ISSN 1 746-7233, England, UK
World Journal of Modelling and Simulation
Vol. 11 (2015) No. 2, pp. 94-103

Approximate analytical solutions of Newell-Whitehead-Segel equation using a new iterative method*

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(*Received November 24 2014, Accepted March 12 2015*)

Abstract. The Newell-Whitehead-Segel equation is an important model arising in fluid mechanics. Various researchers worked on approximate solution of this model by using different methods. In this paper, we use New Iterative Method (NIM) proposed by Daftardar-Gejji and Jafari for the same purpose. We use software Mathematica 10 for computations. The results obtained by NIM are compared with exact solutions and those obtained by other iterative methods such as Laplace Adomian decomposition method (LADM) and Adomian decomposition method (ADM). It is observed that NIM is simple as compared with other methods. The solutions obtained using NIM are in well agreement with exact solution and permit very less error.

Keywords: Newell-Whitehead-Segel equation, new iterative method, Adomian decomposition method, Laplace Adomian decomposition method

1 Introduction

The nonlinear equations play an important role in modelling various phenomena arising in applied science. Several systems are modeled by partial differential equations and most of them are nonlinear. Solving nonlinear system is an important task in mathematical analysis and applications. Researcher have used numerical as well as analytical approaches to solve such systems. Both approaches have merits and demerits. In numerical methods discretization is used which leads to different types of errors and hence accuracy is lost. However in analytical approach we do not use discretization and hence it is free from such errors. The main merit of analytical method is that it produces series converging rapidly to exact solution.

The equation derived by Newell, Whitehead and Segel[33, 38] is in the form :

$$\begin{aligned} u_t(x,t) &= au_{xx}(x,t) + bu(x,t) - cu^m(x,t), \\ u(x,0) &= f(x), \end{aligned}$$

where b, c are real numbers and a, m are positive integers. The Newell-Whitehead-Segel (NWS) equation is used in modeling various phenomena arising in fluid mechanics.

Different methods are utilized to solve NWS equation. Malomed in [31] proposed dispersive NWS equation for the description of traveling waves patterns in binary fluids. Nourazar et al.[34] used homotopy perturbation method (HPM) and Aasaraai[1] used differential transformed method (DTM) to solve these equations. Ezzati and Shakibi[22] applied ADM and multiquadric quasi-interpolation methods for same purpose. Macas-Daz and Ruiz-Ramrez proposed non-standard symmetry-preserving method to compute bounded solutions of NWS equation in [29]. Malik et al. in [30] used $\frac{G'}{G}$ expansion method to get generalized traveling wave

* The authors acknowledges NBHM, Department of Atomic Energy Mumbai, India for funding through Research Project [2148(6)/2013/NBHM(R.P.)/R & D II/689].
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solutions of NWS equation. The comparative study between the reduced DTM and ADM for NWS equation is described in [37].

The new iterative method (NIM) proposed by Daftardar-Gejji and Jafari[19] is a powerful tool used to solve nonlinear equations such as ordinary differential equations, partial differential equations and fractional-order differential equations. NIM series produces analytical solution to such nonlinear equations. In many cases it is observed that the NIM solution are in agreement with exact solution in sufficiently small domain. NIM has been used for solving Helmholtz equation in [5]. A variety of fractional-order differential equations such as diffusion-wave equations[16], partial differential equations[7], boundary value problems[18], evolution equation[8] and fifth-order boundary value problem[32] are solved successfully by NIM. Daftardar-Gejji and Bhalekar have used NIM to solve fractional-order logistic equation [11], system of nonlinear functional equations [6] and some nonlinear dynamical systems [10]. Recently NIM is used to generate a new predictor-corrector method[20] by these authors.

The Adomian decomposition method (ADM)[3] introduced and developed by George Adomian is useful method for solving nonlinear problems. The method is employed to solve nonlinear integro-differential equations in [4]. The reaction-convection-diffusion equation arising in chemistry was successfully solved using ADM in [44]. Wazwaz has used ADM to solve boundary value problem[42], Kadomtsev-Petviashvili equation[40], partial differential equations[39] and diffusion equations[41]. Biazar et al.[12–14] have applied ADM for solving system of Volterra integral equations, ordinary differential equations and systems of integral differential equations. Jafari and Daftardar-Gejji[24–26] have successfully employed ADM to solve fractional boundary value problems, system of fractional differential equations and fractional diffusion-wave equations. Bhalekar and Daftardar-Gejji[17] have applied ADM for solving multi-term linear and non-linear diffusion-wave equations. Convergence of ADM is discussed in [2, 15, 23].

