ISSN 1 746-7233, England, UK
**World Journal of Modelling and Simulation**
Vol. 10 (2014) No. 4, pp. 308-319

# Unsteady squeezing flow of a casson fluid between parallel plates

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(*Received December 15 2013, Accepted August 29 2014*)

**Abstract.** Squeezing flow of Casson fluid has been taken in to account. Nonlinear ordinary differential equation governing the flow is obtained by imposing similarity transformationon conservation laws. Resulting equation has been solved by using two analytical techniques namely variation of parameters method (VPM) and Adomian's decomposition method (ADM). Comparison between both the techniques is provided. A numerical solution is also sought to back the analytical results. They are found to be in excellent agreement. Convergence of solution also discussed. Flow behavior under altering involved physical parameters is also discussed and explained in details with graphical aid.

**Keywords:** squeezing flows, variation of parameters method (VMP), Adomian's decomposition method (ADM), Casson fluid

## 1 Introduction

Squeezing flow between orthogonally moving plates is involved in many industrial and biological situations. Its applications especially in polymer processing, modeling of synthetics transportation inside living bodies, hydro-mechanical machinery and compression/injection processes are of great interest. Many scientists have contributed their efforts towards the better understanding of these types of flows. The basic contribution in this regard can be named to Stefan[30]. His pioneering effort has opened new doors for the researchers and a lot of studies have been carried out following him[4, 9, 20, 28]. Homotopy perturbation solution for Two-dimensional MHD squeezing flow between parallel plates has been determined by Siddiqui et al.[29]. Domairry and Aziz[7] investigated the same problem for the flow between parallel disks. Recently, Mustafa et al.[22] examined heat and mass transfer for squeezing flow between parallel plates using homotopy analysis method (HAM).

In most of realistic models the fluids involved are not simple Newtonian. Complex rheological properties of non-Newtonian fluids cannot be captured by a single model. Different mathematical models have been used to study different types of non-Newtonian fluids. One of such models is known as Casson fluid model. [16, 21] showed that it is most compatible formulation to simulate blood type fluid flow. It is clear from the literature survey that squeezing flow of Casson fluid between plates moving normal to their own surface is yet to be inspected.

Due to inherent nonlinearity of the equations governing fluid flow exact solution are very rare. Even where they are available immense simplification assumptions have been imposed. Those overly imposed suppositions may not be used for more realistic flows. However to deal with this hurdle many analytical approximation techniques have been developed which are commonly used nowadays.

Solution to aforementioned highly nonlinear ordinary differential equation still lacks exactness due to its abstract nature.However, different attempts have been made to approximate its solution in an acceptable and accurate way. Nowadays, several approximation techniques have been developed to fulfill this

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purpose[5, 15, 18, 25, 31]. From them, those are used most often which are easy to apply and require less computational work yet provide reliable results.

Motivated by contributions stated above here we present squeezing flow of a Cassonfluid between parallel plates. Mathematical form of the problem is extracted by using conservation laws along with similarity transformations. Resulting highly nonlinear ordinary differential equation is then solved by two analytical methods namely Adomian's decomposition method (ADM) and variation of parameters method (VPM). Both the techniques have been used to solve several types of abstract problems successfully by different scientist[1, 2, 6, 8, 11–14, 17, 19, 26, 27] . One can see from our work that the obtained analytical solutions not only show compatibility with each other but also with numerical solution obtained by Runge-Kutta order 4 [RK-4]method.

In this study one may clearly see that VPM can successfully be applied to solve the equations governing unsteady squeezing flows between parallel plates. Its main advantage is that it is free from calculation of so called Adomian's polynomials and is more flexible in application.

## 2 Governing equations

We consider an incompressible flow of a Casson fluid between two parallel plates distance $z = \pm l(1 - \alpha t)^{1/2} = \pm h(t)$ apart, where $l$ is the initial position (at time $t = 0$ ). Further $\alpha > 0$ corresponds to squeezing motion of both plates until they touch each other at $t = 1/\alpha$, for $\alpha < 0$ the plates leave each other and dilate. Rheological equation of Casson fluid isdefined as under [3, 10, 23, 24]

$$\tau_{ij} = \left[\mu_B + \left(\frac{p_y}{2\pi}\right)^{1/n}\right]^n 2e_{ij}, \tag{1}$$

