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Revenue management based hospital appointment scheduling*

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Abstract. Hospital resources are limited, a large part of patients could not get timely medical treatment in hospital. Thus, the issue of hospital appointment scheduling is necessary. This research aims at hospital outpatient appointment scheduling which is the key of enhancing hospital efficiency and patients' satisfaction. In this paper, based on the revenue management, we established a hospital outpatient appointment scheduling optimization model with considering both of expert doctors and general doctors. In the model, doctor's salary, hospital's revenue, the patient's needs and doctor's capacity obey normal distribution. By using the expectation operator and chance operator, we make an equivalent transformation of the proposed model to convert the uncertainty mathematical model into a crisp model with full mathematical meaning. Moreover, we use some numerical examples to test the final model in order to show the reasonability and effectiveness.

Keywords: appointment scheduling, revenue management, stochastic programming, expectation operator, chance operator

1 Introduction

In recent years, with the increase of the population and the aging degree, the number of patients has been rapidly rising at the same time. As a result, there exists the "One is hard to find" phenomenon in many large hospitals: patients are usually lined up overnight, even spend more money to buy the "Scalped tickets". However, it is a fact that the hospital resources cannot be used at the same time to serve many people, and also, the unused resources will not generate any revenue. Therefore, hospitals are facing the problem of ensuring the quality of service and increase revenue. As is known to all, the appointment is the first part of treatment. The booking situation directly determines the waiting time for patients' medical treatment. Treatment efficiency is an important factor affecting the entire treatment. Tight treatment schedule causes the patient to wait for a long time, and hospital may lose customers. And if the appointments are too loose, then some doctor will be idle. Therefore, there is a need for hospitals to optimize daily outpatient appointment scheduling, to maximum use of the ability of the doctor, to shorten patients' waiting time, and finally enhance the hospital efficiency and increase the hospital revenue.

The main problem associated with appointment scheduling is to balance patient s' waiting time and doctors' idle time. As Cayirli pointed out that the traditional medical scheduling are designed to improve resource utilization and reduce patients' waiting time[5]. If the demand is greater than the supply of treatment, then waiting happens. Bailey and Lindley studied the hospital appointment system by using queuing theory methods[2, 15]. Gupta and Denton discussed the opportunities and challenges of the reservation applied in clinics, operating rooms and other hospital resources. They proposed that the hospitals need to quickly take treatment for patient and efficiently use all kinds of doctor. In their paper, the doctor's service time is

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divided into multiple slots, reservation staff size can be adjusted according to the patient needs, and a certain amount of time should be left for emergency patients through inventory control[9]. Jebal used two-stage method to deal with scheduling and distribution problem, and put forward a 0-1 linear programming model to minimize the time cost and hospitalization expenses while waiting for surgery[12]. Most of queuing theories are based on the certain steady condition, which is a single queue with the same customer and time of arrival. However, as Bailey pointed out, the certain steady condition is too simple to describe the reality, so these results are usually not suitable the reservation system[3]. Vanden Bosch and Dietz researched the probability of missed appointments during the reservation time[4]. Denton and Gupta considered the patient follow as the general discrete distribution, and proposed a stochastic linear programming model to determine the best appointments[7]. Cayirli and others analyzed the problem from the booking and reservation system to rating criterion, classified the relevant literature of over 50 years, and pointed out the direction of future research[6]. Above all, many scholars have contributed in the area of hospital appointment scheduling[1, 13, 16]. It has been a hot issue in the academic community.

Revenue Management is mainly used in aviation, hotels and other industries, these application industries have a common characteristic: the product will bring no revenue if they are not sold out in a certain period. Therefore, Lieberman proposed that health care industry shares the same feature with the above industries, thus revenue management could also be applied into hospital related problems[14]. Gupta and Wang then showed an application of revenue management in hospital outpatient appointment management[10]. Capability distribution is an important part of the revenue management. According to the price of the product, the reservations amount is determined to meet the needs of a variety of consumer. This theory was first proposed by Littlewood who solved the two price level problems of aircraft positions. Gosavietal, Hamzaee put forward the problem of seats allocation[8, 11]. The study using revenue management in the aviation, hotels and other industries has already showed good performance. Therefore, it's reasonable for us to use revenue management to solve hospital outpatient appointment scheduling problem.

