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On a distribution function and reliability parameters of busy period for pump stations during continuos operation

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Abstract. In this paper, the distribution function of continuous operation time of pump station and their parameters are investigated. The reliability of pump station is calculated in the range of hours and its graphcs is given. It was derived that a reliability of the pump station is very low. The optimal number and reliability of pump units in the station are calculated according to normative documents.

Keywords: reliability, shut-down, distribution law, parameter, continuous operation, optimal number, criterion, accuracy level, critical value

1 Introduction

Subject to natural conditions for different regions, depending on existing requirements for water, the pump stations are widely used in irrigation and hydromeliorative systems. Investigation of these problems is important not only for southern countries but also for any other countries. Mathematical bases of reliability theory, which also can be applied for these systems are given in [2, 4]. However investigation of each problem leads to construction of new mathematical models and methods for effective study of these models. In this paper, new mathematical models taking into consideration local features and new methods for their investigation are suggested.

The reliability parameters of pump stations should correspond to reliability parameters for hydromeliorotive systems. The structure of equipments in pump stations, their structural properties, the type and amount of main and additional equipments, the principles of complex use of water sources, minimality of the cost value should be taken into consideration with the technologial parameters of pump stations at their operation[3].

The low reliability of pump stations will increase their operational costs and decreases their economical efficiency. At the same time, shut-down of low efficiency pump stations causes financial losses.

Mainly, the low level of the effective use of electic power is connected with wrong choice of optimal number and characteristics of pump units.

2 Problem statement

The subject of the investigation is “Yeni Mugan” pump station mounted on Northern collector (NC-5) in Salyan region of Azerbaijan. This pump station consists four pump units (2 pieces 16 NDN and 2 pieces 20 NDN) and has been using since 1978. The operating hours of the pump station for a year is given in the Tab. 1.

As for zero hypothesis, we assume that variation series ($t_1 \prec t_2 \prec \cdots \prec t_r$) of operating hours of pump units to shut-down is subjected to two-parameter Weibull distribution. If the zero hypothesis is rejected, then

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Table 1. Actual operating hours of "Yeni Mugan" pump station

№	By months										Type of pump
	II	III	IV	V	VI	VII	VIII	IX	X	XI	
1	261	642	590	382	217	181	218	117	240	168	16NDN
2	423	420	210	346	480	216	240	154	-	-	16NDN
3	479	362	665	332	198	258	-	-	-	-	20NDN
4	225	126	288	300	-	-	-	-	-	-	20NDN

other distributions, including three parameter Weibull distribution may be considered. Various authors used only the criterions of compliance developed for Weibull distribution[6]. Making the substituion $x_i = \ln t_i (i = 1, 2, \cdots, r)$ the statistics based on the criterion of compliance will take the following form [5, 6].

$$S = \frac{\sum\limits_{i=(r/2)+1}^{r-1} [\frac{x_{i+1}-x_i}{M_i}]}{\sum\limits_{i=1}^{r-1} [\frac{(x_{i+1}-x_i)}{M_i}]}, \tag{1}$$

where r is the number of shut-downs, the values of M_i and S are found from special tables.

NC-5 collector occupies 13 thousand hectare[1]. Therefore, "Yeni Mugan" pump station belongs to the IV group and, an admissible reliability level should be $P_b = 0.900$.

The optimal number of the main pump units can be calculated using the following relation[7, 9]:

$$N = (1/2)(N_1 + N_2), \tag{2}$$

$$N_1 = (Q_{\max}/Q_{\min}) + l, \tag{3}$$

$$N_2 = Q_{\max}/\varDelta\bar{Q}, \tag{4}$$

$$\varDelta\bar{Q} = (l/n) \sum |Q_i - Q_{i+1}|, \tag{5}$$

where N_1, N_2 is the number of the main pumps, $Q_{\max}$, $Q_{\min}$ is maximum and minimum consumption of the collector; Q_i, $Q_{i+1}$are consumptions of the collector by months; $\varDelta\bar{Q}$ is an average gain; n is the number of changing collector's consumption.