$\mu_B$ is dynamic viscosity of the non-Newtonian fluid, $p_y$ is yield stress of fluid and $\pi$ is the product of component of deformation rate with itself, i.e. $\pi = e_{ij}e_{ij}$, where $e_{ij}$ is the $(i, j)th$ component of the deformation rate. The equations governing the flow are:

$$\frac{\partial \hat{u}}{\partial \hat{x}} + \frac{\partial \hat{v}}{\partial \hat{y}} = 0, \tag{2}$$

$$\begin{aligned} \frac{\partial \hat{u}}{\partial t} + \hat{u}\frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{u}}{\partial \hat{y}} &= -\frac{1}{\rho}\frac{\partial \hat{p}}{\partial \hat{x}} + \upsilon\left(1 + \frac{1}{\gamma}\right)\left(2\frac{\partial^2 \hat{u}}{\partial \hat{x}^2} + \frac{\partial^2 \hat{u}}{\partial \hat{y}^2} + \frac{\partial^2 \hat{v}}{\partial \hat{y}\partial \hat{x}}\right), \\ \frac{\partial \hat{v}}{\partial t} + \hat{u}\frac{\partial \hat{v}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{v}}{\partial \hat{y}} &= -\frac{1}{\rho}\frac{\partial \hat{p}}{\partial \hat{y}} + \upsilon\left(1 + \frac{1}{\gamma}\right)\left(2\frac{\partial^2 \hat{v}}{\partial \hat{x}^2} + \frac{\partial^2 \hat{v}}{\partial \hat{y}^2} + \frac{\partial^2 \hat{u}}{\partial \hat{y}\partial \hat{x}}\right). \end{aligned} \tag{3}$$

In the above equations $\hat{u}$ and $\hat{v}$ are the velocity components in $\hat{x}$ and $\hat{y}$-directions respectively, $\hat{p}$ is the pressure, $V$ the kinematic viscosity of the fluid and $\gamma = \mu_B\sqrt{2\pi_c/p_y}$ is the Casson fluid parameter.

Boundary conditions for the flow problem are

$$\begin{aligned} &\hat{u} = 0, \quad \hat{v} = v_w = \frac{dh}{dt} \text{ at} \hat{y} = h(t), \\ &\frac{\partial \hat{u}}{\partial \hat{y}} = 0, \quad \hat{v} = 0 \text{ at} \hat{y} = 0. \end{aligned} \tag{4}$$

We can simplify this system of equations by eliminating pressure terms from Eq. (2) and Eq. (3) and using Eq. (1). After cross differentiation and introducing vorticity $\omega$, we get

$$\frac{\partial \omega}{\partial t} + \hat{u}\frac{\partial \omega}{\partial \hat{x}} + \hat{v}\frac{\partial \omega}{\partial \hat{y}} = \upsilon\left(1 + \frac{1}{\gamma}\right)\left(\frac{\partial^2 \omega}{\partial \hat{x}^2} + \frac{\partial^2 \omega}{\partial \hat{y}^2}\right), \tag{5}$$

where,

$$\omega = \left(\frac{\partial \hat{v}}{\partial \hat{x}} - \frac{\partial \hat{u}}{\partial \hat{y}}\right). \tag{6}$$

Using transform introduced by Wang[31] for a two-dimensional flow

$$\hat{u} = \frac{\alpha \hat{x}}{[2(1-\alpha t)]} F'(\eta), \tag{7}$$

$$\hat{v} = \frac{-\alpha l}{[2(1-\alpha t)^{1/2}]} F(\eta), \tag{8}$$

where,

$$\eta = \frac{\hat{y}}{[l(1-\alpha t)]^{1/2}}. \tag{9}$$

Substituting Eqs. (7)-(9) in Eq. (5) accompanied by Eq. (6), we obtain a nonlinear ordinary differential equation for Casson fluid flow as

$$\left(1+\frac{1}{\gamma}\right) F^{iv}(\eta) - S\left(\eta F(\eta) + 3F''(\eta) + F'(\eta)F''(\eta) - F(\eta)F'''(\eta)\right) = 0, \tag{10}$$

where $S = \alpha l^2/2v$ is the non-dimensional Squeeze number. Using Eqs. (7)-(9) boundary conditions also reduce to

$$F(0) = 0, F''(0) = 0, F(1) = 1, F'(1) = 0. \tag{11}$$

Here, squeeze number $S$ describes movement of the plates ($S > 0$ corresponds to the plates moving apart, while $S < 0$ corresponds to the plates moving together). Skin friction coefficient is defined as:

$$C_f = v(1+\frac{1}{\gamma})\frac{\left(\frac{\partial \hat{u}}{\partial \hat{y}}\right)_{\hat{y}=h(t)}}{v_w^2}.$$

In terms of Eq. (7)-(9), we have

$$l^2/x^2(1-\alpha t)Re_x C_f = (1+\frac{1}{\gamma})F''(1). \tag{12}$$

## 3 Solution procedure

### 3.1 Variation of parameters method (VPM)

Using standard procedure for VPM [11–14, 17, 19, 26, 27], we can write Eq. (10) as

$$F_{n+1}(\eta) = C_1 + C_2\eta + C_3\frac{\eta^2}{2} + C_4\frac{\eta^3}{6} - \int_0^\eta \left(\frac{\eta^3}{3!} - \frac{\eta^2 s}{2!} + \frac{\eta s^2}{2!} + \frac{s^3}{3!}\right)\left(-S\left(\frac{\gamma}{1+\gamma}\right)\left(sF(s) + 3F''(s) + F'(s)F''(s) - F(s)F'''(s)\right)\right) \mathrm{d}s. \tag{13}$$

Consuming boundary conditions given in Eq. (11), above equation becomes

$$F_{n+1}(\eta) = C_2\eta + C_4\frac{\eta^3}{6} - \int_0^\eta \left(\frac{\eta^3}{3!} - \frac{\eta^2 s}{2!} + \frac{\eta s^2}{2!} + \frac{s^3}{3!}\right)\left(-S\left(\frac{\gamma}{1+\gamma}\right)\left(sF(s) + 3F''(s) + F'(s)F''(s) - F(s)F'''(s)\right)\right) \mathrm{d}s, \tag{14}$$

with $n = 0, 1, 2, \cdots$.