Above all, we can conclude our motivation as follows: ① In the past, the appointment scheduling are considered without revenue management, but this paper will combine them to develop a more comprehensive consideration in the hospital outpatient appointment scheduling problem. ② Several scholars considered the patients arrival as random factor. By inheriting that, this paper not will also deem the generated revenue, the doctor capacities and salaries to be random variable. ③ In this paper, the doctor was divided into expert doctor and general doctor, and introduced the part-time doctor, which is more realistic.

This paper is composed by the following sections. In Section 1, a programming model is established, and an equivalent transformation is also proposed. In Section 3, several experiments are conducted to testify the proposed model. And finally, some conclusions are made in Section 4.

2 Modeling

In general, patients want to get treatment as soon as possible, and they don't expect long waiting time for appointments. If the patient is asked to waiting for a long time, especially for the patients' first time treatment, they might turn to other hospitals for further treatment once they experience a long waiting. In other words, the hospital will lose customers in this case.

The following assumptions are made for the study.

(1) The hospital is divided into departments, the hospital's total revenue are generated by any treatment of each department.
(2) The doctor's working time is divided into a number of slots, the patient can make a reservation by choosing one of the slots accordingly.
(3) The doctor is divided into expert doctor and general doctor.
(4) For a treatment, the revenue generated by the expert doctor is higher than that generated by the general doctor.
(5) For practical reasons, the treatment of part-time doctor can be available if necessary, i.e. when the full-time doctor ability is insufficient, part-time doctor can be hired.

(6) Doctors' salaries are divided into basic wage and performance wages which is related to the number of treatments.
(7) Expert doctors are expected to be the priority choice in the scheduling.

2.1 Notations

The following notations will be used in the proposed mathematical models.

(1) Sets

D : Set of doctors

A : Set of appointment slots

T : Set of treatments

M : Set of departments

(2) Indices

$$d \quad : \quad \text{Doctors, and } d = \begin{cases} 1, & \text{full-time expert doctors} \\ 2, & \text{full-time general doctors} \\ 3, & \text{part-time expert doctors} \\ 4, & \text{part-time general doctors} \end{cases}$$

a : Appointment slot, and $a = 1, 2, \cdots, A$

t : Treatments, and $t = 1, 2, \cdots, T$

m : Hospital departments, and $m = 1, 2, \cdots, M$

(3) Parameters

$|D_m|$: The number of doctors available in department m

$|M|$: The number of departments available in Hospital outpatient department

$|T|$: Total number of treatments

$|N_{mtd}|$: The number of d-type doctors who can give treatment t of in department m, $m \in |M|$, $d \in |D|$, $t \in |T|$

Sal_{md} : d-type doctors' basic daily wage in Department m, $m \in |M|$, $d \in |D|$

$\overline{R_{mdt}}$: Revenue from a d-type doctor who give treatment t once in department m, $m \in |M|$, $d \in |D|$, $t \in |T|$, random variable

$\overline{C_{mdt}}$: Wage for a d-type doctor who gives treatment t once in department m, $m \in |M|$, $d \in |D|$, $t \in |T|$, random variable

$\overline{Cap_{mdt}}$: The number of patients who can be given treatment t in department m in a day, $m \in |M|$, $d \in |D|$, random variable

$\overline{Dem_{mt}}$: Daily Demand of treatment t in department m, $m \in |M|$, $t \in |T|$, random variable

(4) Decision Variables

$$X_{mdat} = \begin{cases} 1, & \text{If } d-\text{type doctor give treatment } t \text{ to the patient in department } m \text{ in slot } a \\ 0, & \text{Otherwise.} \end{cases}$$

Y_{md} : Number of d-type doctor in department m

2.2 Hospital appointment scheduling model

As the first step for a patient who needs to see a doctor, the reservation system is so important. Therefore, the hospital should determine a reasonable number of expert doctors and general doctors, in order to ensure the quality of diagnosis and treatment, and eventually to improve the efficiency and increase the total profits.