3 Problem solution

Tab. 2 shows values of observation criterion for the first pump unit. $S_m \approx 0,72$ is obtained according to

Table 2. Operating time of the first pump unit

i	t_i	$x_i = \ln t_i$	M_i	$x_{i+1} - x_i$	$x_{i+1} - x_i/M_i$
1	117	47,622	1,054	0,3618	0,3433
2	168	51,240	0,559	0,0750	0,1342
3	181	51,985	0,399	0,1814	0,4546
4	217	53,799	0,325	0,0046	0,0140
5	218	53,845	0,286	0,0961	0,3360
6	240	54,806	0,269	0,0840	0,3123
7	261	55,645	0,271	0,3809	14,055
8	382	59,454	0,301	0,4347	14,440
9	590	63,801	0,405	0,0850	0,2086
10	642	64,646			

the expression (1). Using special tables [5, 6], it was determined that the critical value of the main criterion is $S_b = 0, 69$, based on accuracy level $r = 10$ and $\alpha = 0, 05$.

For $S_m \succ S_b$ in this case, it was determined that the operating hours of the first pump station between the shut-downs does not correspond to two-parameter Weibull distribution.

It is known that the minimal operating coefficient in three-parameter Weibull distribution law is located between the smallest value of statistics and zero $(0 \leq \delta \leq x_{\min})$. Using this method, we can pass from three-parameter Weibull distribution to two-parameter Weibull distribution and check the compliance[5].

Table 3. Operating hours of a pump unit with regard to minimal parameter δ

i	t_i	$x_i = \ln t_i$	M_i	$x_{i+1} - x_i$	$x_{i+1} - x_i/M_i$
1	12	2,4849	1,054	1,6582	1.5732
2	63	4,1431	0,559	0,1876	0,3356
3	76	4,3307	0,399	0,3878	0.9719
4	112	4,7185	0,325	0,0089	0,0274
5	113	4,7274	0,286	0,1779	0,6220
6	135	4,9053	0,269	0,1446	0,5376
7	156	5,0499	0,271	0,5741	2,1185
8	277	5,6240	0,301	0,5602	1.8611
9	485	6,1842	0,405	0,1418	0.3501
10	537	6,2860			

In this case, δ parameter may be taken as 90% of the minimal value of the series i.e. as $\delta \approx 105$. If we take into consideration the value of δ, for statistics series, we get Tab. 3. According to the Tab. 3, $S_m = 0,54$. For $S_m \prec S_b$, the distribution corresponds to two-parameter Weibull distribution, and the distribution function

$$F(x; a, b) = 1 - e^{-(\frac{t}{a})^b}, (x \geq 0), \tag{6}$$

where a is a scale or resource parameter, b is a form parameter or a slope. Using the substitution $x = \ln t$ we have

$$F(x) = 1 - \exp\left[-\exp(\frac{x-u}{v})\right], (-\infty \prec x \prec +\infty), \tag{7}$$

where $u = \ln a$ and $v = \frac{1}{b}$. If the reliability of the unit is known, for finding the random variable x, we can present the expression (7) in the following form:

$$x = u + v\left[\ln(\ln\frac{1}{P})\right]. \tag{8}$$

For $r = 10$ linear weighted sets found from special tables are given in Tab. 4. Making the substitution $x_i =$

Table 4. For $r = 10$ linear weight sets

t_i	$x_i = \ln t_i$	a_i	c_i
12	2,4849	0,006411	−0,080881
63	4,1431	0,015598	−0,085171
76	4,3307	0,025675	−0,083952
112	4,7185	0,036799	−0,078714
113	4,7274	0,049211	−0,069610
135	4,9053	0,063256	−0,056237
156	5,0499	0,079438	−0,037675
277	5,6240	0,098522	−0,012272
485	6,1842	0,121752	0,022956
537	6,2860	0,503338	0,481555

$\ln t_i$ the coefficients in the expression (7) are calculated as the expression:

$$\overline{u} = \sum_{i=1}^{r} a_i \times x_i, \tag{9}$$

$$\overline{v} = \sum_{i=1}^{r} c_i \times x_i. \tag{10}$$

Here a_i and c_i are linear weight multiplicands. The first, second parameters and reliability of the Veibulla distribution are calculated as follows[5].