The Laplace Adomian Decomposition Method (LADM)[28] was introduced and developed by George Khuri in 1994. Wazwaz has solved nonlinear Volterra integro-differential equations in [43] using LADM. Khan and Muhammad [27] have applied LADM for solving nonlinear partial differential equations. This method is used for solving a model for HIV infection of CD4+ T cells in [35]. In this paper we apply NIM to find approximate analytical solution of Newell-Whitehead-Segel equation. The solutions obtained by NIM are compared with exact solutions and those obtained using ADM or LADM.

The paper is organized as follows: New iterative method and Adomian decomposition methods are described briefly in Sections 2 and 3 respectively. Convergence results of NIM are stated in Section 4. A general technique used to solve NWS equations using NIM is described in Section 5. Section 6 deals with different types of illustrative examples and the conclusions are summarized in Section 7.

2 New iterative method

The new iterative method (NIM) is described as bellow:
Consider the nonlinear equation

$$u = f + L(u) + N(u), \tag{1}$$

where f is a given function, L and N are linear and nonlinear operators respectively. It is assumed that the NIM solution for the Eq. (1) has the form:

$$u = \sum_{i=0}^{\infty} u_i. \tag{2}$$

The convergence of series (2) is proved in [9] and described in Section 4.

Since L is linear

$$L\left(\sum_{i=0}^{\infty} u_i\right) = \sum_{i=0}^{\infty} L(u_i). \tag{3}$$

The nonlinear operator N in Eq. (1) is decomposed by Daftardar-Gejji and Jafari as bellow:

$$\begin{aligned} N\left(\sum_{i=0}^{\infty} u_i\right) &= N(u_0) + \sum_{i=1}^{\infty}\left\{N\left(\sum_{j=0}^{i} u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right)\right\} \\ &= \sum_{i=0}^{\infty} G_i, \end{aligned} \tag{4}$$

where $G_0 = N(u_0)$ and

$$G_i = \left\{N\left(\sum_{j=0}^{i} u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right)\right\}, i \geq 1$$

. Using Eqs. (2), (3) and (4) in Eq. (1), we get

$$\sum_{i=0}^{\infty} u_i = f + \sum_{i=0}^{\infty} L(u_i) + \sum_{i=0}^{\infty} G_i. \tag{5}$$

From Eq. (5), the NIM series terms are generated as bellow:

$$u_0 = f,\ u_{m+1} = L(u_m) + G_m, \quad m = 0, 1, 2, \cdots. \tag{6}$$

3 Adomian decomposition method

The ADM solution of Eq. (1) has same form as in Eq. (2). The nonlinear operator N in Eq. (1) was decomposed by Adomian as

$$N\left(\sum_{i=0}^{\infty} u_i\right) = \sum_{i=0}^{\infty} A_i$$

where

$$A_i = \frac{1}{i!}\frac{d^i}{d\lambda^i} N\left(\sum_{i=0}^{\infty} u_i \lambda^i\right)_{\lambda=0} \tag{7}$$

are called as Adomian polynomials.Eq. (1) becomes

$$\sum_{i=0}^{\infty} u_i = f + \sum_{i=0}^{\infty} L(u_i) + \sum_{i=0}^{\infty} A_i. \tag{8}$$

From Eq. (8), the ADM series term are generated as bellow:

$$\begin{aligned} u_0 &= f, \\ u_m &= L(u_{m-1}) + A_{m-1}, \quad m = 1, 2, \cdots. \end{aligned} \tag{9}$$

In [28], Khuri used Laplace transform along with ADM to solve equation of the form (1). The method was termed as Laplace Adomian decomposition method (LADM).

4 Convergence of NIM

The following convergence results for NIM are described by Daftardar-Gejji and Bhalekar.