Objective Function: As policymakers, hospital managers have a duty to guarantee the quality of medical service, efficiency, and the profits of hospital. Eq. (1) shows the total revenue produced by full-time doctors and part-time doctors.

$$\max \sum_{m=1}^{M}\sum_{d=1}^{4}\sum_{a=1}^{A}\sum_{t=1}^{T}\left(\overline{R_{mdt}}-\overline{C_{mdt}}\right)X_{mdat}-\sum_{m=1}^{M}\sum_{d=1}^{2}Sal_{md}Y_{md}. \tag{1}$$

Constraints:

Doctor's number: Total number of doctors in the reservation system cannot exceed the number of available doctors.

$$\sum_{d=1}^{4}Y_{md}\leq|D_m|\,,\ \ \forall m\in M. \tag{2}$$

Capacity: Total number of patients seen by a doctor cannot exceed the capacity of the doctor.

$$\sum_{a=1}^{A}\sum_{t=1}^{T}X_{mdat}\leq\overline{Cap_{mdt}},\ \ \forall m\in M,\ \ d\in D. \tag{3}$$

Demand: Total number of appointments should not exceed the total demand.

$$\sum_{d=1}^{2}\sum_{a=1}^{A}X_{mdat}\leq\overline{Dem_{mt}}\ \ \forall m\in M,\ \ t\in T. \tag{4}$$

Eq. (5) ensures a patient can choose at least one doctor in the appointment.

$$X_{mdat}\leq N_{mtd},\ \ \forall m\in M,\ \ d\in D,\ \ a\in A,\ t\in T. \tag{5}$$

Eq. (6) ensures that each full-time doctor will be assigned at least one patient.

$$\sum_{a=1}^{A}\sum_{t=1}^{T}X_{mdat}\geq 1,\ \ \forall m\in M,\ \ d=1,2. \tag{6}$$

Eq. (7) ensures that no doctor is assigned to more than one patient at the same time.

$$\sum_{t=1}^{T}X_{mdat}\leq 1,\ \ \forall m\in M,\ \ d\in D,\ \ a\in A. \tag{7}$$

Above all, the whole model (8) is given by

$$\begin{aligned}&\max \sum_{m=1}^{M}\sum_{d=1}^{4}\sum_{a=1}^{A}\sum_{t=1}^{T}\left(\overline{R_{mdt}}-\overline{C_{mdt}}\right)X_{mdat}-\sum_{m=1}^{M}\sum_{d=1}^{2}Sal_{md}Y_{md}\\&s.t.\begin{cases}\sum_{d=1}^{4}Y_{md}\leq|D_m|\,,\ \ \forall m\in M\\ \sum_{a=1}^{A}\sum_{t=1}^{T}X_{mdat}\leq\overline{Cap_{mdt}},\ \ \forall m\in M,\ \ d\in D\\ \sum_{d=1}^{2}\sum_{a=1}^{A}X_{mdat}\leq\overline{Dem_{mt}},\ \ \forall m\in M,\ \ t\in T\\ X_{mdat}\leq N_{mtd},\ \ \forall m\in M,d\in D,a\in A,t\in T\\ \sum_{a=1}^{A}\sum_{t=1}^{T}X_{mdat}\geq 1,\ \ \forall m\in M,\ \ d=1,2\\ \sum_{t=1}^{T}X_{mdat}\leq 1,\ \ \forall m\in M,\ \ d\in D,\ \ a\in A.\end{cases}\end{aligned} \tag{8}$$