$$\overline{a} = e^{\overline{u}}, \tag{11}$$

$$\overline{b} = \frac{1}{\overline{v}}, \tag{12}$$

$$\overline{P}(t) = \exp\left(-\frac{t}{a}\right)^{b}. \tag{13}$$

As from expressions (9) and (10) $\overline{u} = 5,7805$, $\overline{v} = 1,0160$, in the expressions (11), (12) $\overline{a} = 324 hour$, $\overline{b} = 0,98$. Then from the relation (13), in the general case we get

$$\overline{P}_1(t) = \exp\left(-\frac{t}{a}\right)^{b} = \exp\left(-\frac{t}{324}\right)^{0.98}. \tag{14}$$

Tab. 5 shows the law of continuous operation time distribution of the second pump unit. According to the

Table 5. Operating hours of the second pump unit

i	t_i	$x_i = \ln t_i$	M_i	$x_{i+1} - x_i$	$x_{i+1} - x_i / M_i$
1	154	5,037	1,068252	0,31	0,2902
2	210	5,347	0,577339	0,028	0,0485
3	216	5,375	0,422889	0,106	0,2507
4	240	5,481	0,356967	0,365	1,0225
5	346	5,846	0,334089	0,194	0,5807
6	420	6,040	0,349907	0,007	0,0200
7	423	6,047	0,449338	0,127	0,2826
8	480	6,174	-	-	-

expression (1), $S_m = 0,3540$. Taking the number of shut-downs $r = 8$ and accuracy level $\alpha = 0,05$, we find that $S_b = 0,71$. For $S_m \prec S_b$, distribution of continuous operation time of the second pump unit corresponds to the Weibull distribution.

Table 6. For $r = 8$ linear weight sets

t_i	$x_i = \ln t_i$	a_i	c_i
154	5,037	−0,026	−0,109
210	5,347	−0,015	−0,112
216	5,375	−0,001	−0,108
240	5,481	0,016	−0,097
346	5,846	0,036	−0,081
420	6,040	0,060	−0,059
423	6,047	0,088	−0,030
480	6,174	0,843	0,597

From (9) and (10) it follows $\overline{u} = 6,181$, $\overline{v} = 0,415$, from the expression (11) and (12) it follows $\overline{a} = 484$ hour, $\overline{b} = 2,41$. In this case, formula (13) should be calculated from expression:

$$\overline{P}_2(t) = \exp\left(-\frac{t}{484}\right)^{2,41}. \quad (15)$$

Tab. 7 shows the law of continuous operation time distribution of the third pum unit. If we take into account

Table 7. Operating hours of the third pump unit

i	t_i	$x_i = \ln t_i$	M_i	$x_{i+1} - x_i$	$x_{i+1} - x_i/M_i$
1	198	5,2883	1,093929	0,325	0,2971
2	258	5,553	0,612330	0,452	0,7382
3	332	5,805	0,474330	0,087	0,1834
4	362	5,892	0,442920	0,280	0,6322
5	479	6,172	0,522759	0,328	0,6274
6	665	6,500			

the values in Tab. 7 in formula (1), . If the number of shut-downs ($r = 6$) and accuracy level ($\alpha = 0,05$), from the tables it was determined that $S_b = 0,73$. For $S_m \prec S_b$, distribution of continuous operation time of the third pump unit corresponds to Vebulla distribution. To find the continuous operation probability of the third pump unit, at first we should determine the parameters. To this end we obtained Tab. 8.

Table 8. For $r = 6$ linear weight sets

t_i	$x_i = \ln t_i$	a_i	c_i
198	5,037	−0,026	−0,109
258	5,347	−0,015	−0,112
332	5,375	−0,001	−0,108
362	5,481	0,016	−0,097
479	5,846	0,036	−0,081
665	6,040	0,060	−0,059

In the similar way, as from dependences (9) and (10) $\overline{u} = 6,7285$, $\overline{v} = 0,55$, from expressions (11) and (12), $\overline{a} = 836 hour$, $\overline{b} = 1,82$. In this case, the expression (13) should be calculated as follows:

$$\overline{P}_3(t) = \exp(-\frac{t}{836})^{1,82}. \quad (16)$$

The law of distribution of continuous operation time of the fourth pump unit and its parameters was determined and its reliability was calculated in the same way.