Theorem 1. *[9] If N is $C^{(\infty)}$ in a neighborhood of y_0 and $\left\|N^{(n)}(y_0)\right\| \leq L$, for any n and for some real $L > 0$ and $\|y_i\| \leq M < \frac{1}{e}$, $i = 1, 2, \cdots$, then the series $\sum_{n=0}^{\infty} G_n$ is absolutely convergent to N and moreover,*

$$\|G_n\| \leq L M^n e^{n-1}(e-1), \quad n = 1, 2, \cdots.$$

Theorem 2. *[9] If N is $C^{(\infty)}$ and $\left\|N^{(n)}(y_0)\right\| \leq M \leq e^{-1}$, $\forall n$, then the series $\sum_{n=0}^{\infty} G_n$ is absolutely convergent to N.*

5 Applications

Consider the Newell-Whitehead-Segel equation

$$\begin{aligned} u_t(x,t) &= au_{xx}(x,t) + bu(x,t) - cu^m(x,t), \\ u(x,0) &= f(x), \end{aligned} \tag{10}$$

where b, c are real numbers and a, m are positive integer. Eq. (10) can be written equivalently as

$$u = f(x) + \int_0^t (au_{xx} + bu)dt - c\int_o^t u^m dt. \tag{11}$$

This Eq. (11) is of the form Eq. (1), with $L(u) = \int_0^t (au_{xx} + bu)dt$ and $N(u) = -c\int_o^t u^m dt$.

Using Eq. (6), the NIM series terms are generated as bellow:

$$u_0 = f \tag{12}$$

$$u_1 = \int_0^t (a(u_0)_{xx} + bu_0)dt - c\int_0^t u_0^m dt \tag{13}$$

$$\begin{aligned} u_{n+1} &= \int_0^t (a(u_n)_{xx} + bu_n)dt \\ &\quad -c\left[\int_0^t \left(\sum_{i=0}^{n} u_i\right)^m dt - \int_0^t \left(\sum_{i=0}^{n-1} u_i\right)^m dt\right], \quad n = 1, 2, \cdots . \end{aligned} \tag{14}$$

6 Illustrative examples

Example 6.1 Consider the Newell-Whitehead-Segel equation

$$u_t = 5u_{xx} + 2u + u^2, \quad u(x,0) = \lambda. \tag{15}$$

The equivalent integral equation is

$$u = \lambda + \int_0^t (5u_{xx} + 2u)dt + \int_0^t u^2 dt. \tag{16}$$

Using NIM we get

$$u_0 = \lambda, \tag{17}$$

$$u_1 = 2t\lambda + t\lambda^2, \tag{18}$$

$$\begin{aligned} u_2 &= 2t^2\lambda - t\lambda^2 + t^2\lambda^2 + \frac{\lambda^2\left(-1 + (1 + t(2+\lambda))^3\right)}{3(2+\lambda)}, \\ u_3 &= \frac{1}{6}t^3\lambda(2+\lambda)\left(4 + 2(2+t)\lambda + t\lambda^2\right) - \frac{\lambda^2\left(-1 + (1 + t(2+\lambda))^3\right)}{3(2+\lambda)} \\ &\quad + \frac{1}{9}\lambda^2\big(9t + 9t^2(2+\lambda) + t^6\lambda(1+\lambda)(2+\lambda)^3 \\ &\quad + \frac{1}{7}t^7\lambda^2(2+\lambda)^4 + 3t^3(2+\lambda)(4+3\lambda) \\ &\quad + \frac{3}{2}t^4(2+\lambda)^2(3+4\lambda) + \frac{3}{5}t^5(2+\lambda)^2(3+5\lambda(2+\lambda))\big), \end{aligned} \tag{19}$$

and so on.

The four-term NIM solution is

$$\begin{aligned} u \ = \ & \lambda + 2t\lambda + 2t^2\lambda + t^2\lambda^2 + \frac{1}{6}t^3\lambda(2+\lambda)\left(4 + 2(2+t)\lambda + t\lambda^2\right) \\ & + \frac{1}{9}\lambda^2\big(9t + 9t^2(2+\lambda) + t^6\lambda(1+\lambda)(2+\lambda)^3 + \frac{1}{7}t^7\lambda^2(2+\lambda)^4 + 3t^3(2+\lambda)(4+3\lambda) \\ & + \frac{3}{2}t^4(2+\lambda)^2(3+4\lambda) + \frac{3}{5}t^5(2+\lambda)^2(3+5\lambda(2+\lambda))\big). \end{aligned} \tag{20}$$

The exact solution of Eq. (15) is

$$u = \frac{2e^{2t}\lambda}{2 + (1 - e^{2t})\lambda}. \tag{21}$$

The four-term LADM solution of Eq. (15) is described in [28] given by

$$u = e^{2t}\left(\lambda + \frac{1}{2}\left(-1 + e^{2t}\right)\lambda^2 + \frac{1}{4}\left(-1 + e^{2t}\right)^2\lambda^3 + \frac{1}{8}\left(-1 + e^{2t}\right)^3\lambda^4\right). \tag{22}$$

We compare these solutions for $\lambda = 12$ in Fig. 1 and Fig. 2, where NIM solution, LADM solution and exact solution are shown by green, blue and red colors respectively.