Where $C_2$ and $C_4$ are constants which can be computed using boundary conditions $F(1) = 1$ and $F'(1) = 0$, respectively.

First few terms of the solution are given as

$$\begin{aligned}
F_0(\eta) &= \left(\frac{\gamma}{1+\gamma}\right)\left(C_2\eta + C_4\frac{\eta^3}{6}\right),\\
F_1(\eta) &= \left(\frac{\gamma}{1+\gamma}\right)\left(C_2\eta + C_4\frac{\eta^3}{6}\right) - \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{1}{30}SC_4\right)\eta^5 - \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{1}{2530}SC_4^2\right)\eta^7,\\
F_2(\eta) &= \left(\frac{\gamma}{1+\gamma}\right)\left({}_2\eta + C_4\frac{\eta^3}{6}\right) - \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{1}{30}SC_4\right)\eta^5\\
&\quad - \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{1}{210}S^2C_4 + \frac{1}{2520}SC_4^2 - \frac{1}{630}S^2C_2C_4\right)\eta^7\\
&\quad - \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{1}{11340}S^2C_4^2 - \frac{1}{45360}S^2C_2C_4^2\right)\eta^9\\
&\quad - \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{1}{178200}S^3C_4^2 - \frac{1}{2494800}S^2C_4^3\right)\eta^{11} + \ldots.
\end{aligned} \tag{15}$$

Similarly, other iterations of the solution can also be computed.

### 3.2 Adomian's decomposition method (ADM)

Following proposed procedure for ADM[1, 2, 6, 8, 23] , Eq. (12) in operator form gives

$$\begin{aligned}
LF(\eta) =& A_1 + A_2\eta + A_3\frac{\eta^2}{2} + A_4\frac{\eta^3}{6}\\
&+ S\left(\frac{\gamma}{1+\gamma}\right)\left(\eta F(\eta) + 3F''(\eta) + F'(\eta)F''(\eta) - F(\eta)F'''(\eta)\right),
\end{aligned} \tag{16}$$

where $L = \frac{d^4}{\mathrm{d}\eta^4}$.

Applying $L^{-1}$ on both sides of above equation, we get

$$\begin{aligned}
F(\eta) =& F(0) + F'(0)\eta + F''(0)\frac{\eta^2}{2} + F'''(0)\frac{\eta^3}{6} +\\
& L^{-1}\left[S\left(\frac{\gamma}{1+\gamma}\right)\left(\eta F(\eta) + 3F''(\eta) + F'(\eta)F''(\eta) - F(\eta)F'''(\eta)\right)\right],
\end{aligned} \tag{17}$$

where,

$$L^{-1} = \int_0^\eta\int_0^\eta\int_0^\eta\int_0^\eta (.)\mathrm{d}\eta\mathrm{d}\eta\mathrm{d}\eta\mathrm{d}\eta.$$

Above Eq. (17) can also be written as

$$\begin{aligned}
F(\eta) =& F(0) + F'(0)\eta + F''(0)\frac{\eta^2}{2} + F'''(0)\frac{\eta^3}{6}\\
&+ \int_0^\eta\int_0^\eta\int_0^\eta\int_0^\eta S\left(\frac{\gamma}{1+\gamma}\right)\left(\eta F(\eta) + 3F''(\eta) + F'(\eta)F''(\eta) - F(\eta)F'''(\eta)\right)\mathrm{d}\eta\mathrm{d}\eta\mathrm{d}\eta\mathrm{d}\eta,
\end{aligned} \tag{18}$$

Using boundary conditions given in Eq. (11) above equation reduces to

$$\begin{aligned}
F(\eta) =& F'(0)\eta + F'''(0)\frac{\eta^3}{6}\\
&+ \int_0^\eta\int_0^\eta\int_0^\eta\int_0^\eta S\left(\frac{\gamma}{1+\gamma}\right)\left(\eta F(\eta) + 3F''(\eta) + F'(\eta)F''(\eta) - F(\eta)F'''(\eta)\right)\mathrm{d}\eta\mathrm{d}\eta\mathrm{d}\eta\mathrm{d}\eta.
\end{aligned} \tag{19}$$

Replacing $F'(0)$ by $C_2$ and $F''(0)$ by $C_4$ we get

$$F(\eta) = C_2\eta + C_4\frac{\eta^3}{6} + \int_0^\eta\int_0^\eta\int_0^\eta\int_0^\eta S\left(\frac{\gamma}{1+\gamma}\right)\left(\eta F(\eta) + 3F''(\eta) + F'(\eta)F''(\eta) - F(\eta)F'''(\eta)\right)\mathrm{d}\eta\mathrm{d}\eta\mathrm{d}\eta\mathrm{d}\eta, \quad (20)$$

where $C_2$ and $C_4$ are constants which can be computed using boundary conditions $F(1) = 1$ and $F'(1) = 0$, respectively.