Because $\overline{R_{mdt}}, \overline{C_{mdt}}, \overline{Cap_{md}}, \overline{Dem_{mt}}$ are random variables, model (8) can not be solved directly. Therefore, we employ the expectation operator and the chance operator to handle the model as follows:

For objective function (1), we maximize the expected value of the total revenue.

$$\max \mathrm{E}\left[\sum_{m=1}^{M}\sum_{d=1}^{4}\sum_{a=1}^{A}\sum_{t=1}^{T}\left(\overline{R_{mdt}}-\overline{C_{mdt}}\right)X_{mdat}-\sum_{m=1}^{M}\sum_{d=1}^{2}Sal_{md}Y_{md}\right]. \tag{9}$$

For constraint (3), we know that $\sum_{a=1}^{A}\sum_{t=1}^{T}X_{mdat}\leq\overline{Cap_{mdt}}$ is actually a random event because of the random coefficient $\overline{Cap_{md}}$, and we can guarantee the possibility of the event under a given confidence level α, that is,

$$P\left(\sum_{a=1}^{A}\sum_{t=1}^{T}X_{mdat}\leq\overline{Cap_{mdt}}\right)\geq\alpha,\ \ \forall m\in M,\ d\in D. \tag{10}$$

For constraint (4), we can obtain the following Eq. (11) similarly.

$$P\left(\sum_{d=1}^{2}\sum_{a=1}^{A}X_{mdat}\leq\overline{Dem_{mt}}\right)\geq\beta,\ \ \forall m\in M,\ t\in T. \tag{11}$$

Above all, we have the following model (12) which has a clear mathematical meaning:

$$\begin{aligned}&\max \mathrm{E}\left[\sum_{m=1}^{M}\sum_{d=1}^{4}\sum_{a=1}^{A}\sum_{t=1}^{T}\left(\overline{R_{mdt}}-\overline{C_{mdt}}\right)X_{mdat}-\sum_{m=1}^{M}\sum_{d=1}^{2}Sal_{md}Y_{md}\right]\ ①\\&s.t.\begin{cases}\sum_{d=1}^{4}Y_{md}\leq|D_m|,\ \ \forall m\in M\\P\left(\sum_{a=1}^{A}\sum_{t=1}^{T}X_{mdat}\leq\overline{Cap_{mdt}}\right)\geq\alpha,\ \ \forall m\in M,\ d\in D\ ②\\P\left(\sum_{d=1}^{2}\sum_{a=1}^{A}X_{mdat}\leq\overline{Dem_{mt}}\right)\geq\beta,\ \ \forall m\in M,\ t\in T\ ③\\X_{mdat}\leq N_{mtd},\forall m\in M,d\in D,a\in A,t\in T\\\sum_{a=1}^{A}\sum_{t=1}^{T}X_{mdat}\geq 1,\ \ \forall m\in M,\ d=1,2\\\sum_{t=1}^{T}X_{mdat}\leq 1,\ \ \forall m\in M,\ d\in D,\ a\in A.\end{cases}\end{aligned} \tag{12}$$

Model (12) optimizes the expected objective value which is subject to several chance constraints.

2.3 Equivalent transformation

After investigation and statistical tests, we can conclude that $\overline{R_{mdt}}$, $\overline{C_{mdt}}$, $\overline{Cap_{md}}$, $\overline{Dem_{mt}}$ obey the normal distributions: $\overline{R_{mdt}}\sim N\left(\mu_{\overline{R_{mdt}}},\sigma^2_{\overline{R_{mdt}}}\right)$, $\overline{C_{mdt}}\sim N\left(\mu_{\overline{C_{mdt}}},\sigma^2_{\overline{C_{mdt}}}\right)$, $\overline{Cap_{mdt}}\sim N\left(\mu_{\overline{Cap_{mdt}}},\sigma^2_{\overline{Cap_{mdt}}}\right)$, $\overline{Dem_{mt}}\sim N\left(\mu_{\overline{Dem_{mt}}},\sigma^2_{\overline{Dem_{mt}}}\right)$. We assume that they mutually independent, the following lemmas are introduced to transform the models.