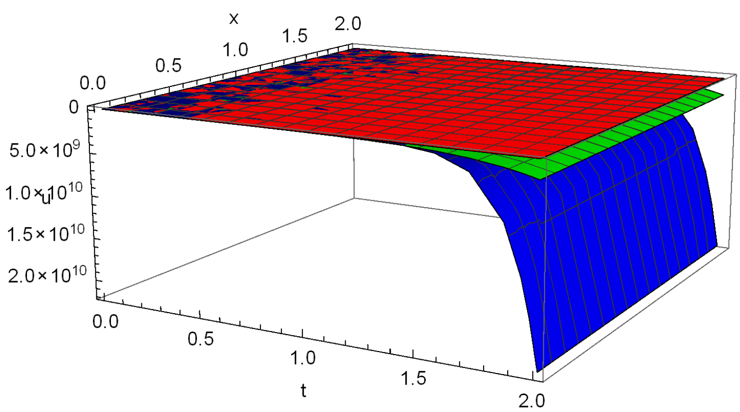

Fig. 1. Comparison of solutions of Eq. 15

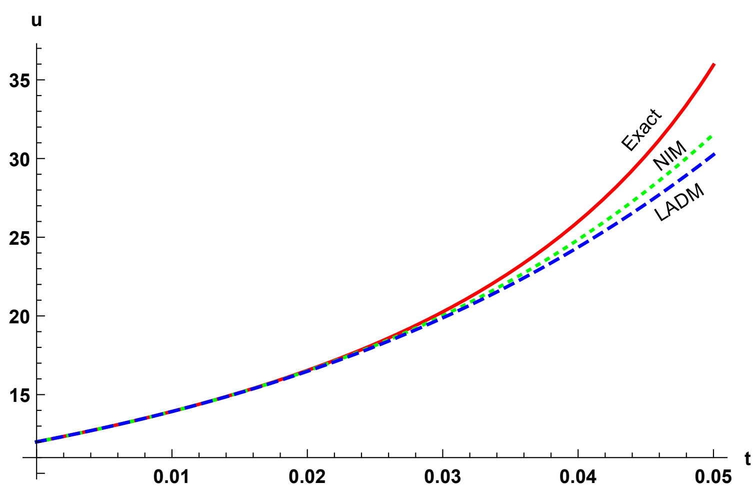

Fig. 2. Comparison for $x = 1$

It can be checked that NIM solution is better than LADM solution.

Example 6.2 Consider the Newell-Whitehead-Segel equation

$$u_t = u_{xx} + 2u - 3u^2, \quad u(x,0) = \lambda. \tag{23}$$

The equivalent integral equation is

$$u = \lambda + \int_0^t (u_{xx} + 2u)dt - 3\int_0^t u^2 dt. \tag{24}$$

Using NIM we get

$$u_0 = \lambda, \tag{25}$$
$$u_1 = 2t\lambda - 3t\lambda^2, \tag{26}$$
$$u_2 = 2t^2\lambda + 3t\lambda^2 - 3t^2\lambda^2 - \frac{3\lambda^2\left(1 + (-1 + t(-2+3\lambda))^3\right)}{-6+9\lambda}, \tag{27}$$
$$\begin{aligned} u_3 = {} & \frac{4t^3\lambda}{3} - 6t^3\lambda^2 - 2t^4\lambda^2 + 6t^3\lambda^3 + 6t^4\lambda^3 - \frac{9t^4\lambda^4}{2} + \frac{3\lambda^2\left(1 + (-1 + t(-2+3\lambda))^3\right)}{-6+9\lambda} \\ & -3\lambda^2\left(t + t^2(2-3\lambda) + \frac{1}{7}t^7(2-3\lambda)^4\lambda^2 - \frac{1}{3}t^6\lambda(-2+3\lambda)^3(-1+3\lambda)\right. \\ & -\frac{1}{2}t^4(2-3\lambda)^2(-1+4\lambda) + \frac{1}{5}t^5(2-3\lambda)^2(1+5\lambda(-2+3\lambda)) \\ & \left.+t^3\left(\frac{8}{3} + \lambda(-10+9\lambda)\right)\right), \end{aligned} \tag{28}$$