To solve above equation by ADM, we let

$$F(\eta) = \sum_{n=0}^{\infty} F_n(\eta), \quad (21)$$

and

$$NF(\eta) = \sum_{n=0}^{\infty} A_n. \quad (22)$$

In Eq. (20) non-linear terms are

$$NF(\eta) = S\left(\frac{\gamma}{1+\gamma}\right)(\eta F(\eta) + 3F''(\eta) + F'(\eta)F''(\eta) - F(\eta)F'''(\eta)), \quad (23)$$

Thus, Eq. (20) finally takes the form:

$$F(\eta) = F_0(\eta) + \int_0^\eta\int_0^\eta\int_0^\eta\int_0^\eta (\sum_{n=0}^{\infty} A_n)\mathrm{d}\eta\mathrm{d}\eta\mathrm{d}\eta\mathrm{d}\eta,$$

here $A_n$ are Adomian's polynomials which can be calculated using different approaches [10, 24]. Now following typicalpractice, we can take

$$F_0(\eta) = C_2\,\eta + C_4\frac{\eta^3}{6}. \quad (24)$$

The other components of $F_n(\eta)$ are given by

$$F_{n+1}(\eta) = \int_0^\eta\int_0^\eta\int_0^\eta\int_0^\eta (A_n)\mathrm{d}\eta\mathrm{d}\eta\mathrm{d}\eta\mathrm{d}\eta, \quad (25)$$

Adomian's polynomials $A_n$ are obtained as under

$$A_0 = S\left(\frac{\gamma}{1+\gamma}\right)\left(\eta F_0(\eta) + 3F_0''(\eta) + F_0'(\eta)F_0''(\eta) - F_0(\eta)F_0'''(\eta)\right), \quad (26)$$

$$A_1 = S\left(\frac{\gamma}{1+\gamma}\right)\left\{\begin{array}{l}\eta F_1(\eta) + 3F_1''(\eta) + (F_1'(\eta)F_0''(\eta) + F_0'(\eta)F_1''(\eta)) \\ -(F_1(\eta)F_0'''(\eta) + F_0(\eta)F_1'''(\eta))\end{array}\right\}, \quad (27)$$

$$A_2 = S\left\{\frac{\gamma}{1+\gamma}\right)\left\{\begin{array}{l}\eta F_2(\eta) + 3F_2''(\eta) + (F_2'(\eta)F_0''(\eta) + F_1'(\eta)F_1''(\eta) + F_0'(\eta)F_2''(\eta)) \\ -(F_2(\eta)F_0'''(\eta) + F_1(\eta)F_1'''(\eta) + F_0(\eta)F_2'''(\eta))\end{array}\right\}. \quad (28)$$

And so on.

First three terms of solution are given by

$$F_0(\eta) = \left(\frac{\gamma}{1+\gamma}\right)\left(C_2\,\eta + C_4\frac{\eta^3}{6}\right), \quad (29)$$

$$F_1(\eta) = \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{21}{830}SC_4\eta^4 + \frac{1}{2520}SC_4^2\eta^6\right), \quad (30)$$

$$F_2(\eta) = \left(\frac{\gamma}{1+\gamma}\right)\left\{\begin{array}{l}\frac{396}{8237}\left(S^2 + S^2C_2C_4\right)\eta^4 - \frac{11}{498460}\left(40S^2C_4^2 - S^2C_2C_4^2\right)\eta^6 \\ +\frac{1}{2494800}S^2C_4^3\eta^8\end{array}\right\}. \quad (31)$$

Other approximations of the solution can also be calculated in a similar way. Final solution will be of the form

$$F(\eta) = (\frac{\gamma}{1+\gamma})\left\{\begin{array}{l} C_2\eta + C_4\frac{\eta^3}{6} + \frac{21}{830}SC_4\eta^4 + \frac{1}{2520}SC_4^2\eta^6 + \frac{396}{8237}\left(S^2 + S^2C_2C_4\right)\eta^4 \\ -\frac{11}{498460}\left(40S^2C_4^2 - S^2C_2C_4^2\right)\eta^6 + \frac{1}{2494800}S^2C_4^3\eta^8\cdots \end{array}\right\} \tag{32}$$

## 4 Results and discussions

The behavior of Squeeze number $S$ and Casson fluid parameter $\gamma$ on axial $(F(\eta))$and radial $(F'(\eta))$ velocities is described in this section. For this purpose Figs. 1-8 are displayed. Fig. 1 depicts the effects of increasing values for squeeze number Son axial velocity $F(\eta)$. It is clear that increasing $S$ results in lower values of $F(\eta)$. Alike effects for increasing $S$ on radial velocity can also be seen in a part of Fig. 2. Decrease in is observed for a rapid increase in squeeze number $S$ for $\eta \leq 0.5$. However, there is an increase in $F'(\eta)$ for $0.5 < \eta \leq 1$ .