Table 9. Operating hours of the fourth pump unit

i	t_i	$x_i = \ln t_i$	M_i	$x_{i+1} - x_i$	$x_{i+1} - x_i/M_i$
1	126	4,84	1,150727	0,58	0,504
2	225	5,42	0,706698	0,24	0,340
3	288	5,66	0,679596	0,04	0,059
4	300	5,70	-	-	-

According to expression (1), $S_m \approx 0,44$. According to the number of shut-downs $r = 4$ and accuracy level $\alpha = 0,05$, from the special tables it was determined $S_b = 0,76$. For $S_m \prec S_b$, distribution of continuous operating time of the second pump unit corresponds to the Veibulla distribution. Tab. 10 was composed for determining Veibulla distribution parameters and continuous operation probability of the fourth pump unit.

Similarly, as from (13) and (14), $\overline{u} = 6$, $\overline{v} = 028$, from the expressions (15) and (16), $\overline{a} = 403$ hour, $\overline{b} \approx 3,6$. In this case the expression (17) has the following:

Table 10. For $r = 4$ linear weight sets

t_i	$x_i = \ln t_i$	a_i	c_i
126	4,84	−0,267	−0,239
225	5,42	−0,234	−0,231
288	5,66	−0,178	−0,204
300	5,7	1,679	0,674

$$\overline{P}_4(t) = \exp\left(-\frac{t}{403}\right)^{3,6}. \quad (17)$$

The reliability and shut-down probability of the pump station has the following form:

$$\overline{P}_{n/st} = \prod_{i=1}^{4} \overline{P}_i, q \approx 1 - P_{n/st}(t). \quad (18)$$

For spare pump unit we have

$$\overline{P}_{n/st} = \sum_{i=0}^{N-n} C_N^i (1-P_0)^i \cdot P_0^{N-1}, q_{n,N} = C_N^n P_0^{N-n} (1-P_0)^n. \quad (19)$$

Giving various values to t in the expressions (14), (15), (16) and (17), we can calculate a reliability of pump units of a pump station according to (18). Estmating the function by 50 hours steps at the range of 0-400, we get the following table. The plot of continuous operation probabilities of pump units and a pump station is given in Fig. 1. The numbers of plots corresponds to the numbers of the pump units. N is the plot of continuous operation probability of the system.

Table 11. Continuous operation probability of pump station within 0-400 hours

$t_i, hour$	$P_1(t)$	$P_2(t)$	$P_3(t)$	$P_4(t)$	$P_{n/st}(t)$
0	1	1	1	1	1
50	0,85	0,996	0,994	0,999	0,841
100	0,73	0,98	0,98	0,99	0,69
150	0,63	0,94	0,96	0,97	0,55
200	0,54	0,89	0,93	0,92	0,41
250	0,46	0,82	0,90	0,84	0,29
300	0,40	0,73	0,86	0,71	0,19
350	0,34	0,63	0,82	0,55	0,10
400	0,29	0,53	0,80	0,38	0,05