and so on. The four-term NIM solution is

$$\begin{aligned} u = {} & \lambda + 2t\lambda + 2t^2\lambda + \frac{4t^3\lambda}{3} - 3t^2\lambda^2 - 6t^3\lambda^2 - 2t^4\lambda^2 + 6t^3\lambda^3 + 6t^4\lambda^3 - \frac{9t^4\lambda^4}{2} \\ & -3\lambda^2\left(t + t^2(2-3\lambda) + \frac{1}{7}t^7(2-3\lambda)^4\lambda^2 - \frac{1}{3}t^6\lambda(-2+3\lambda)^3(-1+3\lambda)\right. \\ & -\frac{1}{2}t^4(2-3\lambda)^2(-1+4\lambda) + \frac{1}{5}t^5(2-3\lambda)^2(1+5\lambda(-2+3\lambda)) . \\ & \left.+t^3\left(\frac{8}{3} + \lambda(-10+9\lambda)\right)\right) \end{aligned} \tag{29}$$

The exact solution of Eq. (23) is

$$u = -\frac{2e^{2t}\lambda}{-2 + 3\left(1 - e^{2t}\right)\lambda}. \tag{30}$$

The following four-term ADM solution of Eq. (23) is described in [37].

$$u = \lambda + t^2(1-3\lambda)(2-3\lambda)\lambda + t\left(2\lambda - 3\lambda^2\right) + \frac{1}{3}t^3(2-3\lambda)\lambda\left(2 - 18\lambda + 27\lambda^2\right). \tag{31}$$

We compare these solutions for $\lambda = 0.1$ in Fig. 3 and Fig. 4, where NIM solution, ADM solution and exact solution are shown by green, blue and red colors respectively.

Example 6.3 Consider the Newell-Whitehead-Segel equation

$$u_t = u_{xx} + 2u - 3u^3, \quad u(x,0) = \sqrt{\frac{2}{3}}\frac{e^{2x}}{e^x + e^{2x}}. \tag{32}$$

The equivalent integral equation is

$$u = \sqrt{\frac{2}{3}}\frac{e^{2x}}{e^x + e^{2x}} + \int_0^t (u_{xx} + 2u)dt - 3\int_0^t u^3 dt. \tag{33}$$