Fig. 3 depicts the behavior of Casson fluid parameter $\eta$ on $F(\eta)$. Axial velocity is found to be a decreasing function of $\gamma$. Effects of rising $\gamma$ on radial velocity are shown in Fig. 4 and a similar behavior to that positive value for $S$ is observed. That is, increase in $\gamma$ decreases $F'(\eta)$ for $\eta \leq 0.5$ and a decrease in $F'(\eta)$ is observed for $0.5 < \eta \leq 1$ .

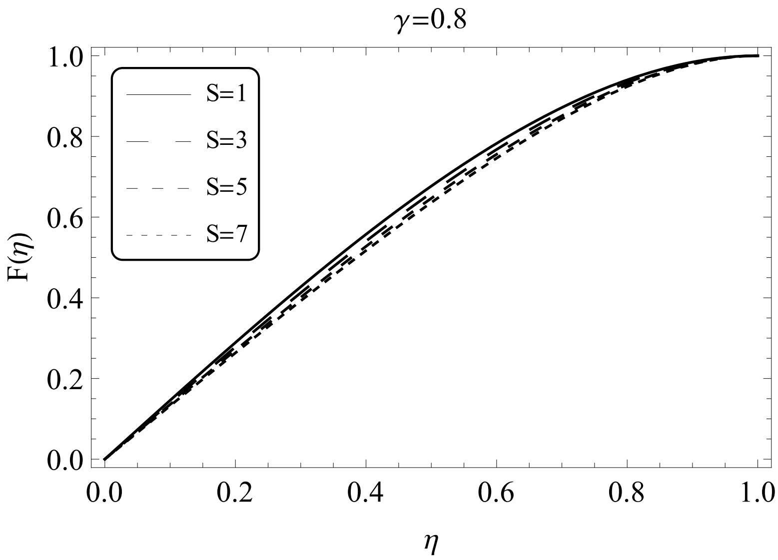


**Fig. 1.** Effects of positive values of $S$ on $F(\eta)$

Figs. 5-8 are presented to analyze the effects of parameters when the plates are moving toward each other ($S < 0$). In Fig. 5, considerable increase in axial velocity is observed with a decrease in squeeze number $S$. Fig. 6 demonstrate the effects of decreasing squeeze number on radial velocity. It is clear that $F'(\eta)$ increases with squeeze rate for $\eta \leq 0.4$. A sudden change in $F'(\eta)$ is observed when $0.4 < \eta \leq 1$, i.e. for decreasing values of squeeze number, there is a rapid decrease in radial velocity of the fluid. Figs. 7-8 show the effects of Casson fluid parameter on axial and radial velocities respectively. Almost identical behavior is observed for Casson fluid parameter $\gamma$ and squeeze number $S$ when plates are coming together.

It is important to check the convergence of series solutions obtained in Eq. (15) and Eq. (32). For this purpose, numerical values for unknown constants $C_2$ and $C_4$ are computed in Tabs. 1 and 2. In case of expanding movement of the plates, values of $C_2$ and $C_4$ are shown in Tab. 1. It is pertinent to mention that VPM converges faster as compared to ADM. Only 5 approximations are enough for a convergent solution for VPM while ADM require 7 approximations of the solution to converge for positive values of Squeeze number $S$. Results obtained are also compared with the ones obtained by solving the Eq. (10) by using RK-4 method and an excellent agreement is found.