Lemma 1. $\mathrm{E}\left[\sum_{m=1}^{M}\sum_{d=1}^{4}\sum_{a=1}^{A}\sum_{t=1}^{T}\left(\overline{R_{mdt}}-\overline{C_{mdt}}\right)X_{mdat}-\sum_{m=1}^{M}\sum_{d=1}^{2}Sal_{md}Y_{md}\right]$ *is equivalent to* $\sum_{m=1}^{M}\sum_{d=1}^{4}\sum_{a=1}^{A}\sum_{t=1}^{T}\left(\mu_{\overline{R_{mdt}}}-\mu_{\overline{C_{mdt}}}\right)X_{mdat}-\sum_{m=1}^{M}\sum_{d=1}^{2}Sal_{md}Y_{md}$ ④.

Proof. According to the expected value theorem of normal distribution, it can be proved as follows:

$$
\begin{aligned}
&\mathrm{E}\left[\sum_{m=1}^{M}\sum_{d=1}^{4}\sum_{a=1}^{A}\sum_{t=1}^{T}\left(\overline{R_{mdt}}-\overline{C_{mdt}}\right)X_{mdat}-\sum_{m=1}^{M}\sum_{d=1}^{2}C_{md}Y_{md}\right]\\
&=\sum_{m=1}^{M}\sum_{d=1}^{4}\sum_{t=1}^{T}\mathrm{E}\left(\overline{R_{mdt}}-\overline{C_{mdt}}\right)\sum_{m=1}^{M}\sum_{d=1}^{4}\sum_{a=1}^{A}\sum_{t=1}^{T}X_{mdat}-\sum_{m=1}^{M}\sum_{d=1}^{2}C_{md}Y_{md}\\
&=\sum_{m=1}^{M}\sum_{d=1}^{4}\sum_{t=1}^{T}\left(\mathrm{E}[\overline{R_{mdt}}]-\mathrm{E}[\overline{C_{mdt}}]\right)\sum_{m=1}^{M}\sum_{d=1}^{4}\sum_{a=1}^{A}\sum_{t=1}^{T}X_{mdat}-\sum_{m=1}^{M}\sum_{d=1}^{2}C_{md}Y_{md}\\
&=\sum_{m=1}^{M}\sum_{d=1}^{4}\sum_{a=1}^{A}\sum_{t=1}^{T}\left(\mu_{\overline{R_{mdt}}}-\mu_{\overline{C_{mdt}}}\right)X_{mdat}-\sum_{m=1}^{M}\sum_{d=1}^{2}C_{md}Y_{md}.
\end{aligned}
$$

Lemma 2. *$P\left(\sum_{a=1}^{A}\sum_{t=1}^{T}X_{mdat}\le\overline{Cap_{mdt}}\right)\ge\alpha$ is equivalent to $\sum_{a=1}^{A}\sum_{t=1}^{T}X_{mdat}\le\sigma_{\overline{Cap_{mdt}}}\phi^{-1}_{(1-\alpha)}+\mu_{\overline{Cap_{mdt}}}$ ⑤.*