Using NIM we get

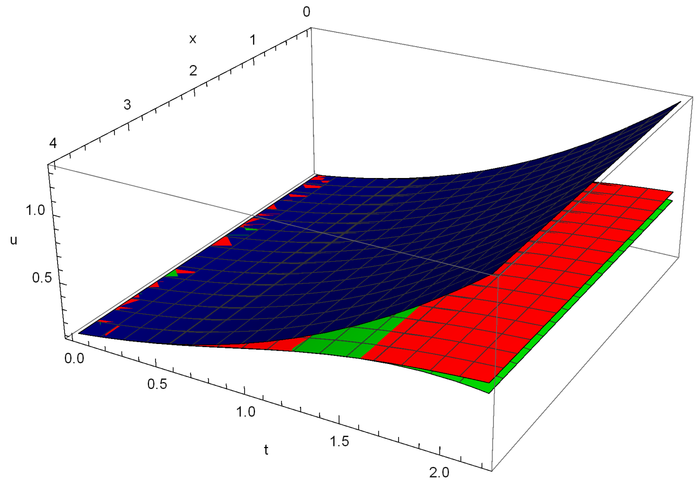

Fig. 3. Comparison of solutions of Eq. 23

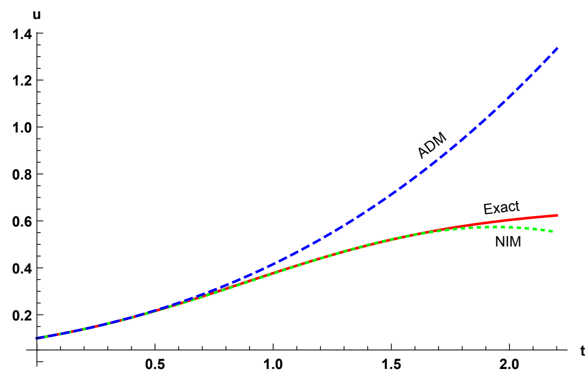

Fig. 4. Comparison for $x = 2$

$$u_0 = \sqrt{\frac{2}{3}} \frac{e^{2x}}{e^x + e^{2x}}, \tag{34}$$

$$\begin{aligned} u_1 = & -2\sqrt{\frac{2}{3}} \frac{e^{6x}t}{(e^x + e^{2x})^3} + \left(\frac{2\sqrt{6}e^{2x}}{e^x + e^{2x}} - \frac{4\sqrt{\frac{2}{3}}e^{2x}\left(e^x + 2e^{2x}\right)}{(e^x + e^{2x})^2} \right. \\ & \left. + \sqrt{\frac{2}{3}}e^{2x}\left(\frac{2\left(e^x + 2e^{2x}\right)^2}{(e^x + e^{2x})^3} - \frac{e^x + 4e^{2x}}{(e^x + e^{2x})^2} \right) \right) t, \end{aligned} \tag{35}$$

$$\begin{aligned} u_2 = & \frac{2\sqrt{\frac{2}{3}}e^{6x}t}{(e^x + e^{2x})^3} + \frac{4\sqrt{6}e^{6x}t^2}{(e^x + e^{2x})^5} + \frac{32\sqrt{6}e^{7x}t^2}{(e^x + e^{2x})^5} + \frac{92\sqrt{6}e^{8x}t^2}{(e^x + e^{2x})^5} + \frac{112\sqrt{6}e^{9x}t^2}{(e^x + e^{2x})^5} \\ & + \frac{48\sqrt{6}e^{10x}t^2}{(e^x + e^{2x})^5} - \frac{14\sqrt{6}e^{5x}t^2}{(e^x + e^{2x})^4} - \frac{96\sqrt{6}e^{6x}t^2}{(e^x + e^{2x})^4} - \frac{203\sqrt{6}e^{7x}t^2}{(e^x + e^{2x})^4} - \frac{132\sqrt{6}e^{8x}t^2}{(e^x + e^{2x})^4} \\ & + \frac{2\sqrt{\frac{2}{3}}e^{4x}t^2}{(e^x + e^{2x})^3} + \frac{19\sqrt{6}e^{4x}t^2}{(e^x + e^{2x})^3} + \frac{8\sqrt{\frac{2}{3}}e^{5x}t^2}{(e^x + e^{2x})^3} + \frac{104\sqrt{6}e^{5x}t^2}{(e^x + e^{2x})^3} + \frac{126\sqrt{6}e^{6x}t^2}{(e^x + e^{2x})^3} \\ & - \frac{5\sqrt{\frac{2}{3}}e^{3x}t^2}{(e^x + e^{2x})^2} - \frac{25\sqrt{\frac{3}{2}}e^{3x}t^2}{(e^x + e^{2x})^2} - \frac{48\sqrt{6}e^{4x}t^2}{(e^x + e^{2x})^2} + \frac{6\sqrt{6}e^{2x}t^2}{e^x + e^{2x}} \\ & - \frac{e^{3x}\left(-(1 + e^x)^4 + (1 + e^x + 3t)^4\right)}{3\sqrt{6}(1 + e^x)^6}, \end{aligned} \tag{36}$$

and so on. Using ADM we get

$$u_0 = \sqrt{\frac{2}{3}}\frac{e^{2x}}{e^x+e^{2x}}, \tag{37}$$

$$\begin{aligned} u_1 = & -\frac{2\sqrt{\frac{2}{3}}e^{6x}t}{(e^x+e^{2x})^3}+\sqrt{\frac{2}{3}}\left(\frac{6e^{2x}}{e^x+e^{2x}}-\frac{4e^{2x}\left(e^x+2e^{2x}\right)}{(e^x+e^{2x})^2}\right. \\ & \left.+e^{2x}\left(\frac{2\left(e^x+2e^{2x}\right)^2}{(e^x+e^{2x})^3}-\frac{e^x+4e^{2x}}{(e^x+e^{2x})^2}\right)\right)t, \end{aligned} \tag{38}$$