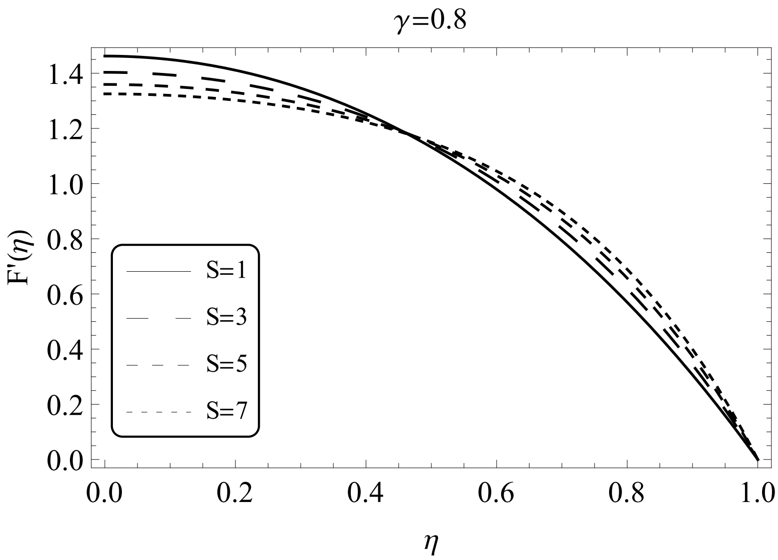


**Fig. 2.** Effects of positive values of $S$ on $F'(\eta)$

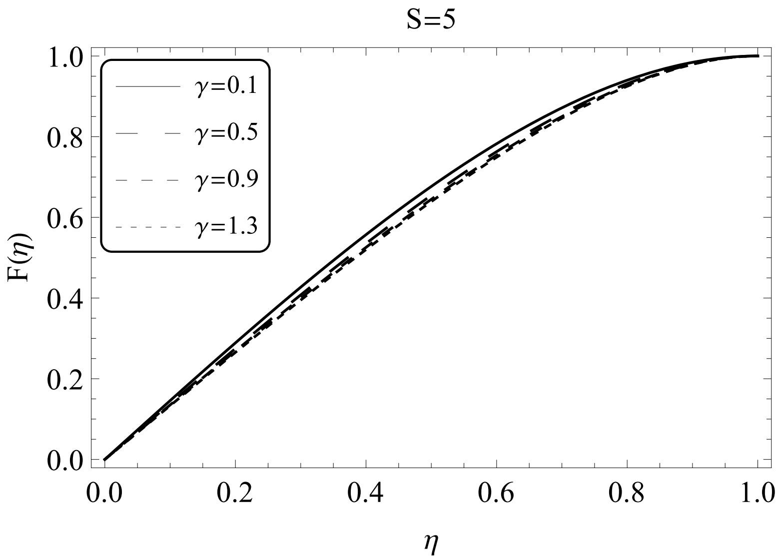


**Fig. 3.** Effects of $\gamma$ on $F(\eta)$ for $S = 5$

In Table numerical values of $C_2$ and $C_4$ are computed for the case of embracing plates. Interestingly, for this case convergence of series solution obtained by ADM is quite slow. There seems to be no effect on the convergence of VPM solution. For this case 5 approximations guarantee the convergent solution for VPM. On the other hand ADM requires 9 approximations to ensure a convergent solution.

In Tab. 4, numerical values for skin friction coefficient are also tabulated. It can be seen that magnitude of skin friction coefficient increase with an increase in both squeeze number and the Casson fluid parameter.

Same problem is solved by a well-known numerical technique Runge-Kutta coupled with shooting. A comparison of all solutions is shown in Tab. 3 for radial velocity. It can be observed that all three solutions agree well with each other.

## 5 Conclusions

Squeezing flow of a non-Newtonian fluid namely Casson fluid is considered between parallel plates. Appropriate steps have been taken to obtain governing nonlinear ordinary differential equation. Solution to the problem is sought using two analytical techniques (VPM, ADM) and one numerical technique (RK-4

**Table 1.** Convergence of VPM and ADM solutions for $S = 5$ and $\gamma = 0.8$

| Order of approximation | $C_2 = f(0)$(VPM) | $C_4 = f(0)$(VPM) | $C_2 = f(0)$(ADM) | $C_4 = f(0)$(ADM) |
|---|---|---|---|---|
| 1 | 1.385311 | $-1.610018$ | 1.385331 | $-1.610018$ |
| 3 | 1.362382 | $-1.43923$ | 1.359495 | $-1.421565$ |
| 5 | 1.359393 | $-1.420987$ | 1.359392 | $-1.420984$ |
| 7 | 1.359393 | $-1.420987$ | 1.359393 | $-1.420987$ |
| 9 | 1.359393 | $-1.420987$ | 1.359393 | $-1.420987$ |
| 11 | 1.359393 | $-1.420987$ | 1.359393 | $-1.420987$ |
| 13 | 1.359393 | $-1.420987$ | 1.359393 | $-1.420987$ |
| Numerical (RK-4) | 1.359393 | $-1.420987$ | 1.359393 | $-1.420987$ |

**Table 2.** Convergence of VPM and ADM solutions for $S = -5$ and $\gamma = 0.8$

| Order of approximation | $C_2 = f(0)$(VPM) | $C_4 = f(0)$(VPM) | $C_2 = f(0)$(ADM) | $C_4 = f(0)$(ADM) |
|---|---|---|---|---|
| 1 | 2.09017 | $-10.68626$ | 2.09017 | $-10.68626$ |
| 3 | 1.900594 | $-8.435428$ | 1.896113 | $-8.389618$ |
| 5 | 1.90024 | $-8.431932$ | 1.90015 | $-8.431048$ |
| 7 | 1.90024 | $-8.431932$ | 1.900256 | $-8.431938$ |
| 9 | 1.90024 | $-8.431932$ | 1.90024 | $-8.431932$ |
| 11 | 1.90024 | $-8.431932$ | 1.90024 | $-8.431932$ |
| 13 | 1.90024 | $-8.431932$ | 1.90024 | $-8.431932$ |
| Numerical (RK-4) | 1.90024 | $-8.431932$ | 1.90024 | $-8.431932$ |

