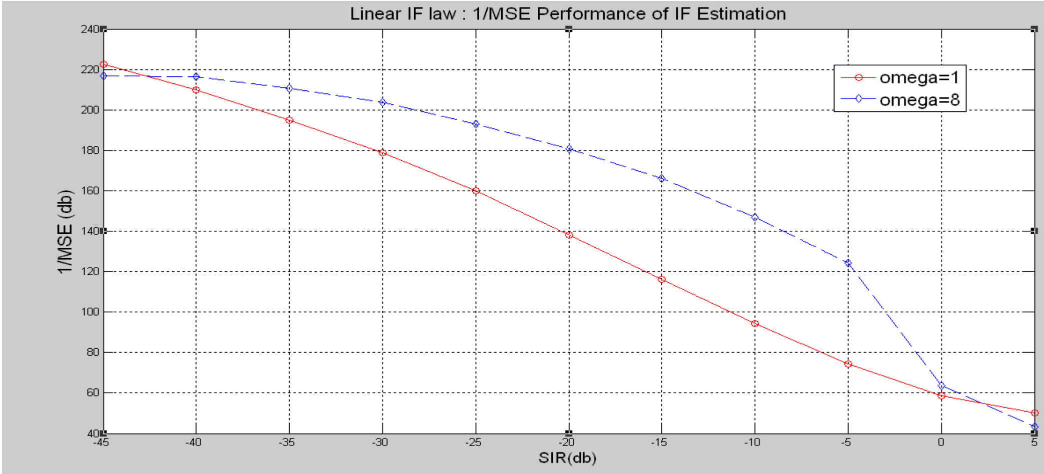

Fig. 6: MSE vs. SIR for IF Estimation of a linear FM signal: TVAR (solid) and DTVAR (dashed)

$$i_n = e^{j2\pi[(-0.01+0.004\sin(0.0009n))n+\varphi]}. \tag{62}$$

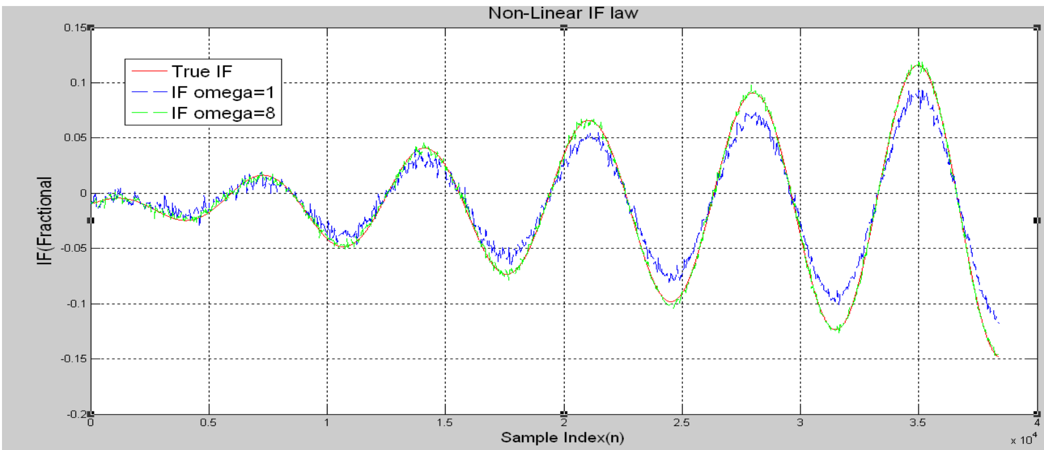

Fig. 7: Non-linear IF law: True (solid), TVAR (Dashed), and DTVAR (plus)

In Fig. 7, we show the true and estimated IF with and without a decorrelation delay at a SIR of -5 dB. From Fig. 7, we see that the DTVAR based IF estimates follow the true IF much more closely than the conventional TVAR based IF estimates. The mean square error (MSE) among the true IF and estimated IF is calculated to -84.651dB for TVAR Model and $-128.~351$dB for DTVAR Model. The reciprocal of the MSE plotted against SIR is shown in Fig. 8 for both of the approaches. In Fig. 8, notable performance gains are observed for the DTVAR for SIR between -20 dB and -5 dB. Although the DTVAR based IF estimator provides superior results at moderate to high SIRs, when the SIR falls below a threshold, the TVAR based approach starts providing better IF estimates than the DTVAR based IF estimator.

7 Conclusion

The TVAR based IF estimator is adequately good and performs fine when used in an uncorrelated environment. nevertheless, its performance degrades when TVAR modeling is applied to a signal containing a finitely correlated signal in addition to the white noise. The performance degradation worsens as the power

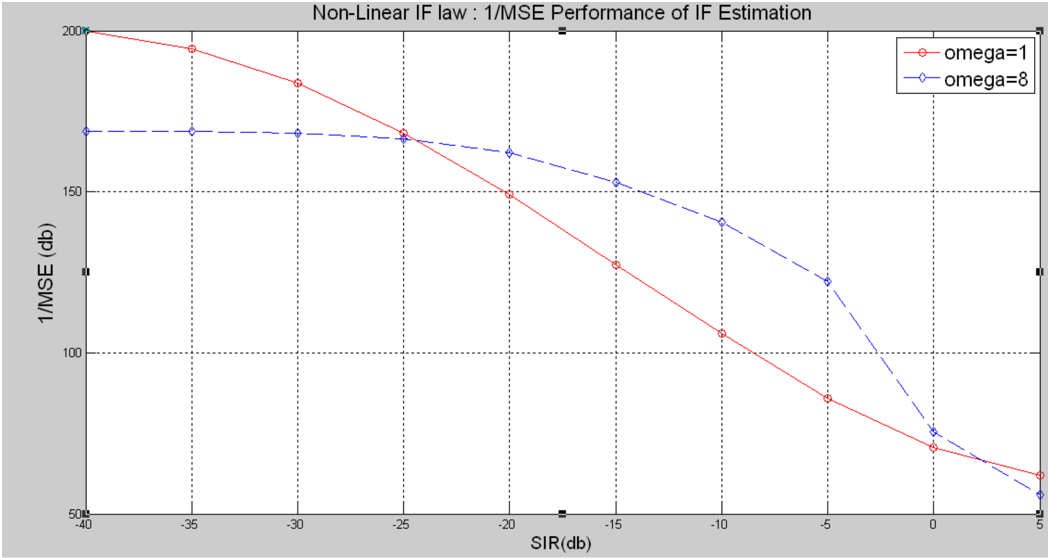

Fig. 8: MSE vs. SIR for IF Estimation of a Non-linear FM signal: TVAR (solid) and DTVAR (dashed)

of the correlated signal starts dominating. The projected Decorrelating TVAR model minimizes this performance degradation and helps restore the IF estimate to satisfactory levels. We showed through simulations for a linear and for a non-linear FM signal, containing a finitely correlated CPM signal, IF estimation using a Decorrelating TVAR model outperforms, IF estimation using the conventional TVAR model when SIR is moderate to high. we also discussed the DTVAR Model order determination through Maximum Likelihood Estimation (MLE) Algorithm.

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