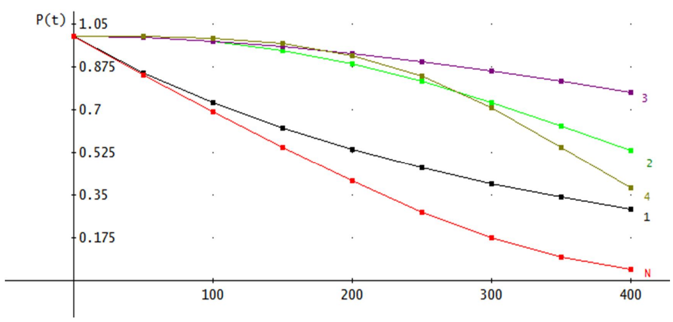
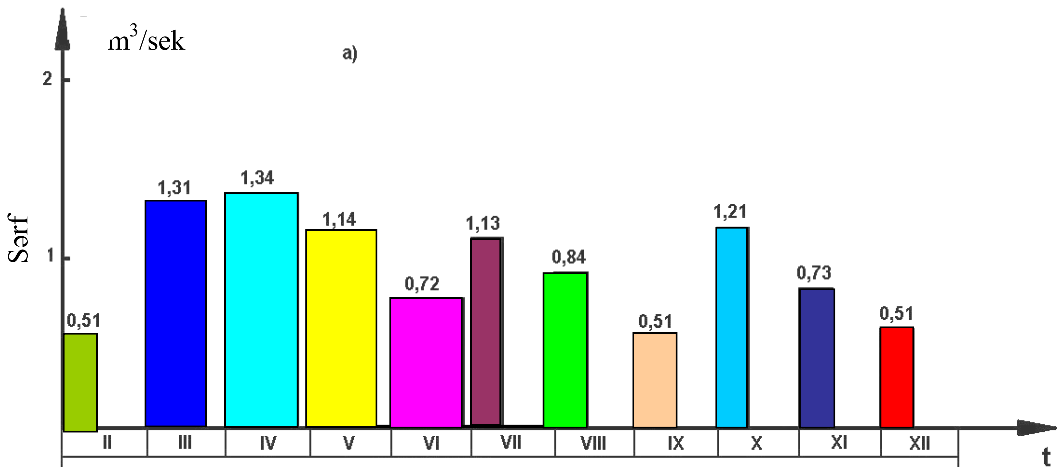


Fig. 1. Continuous operation probabilities of pump units and a pump station

As is seen from Fig. 1, the reliability of the pump station is very low. Now, let's calculate the mean value of the continuous operation time of the system[10].

$$T=\int_0^{400} e^{-(t/324)^{0,98}}\times e^{-(t/484)^{2,41}}\times e^{-(t/836)^{1,82}}\times e^{-(t/403)^{3,6}}dt\approx 178\ haur$$



NA mounts ***I variant*** *t, hours*

NA	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	t	NM
1	261	479	642	590		210		346	217		270	3015	16НДН
2		479	642	590	665	210	288		217	187		3278	20НДН

NA ***II variant*** *t, saat* *NM*

NA	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	t, saat	NM
1	261		642	590		210			217			1920	16НДН
2		479		590		210		346			270	1895	16НДН
3		479	642		665		288		217	187		2478	20НДН

Fig. 2. Consumption (a) and calculation variants of NC -5 collector by months

When determining the type of pumps, the variety of equipments enables to choose them from the equipments that have simple operation, are efficient from economic point of view and consume less energy. By determining the number of pumps, the following cases should be taken into account. If $Q_{\min} < \bar{\Delta Q}$, there should be chosen the pumps that one of them satisfes the condition $Q \le \bar{\Delta Q}$, the remainings the condition $Q_{\min} < Q \le \bar{\Delta Q}$; If $\bar{\Delta Q} < Q_{\min}$, one of the pumps should satisfy the condition $Q < \bar{\Delta Q}$, the other ones the condition $\bar{\Delta Q} < Q < Q_{\min}$. At the same time, by comparing the mentioned variants, the same type equipments should be taken[8].

According to consumption of NC-5 collector by months (Fig. 2) and $\bar{\Delta Q} = 0,38\ m^3/sec$. As the maximum consumption of the collector is $Q = 1,34\ m^3/se$ according to formulas (3), (4) and (5), the number of pump units, in the pump station becomes 3 (three). For estimating the optimal numbers and reliability of pump units, two calculation variant was considered. In variant I, in the pump station 16 NDN and 20 NDN 2 pump units, in variant II, 2 pieces 16 NDN, and one 20 NDN pump units were mounted.