Proof. By the definition of normal distribution function, we can get

$$
\begin{aligned}
&P\left(\sum_{a=1}^{A}\sum_{t=1}^{T}X_{mdat}\le\overline{Cap_{mdt}}\right)\ge\alpha\\
&\Leftrightarrow P\left(\frac{\sum_{a=1}^{A}\sum_{t=1}^{T}X_{mdat}-\mu_{\overline{Cap_{mdt}}}}{\sigma_{\overline{Cap_{mdt}}}}\le\frac{\overline{Cap_{mdt}}-\mu_{\overline{Cap_{mdt}}}}{\sigma_{\overline{Cap_{mdt}}}}\right)\ge\alpha\\
&\Leftrightarrow 1-\phi\left(\frac{\sum_{a=1}^{A}\sum_{t=1}^{T}X_{mdat}-\mu_{\overline{Cap_{mdt}}}}{\sigma_{\overline{Cap_{mdt}}}}\right)\ge\alpha\\
&\Leftrightarrow \phi\left(\frac{\sum_{a=1}^{A}\sum_{t=1}^{T}X_{mdat}-\mu_{\overline{Cap_{mdt}}}}{\sigma_{\overline{Cap_{mdt}}}}\right)\le 1-\alpha\\
&\Leftrightarrow \frac{\sum_{a=1}^{A}\sum_{t=1}^{T}X_{mdat}-\mu_{\overline{Cap_{mdt}}}}{\sigma_{\overline{Cap_{mdt}}}}\le\phi^{-1}_{(1-\alpha)}\\
&\Leftrightarrow \sum_{a=1}^{A}\sum_{t=1}^{T}X_{mdat}\le\sigma_{\overline{Cap_{mdt}}}\phi^{-1}_{(1-\alpha)}+\mu_{\overline{Cap_{mdt}}}.
\end{aligned}
$$

Lemma 3. *$P\left(\sum_{d=1}^{2}\sum_{a=1}^{A}X_{mdat}\le\overline{Dem_{mt}}\right)\ge\beta$ is equivalent to $\sum_{d=1}^{2}\sum_{a=1}^{A}X_{mdat}\le\sigma_{\overline{Dem_{mt}}}\phi^{-1}_{(1-\beta)}+\mu_{\overline{Dem_{mt}}}$ ⑥.*

Proof. The proof is similar to lemma 2, and thus omitted.

Hence, we have the equivalent transformation model (13):

$$
\begin{aligned}
&\max E\left[\sum_{m=1}^{M}\sum_{d=1}^{4}\sum_{a=1}^{A}\sum_{t=1}^{T}\left(\mu_{\overline{R_{mdt}}}-\mu_{\overline{C_{mdt}}}\right)X_{mdat}-\sum_{m=1}^{M}\sum_{d=1}^{2}Sal_{md}Y_{md}\right] \quad ④\\
&s.t.\begin{cases}
\sum_{d=1}^{4}Y_{md}\le|D_m|\,,\ \forall m\in M\\
\sum_{a=1}^{A}\sum_{t=1}^{T}X_{mdat}\le\sigma_{\overline{Cap_{mdt}}}\phi^{-1}_{(1-\alpha)}+\mu_{\overline{Cap_{mdt}}},\ \forall m\in M,\ d\in D \quad ⑤\\
\sum_{d=1}^{2}\sum_{a=1}^{A}X_{mdat}\le\sigma_{\overline{Dem_{mt}}}\phi^{-1}_{(1-\beta)}+\mu_{\overline{Dem_{mt}}},\ \forall m\in M,\ t\in T \quad ⑥\\
X_{mdat}\le N_{mtd},\forall m\in M,d\in D,a\in A,\ t\in T\\
\sum_{a=1}^{A}\sum_{t=1}^{T}X_{mdat}\ge 1,\ \forall m\in M,\ d=1,2\\
\sum_{t=1}^{T}X_{mdat}\le 1,\ \forall m\in M,\ d\in D,\ a\in A.
\end{cases}
\end{aligned}
\tag{13}
$$

3 Experimentation

We employ a series of experimentations to test the model discussed above. The text can be divided into two cases: ① the patients' demands are larger than the capacity of the hospital doctors, and ② the patients' demand is less than the capacity of the hospital doctors. ③ The patients' demands are larger than the doctors' capacities, but the diagnosis and treatments can be increased.