$$\begin{aligned} u_2 = & \frac{4\sqrt{6}e^{6x}t^2}{(e^x+e^{2x})^5}+\frac{32\sqrt{6}e^{7x}t^2}{(e^x+e^{2x})^5}+\frac{92\sqrt{6}e^{8x}t^2}{(e^x+e^{2x})^5}+\frac{112\sqrt{6}e^{9x}t^2}{(e^x+e^{2x})^5} \\ & +\frac{48\sqrt{6}e^{10x}t^2}{(e^x+e^{2x})^5}-\frac{14\sqrt{6}e^{5x}t^2}{(e^x+e^{2x})^4}-\frac{96\sqrt{6}e^{6x}t^2}{(e^x+e^{2x})^4}-\frac{203\sqrt{6}e^{7x}t^2}{(e^x+e^{2x})^4} \\ & -\frac{132\sqrt{6}e^{8x}t^2}{(e^x+e^{2x})^4}+\frac{2\sqrt{\frac{2}{3}}e^{4x}t^2}{(e^x+e^{2x})^3}+\frac{19\sqrt{6}e^{4x}t^2}{(e^x+e^{2x})^3}+\frac{8\sqrt{\frac{2}{3}}e^{5x}t^2}{(e^x+e^{2x})^3} \\ & +\frac{104\sqrt{6}e^{5x}t^2}{(e^x+e^{2x})^3}+\frac{126\sqrt{6}e^{6x}t^2}{(e^x+e^{2x})^3}-\frac{5\sqrt{\frac{2}{3}}e^{3x}t^2}{(e^x+e^{2x})^2}-\frac{25\sqrt{\frac{3}{2}}e^{3x}t^2}{(e^x+e^{2x})^2} \\ & -\frac{48\sqrt{6}e^{4x}t^2}{(e^x+e^{2x})^2}+\frac{6\sqrt{6}e^{2x}t^2}{e^x+e^{2x}}-3\left(\frac{2\sqrt{\frac{2}{3}}e^{8x}t^2}{(e^x+e^{2x})^5}+\frac{8\sqrt{\frac{2}{3}}e^{9x}t^2}{(e^x+e^{2x})^5}\right. \\ & \left.+\frac{2\sqrt{6}e^{10x}t^2}{(e^x+e^{2x})^5}-\frac{5\sqrt{\frac{2}{3}}e^{7x}t^2}{(e^x+e^{2x})^4}-\frac{4\sqrt{6}e^{8x}t^2}{(e^x+e^{2x})^4}+\frac{2\sqrt{6}e^{6x}t^2}{(e^x+e^{2x})^3}\right), \end{aligned} \tag{39}$$

and so on. We compare five term solution in Fig. 5 and absolute error in Fig. 6 for $x = 2$.

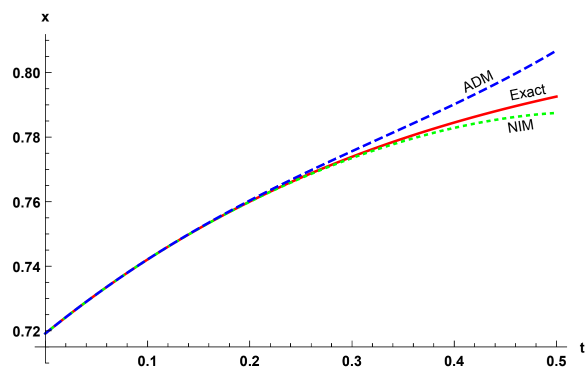

Fig. 5. Comparison of solutions of Eq. (32) for $x = 2$

7 Conclusions

We have successfully utilized new iterative method (NIM) to obtain analytical solutions of Newell-Whitehead-Segel (NWS) equation arising in fluid mechanics. We have discussed different variations of NWS equation in this paper. The solutions obtained and the error functions are plotted using software Mathematica. It is observed that the NIM is simple but powerful technique used to solve such nonlinear equations. The approximate solutions obtained using few series terms of NIM produce better results as compared with

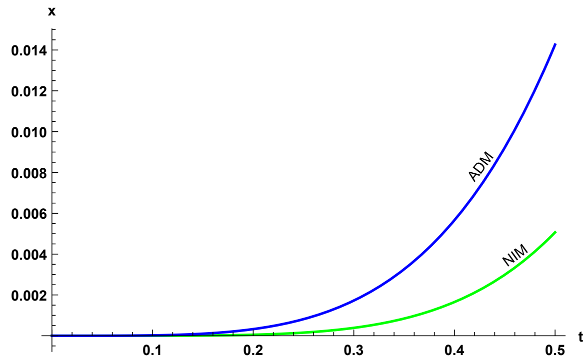

Fig. 6. Comparison for $x = 2$

other iterative methods such as Adomian decomposition method (ADM) and Laplace Adomian decomposition method (LADM).

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