As is seen from Fig. 2, in both variants, water injection of required water from the collector by pumps is possible. But, in the variant I, as there is no spare pump unit, in time when maximum water injection from the collector is required, the both pump units will operate with full power. As is seen from the table, the pumps have worked and hours, respectively. According to formula (13) and [7, 10].

$$P_1(1382) = \exp[-(1382/6600)^{3,1}] = 0,992,$$
$$P_2(1121) = \exp[-(1121/6600)^{3,1}] = 0,984.$$

Because of nonavaibility of a spae unit, according to dependence , the continuous operation probabilit of the pump station.

$$P_{nst}(t) = \prod_{i=1}^{N} P_{ai}(t) = 0,992 \times 0,984 = 0,976 > P_b = 0,900.$$

In variant I, the shut-down probability of one pump unit

$$q_d = P_{1,2} = 1 - 0,976 = 0,024.$$

In variant II, to the end of maximum water injection, the maximum operation time of pump units is 1121 hours. According to formula

$$P(1121) = \exp[-(1121/6600)^{3,1}] = 0,992.$$

Accepting $P_0 = P(1121) = 0,992$, according to formula (19), we calculate maximum water injection probability of the pump station as follows:

$$P_{nst}(t) = 0,992^3 + 3 \times 0,992^2 \times (1 - 0,992) = 0,999 > P_b = 0,900.$$

In variant II the shut-down probability of the pump unit

$$q_d = P_{1,3}(t) = 3 \times 0,992^2(1 - 0,992) = 0,023.$$

It is seen from the calculations that in both variants the required amount of water injection is possible. But consumption of collectors considerably increases at washing the soil, at irrigation conducted for creating humidity and at such random cases as flood happening in the region.

In such cases, in variant I in the case of two pump units it becomes impossible to dischange the excessive waters from the collector. So in variant II, the advantage should be given to reliably operating pump stations with three pump units. If in several variants the continuous operation probability of the pump station is higher than admissible limit, this time the pump station with lower operation expenses should be chosen[11].

4 Conclusion

Our investigations confirmed hypothesis that a reliability of pump station units is lower than the admissible limit. In “Yeni Mugan” pump station, four pump units have been used. According to our investigations, in the existing pump station exploration of three pump units including one reserve pump satysfy to all necessary requirments.

The design of “Yeni Mugan” pump station was carried out according to the normative documents - 18.76 and CH$\prod$ 2.06.03-85. In these documents reliability of the object was not estimated, i.e. the admissible reliability level of the units with respect to reliability groups has not been elaborated. In recomendations for design of pump stations up today these is not a method for calculating the harm done on economy in the case of a random shut-downs of pump stations. Hence, this issue becomes important for investigations of pump stations but it will be the subject of our future research.

References

[1] A. Alimov. Mode and balance of ground waters of Northern Mugan in connection with melioration. *Nauka*, 1997. In Russian.

[2] R. Barlow, F. Proshan. Mathematical theory of probability. *Sov. Radio*, 1969, pp 488. In Russian.

[3] Building code and rules. Water supply. External networks and constructions/State Building of the USSR. *Stroyizdat*, 1985, pp 136. In Russian.
[4] B. Gnedenko, Y. Belyayev, A. Solovyev. Mathematical methods of reliability theory. *Nauka*, 1965, pp 524. In Russian.
[5] K. Kapoor, L. Lamberson. Reliability and design of systems. 1980. *Mir*, 1980, pp 604. In Russian.
[6] R. Mann, K. Fertig, E. Scheuer. Tolerance bounds and a new goodness-of-fit for two-parameter weibull or extreme-value distribution. *Aerospace Research Laboratories*, 1971.
[7] K. Minvodkhoz. Guidance to choice of quantity and type of pump units of pump stations of irrigation systems with regard to reliability indices. *Ukr SSR*, 1987, pp 29. In Russian.
[8] I. Naumenko, A. Podlasov, et al. Increase of reliabiliy of irrigation systems. *Kiev: Urozhay*, 1989, pp 95. In Russian.
[9] I. Naumenko. Reliaility of facilities of hydromeliorative systems. **in:** *: Visha Schkola* (K. Manual), 1990, pp 239. In Russian.
[10] A. Polovko, S. Gurov. Bases of reliability theory, 1964. *St. Petersburg*, 2008, pp 702. In Russian.
[11] Y. Rustamov. *Estimation of reliability of meliorative pump stations*. 2010. *Izvestia Agrarnoy Nauki*, 2010, (2): 69–72. In Russian.