**Table 3.** Comparison of numerical solution, VPM and ADM solutions for $\gamma = 0.8$

| $\eta$ | | $S = 5$ | | | $S = -5$ | |
|---|---|---|---|---|---|---|
| | VPM | ADM | Numerical | VPM | ADM | Numerical |
| 0 | 1.359393 | 1.359393 | 1.359393 | 1.90024 | 1.90024 | 1.90024 |
| 0.1 | 1.352236 | 1.352236 | 1.352236 | 1.858391 | 1.858391 | 1.858391 |
| 0.2 | 1.330124 | 1.330124 | 1.330124 | 1.736537 | 1.736537 | 1.736537 |
| 0.3 | 1.291101 | 1.291101 | 1.291101 | 1.545418 | 1.545418 | 1.545418 |
| 0.4 | 1.231763 | 1.231763 | 1.231763 | 1.30185 | 1.30185 | 1.30185 |
| 0.5 | 1.147043 | 1.147043 | 1.147043 | 1.027213 | 1.027213 | 1.027213 |
| 0.6 | 1.029894 | 1.029894 | 1.029894 | 0.745515 | 0.745515 | 0.745515 |
| 0.7 | 0.870872 | 0.870872 | 0.870872 | 0.481234 | 0.481234 | 0.481234 |
| 0.8 | 0.657596 | 0.657596 | 0.657596 | 0.257123 | 0.257123 | 0.257123 |
| 0.9 | 0.374091 | 0.374091 | 0.374091 | 0.092182 | 0.092182 | 0.092182 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |

**Table 4.** Numerical values for skin friction coefficient

| $S$ | $\gamma$ | $(1+\frac{1}{\gamma})F''(1)$ |
|---|---|---|
| $-5$ | 0.3 | $-8.73184$ |
| $-3$ | - | $-10.635597$ |
| $-1$ | - | $-12.263611$ |
| 1 | - | $-13.644188$ |
| 3 | - | $-14.976768$ |
| 5 | - | $-16.144394$ |
| $-2$ | 0.1 | $-31.536967$ |
| - | 0.3 | $-11.47829$ |
| - | 0.5 | $-7.430105$ |
| 2 | 0.1 | $-34.396598$ |
| - | 0.3 | $-14.351622$ |
| - | 0.5 | $-10.321463$ |

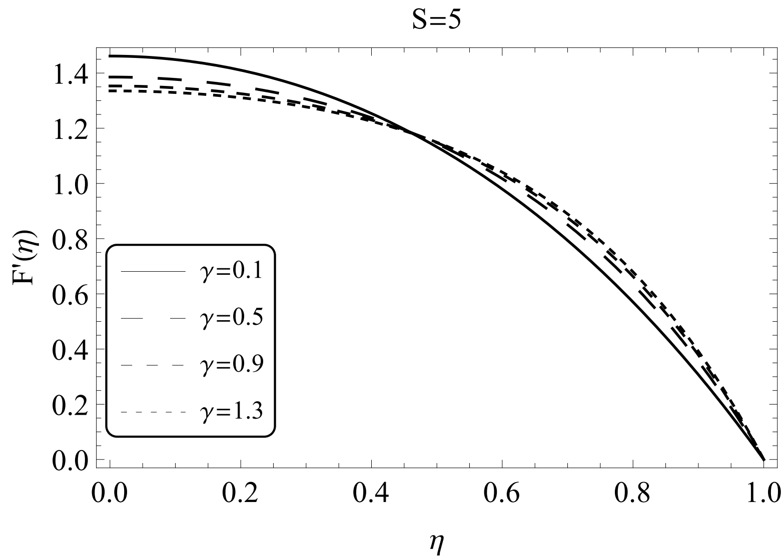


**Fig. 4.** Effects of $\gamma$ on $F'(\eta)$ for $S = 5$

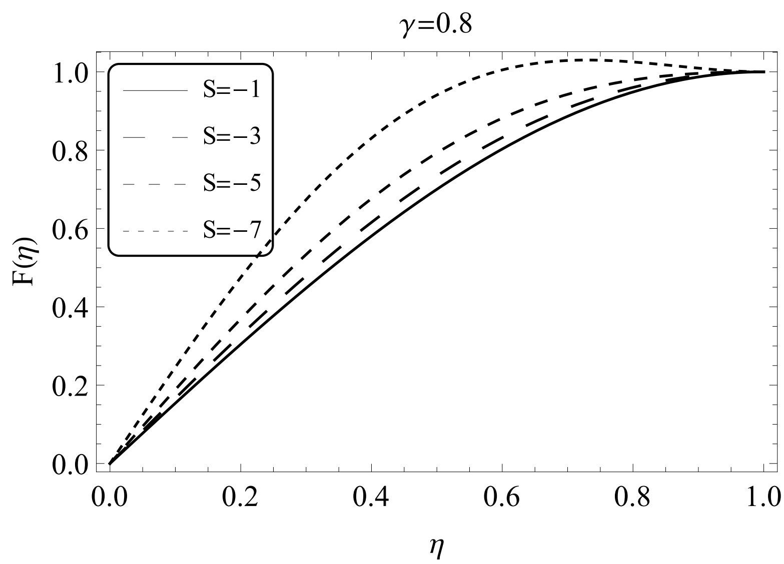


**Fig. 5.** Effects of negative values of $S$ on $F(\eta)$

coupled with shooting). Comparison between analytical solutions tells us that both solutions are compatible; however, variation of parameters method converges rapidly as compared to Adomian's decomposition method. It is also worth mentioning that VPM is free from so called Adomian's polynomials as well. Also, VPM gives as desired results at less computational cost as compared to ADM. Numerical solution also agrees with both the analytical solutions after certain order of approximations as mentioned above.