Suppose that there are four departments in a professional maternal and child health care hospital, among them $m = 1$ is the gynecology department, $m = 2$ is the obstetric department, $m = 3$ is the rehabilitation center, and $m = 4$ is pediatric center for baby care. Doctors in each department are divided into two kinds, one is expert doctors and we use $d = 1$ to represent; the other is a general doctor for $d = 2$. Each department only makes its professional treatment, so $t = 1$. The parameters are shown in Tab. 1.

Table 1: Parameters

m	$m=1$		$m=2$		$m=3$		$m=4$	
d	$d=1$	$d=2$	$d=1$	$d=2$	$d=1$	$d=2$	$d=1$	$d=2$
Y_{11}	3	3	7	6	2	2	4	5
Sal_{md}	100	80	150	120	100	80	200	180
$\mu_{\overline{C_{mdt}}}$	20	10	40	30	20	10	30	20
$\mu_{\overline{R_{mdt}}}$	200	180	360	300	280	250	380	320
$\mu_{\overline{Cap_{mdt}}}$	150	120	28	18	30	20	40	30
$\sigma_{\overline{Cap_{mdt}}}$	20	20	5	5	5	5	10	10

Hospital is aware of the information of their full-time doctors, such as, they know each full-time doctor's capacity, and they will put the capacities of every doctor in full consideration first when they arrange scheduling. We set the confidence coefficients $\alpha = \beta = 0.95$ and conduct the following examinations:

1. The patients' demands are less than the treatment capacities of the total doctors. Specifically, the demands of patients for four departments is $\mu_{\overline{Dem_{11}}} = 200$, $\sigma_{\overline{Dem_{11}}} = 20$; $\mu_{\overline{Dem_{21}}} = 60$, $\sigma_{\overline{Dem_{21}}} = 10$; $\mu_{\overline{Dem_{31}}} = 30$, $\sigma_{\overline{Dem_{31}}} = 10$; and $\mu_{\overline{Dem_{41}}} = 60$, $\sigma_{\overline{Dem_{41}}} = 20$ respectively. We use the seven steps iterative method in the of software Matlab to obtain the maximized benefit, which is 49740 RMB.

When we reduce the confidence levels to $\alpha = 0.95$, $\beta = 0.80$ and can obtain the optimized benefit is 56620 RMB, the number of appointments and doctors are both increased. On the contrary, when we enhance the confidence levels to $\alpha = 0.95$, $\beta = 0.99$, the maximized benefit is 41490 RMB, and the number of appointments and doctors are both declined. The optimization results are shown in Tab. 2:

Table 2: Hospital appointment scheduling (When the patients' demand are less than the doctors' capacities)

Degree of confidence	Department / Doctor type	Gynecology Expert doctor	Gynecology General doctor	Obstetrics Expert doctor	Obstetrics General doctor	Rehabilitation Expert doctor	Rehabilitation General doctor	Pediatrics Expert doctor	Pediatrics General doctor
$\alpha = 0.95$ $\beta = 0.95$	Number of appointment	118	50	20	10	13	1	24	4
	Number of doctor	3	2	7	6	1	1	3	1
$\alpha = 0.95$ $\beta = 0.99$	Number of appointment	118	66	20	10	21	1	24	14
	Number of doctor	3	2	7	6	2	1	4	5
$\alpha = 0.95$ $\beta = 0.80$	Number of appointment	118	36	20	10	6	1	13	1
	Number of doctor	3	1	7	3	1	1	1	1

2. The patients' demands are more than the doctors' capacities. we assume that the demands of patients for four departments are $\mu_{\overline{Dem_{11}}} = 300$, $\sigma_{\overline{Dem_{11}}} = 20\mu_{\overline{Dem_{21}}} = 60$, $\sigma_{\overline{Dem_{21}}} = 10$; $\mu_{\overline{Dem_{31}}} = 50$, $\sigma_{\overline{Dem_{31}}} = 10$; and $\mu_{\overline{Dem_{41}}} = 80$, $\sigma_{\overline{Dem_{41}}} = 20$ respectively. We can get that the maximized benefits are 62990 RMB (with confidence levels $\alpha = \beta = 0.95$), 60760 RMB (with confidence levels $\alpha = 0.95$, $\beta = 0.99$), and 62990 RMB (with confidence levels $\alpha = 0.95$, $\beta = 0.80$), respectively. The optimization results are shown in Tab. 3:

Table 3: Hospital appointment scheduling at every confidence (When the patients' demands are larger than the doctors' capacities)

Degree of confidence	Department	Gynecology		Obstetrics		Rehabilitation		Pediatrics	
	Doctor type	Expert doctor	General doctor	Expert doctor	General doctor	Expert doctor	General doctor	Expert doctor	General doctor
$\alpha = 0.95$ $\beta = 0.95$	Number of appointment	118	88	20	10	22	12	24	14
	Number of doctor	3	3	7	6	2	2	4	5
$\alpha = 0.95$ $\beta = 0.99$	Number of appointment	118	88	20	10	22	5	24	10
	Number of doctor	3	3	7	6	2	1	4	2
$\alpha = 0.95$ $\beta = 0.80$	Number of appointment	118	88	20	10	22	12	24	14
	Number of doctor	3	3	7	6	2	2	4	5

3. When the patients' demands are larger than the doctors' capacities, we can improve the doctors' capacities or employ some part-time doctors to increase diagnosis and treatments. We set the confidence levels, and repeat the experiments.

① We improve the full-time doctors' capacity through training or other ways to enhance the amount of treatments. We assume that the capacity of doctors for gynecology departments is $\mu_{\overline{Cap_{11}}} = 160$, $\sigma_{\overline{Cap_{11}}} = 20$; $\mu_{\overline{Cap_{12}}} = 130$, $\sigma_{\overline{Cap_{12}}} = 20$, and we obtain the optimized benefit is 66490 RMB, and the appointment scheduling results are shown in Tab. 4:

Table 4: Hospital appointment scheduling ($\alpha = 0.95$, $\beta = 0.80$)

Department	Gynecology		Obstetrics		Rehabilitation		Pediatrics	
Doctor type	Expert doctor	General doctor	Expert doctor	General doctor	Expert doctor	General doctor	Expert doctor	General doctor
Number of appointment	128	98	20	10	22	12	24	14
Number of doctor	3	3	7	6	2	2	4	5

② We increase the number of doctors by hiring the part-time doctors to enhance the amount of treatments. In the following case, the gynecology department hire one expert part-time expert doctor, whose average revenue and wage are $\mu_{\overline{R131}} = 500$, $\mu_{\overline{C_{131}}} = 100$ respectively. We solve the model again and can get that the maximized benefit is 82990RMB. The appointment scheduling results are shown in Tab. 5:

4 Conclusion

In this paper, based on revenue management, an appointment scheduling model with random coefficients is built. And then we convert the unsolvable uncertain mathematical model into a crisp model with clear

Table 5: Hospital appointment scheduling ($\alpha = 0.95$, $\beta = 0.80$)

Department	Gynecology		Obstetrics		Rehabilitation		Pediatrics	
Doctor type	Expert doctor	General doctor	Expert doctor	General doctor	Expert doctor	General doctor	Expert doctor	General doctor
Number of appointment	128	98	20	10	22	12	24	14
Number of doctor	3	3	7	6	2	2	4	5

mathematical meaning. Finally, in the optimization experiments, we obtain the number of working doctors, appointment and the maximum hospital revenue. According to the experimentation, we can conclude that: on one hand, when the demand is bigger, doctors can improve their capacities to increase the number of diagnosis and treatments. On the other hand, if it is hard for doctors to improve capacities, then we can hire more external part-time doctors to raise the hospital's avenue. In addition, we can also get that the confidence levels of the decision maker will affect the hospital's appointment scheduling.

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