Effects of different parameters on the flow behavior are shown with the help of graphical figures accompanied by comprehensive discussion.

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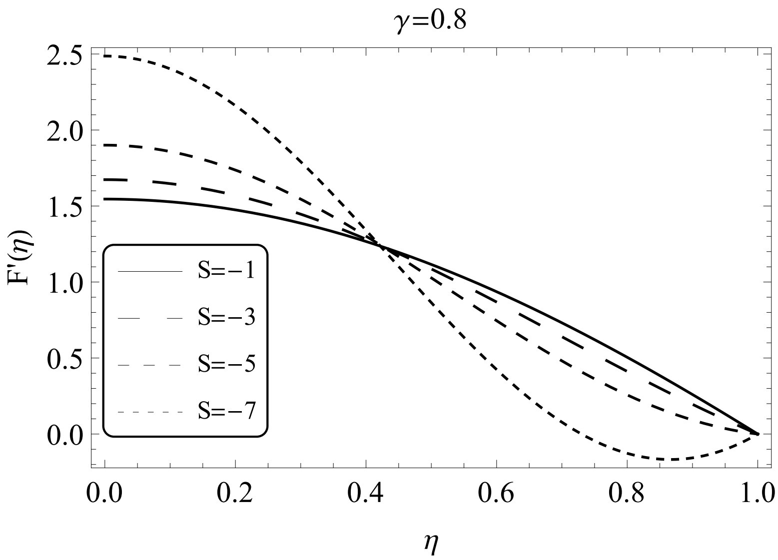


**Fig. 6.** Effects of positive values of $S$ on $F'(\eta)$

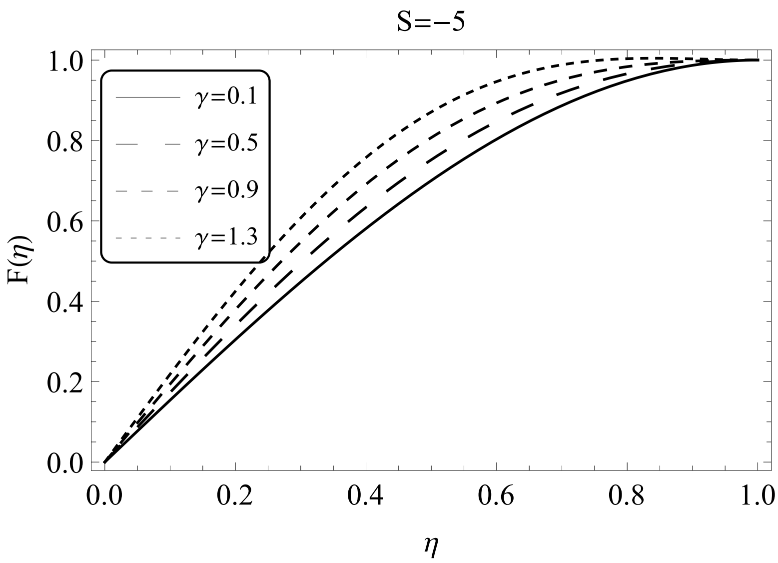


**Fig. 7.** Effects of $F(\eta)$ on for $S = -5$


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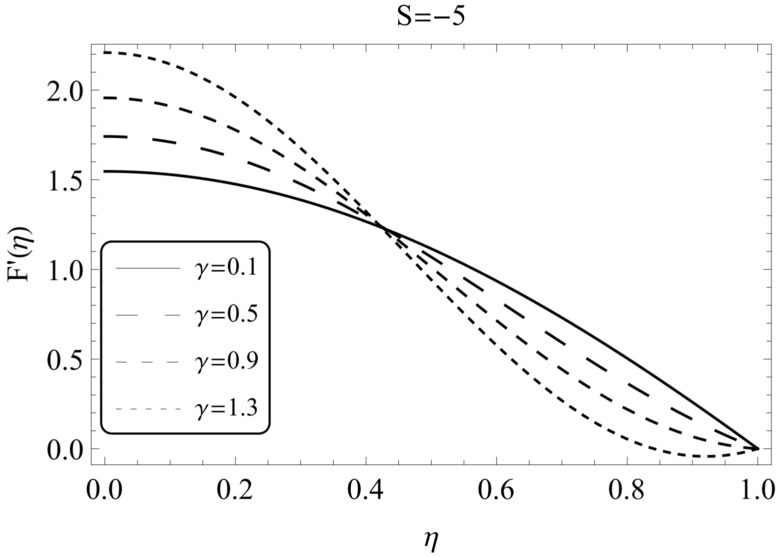


**Fig. 8.** Effects of $F'(\eta)$ on for $S = -5$


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