

Stability analysis of a novel VEISV propagation model of computer worm attacks

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Abstract. In this paper, an VEISV (vulnerable exposed infectious secured vulnerable) network worm attack model with saturated incidence rate is considered. The basic reproduction number R_0 is found. If $R_0 \leq 1$, the worm-free equilibrium is globally asymptotically stable; if $R_0 > 1$, worm-epidemic state is globally asymptotically stable and uniformly persistent.

Keywords: VEISV model, stability, basic reproduction number, numerical simulation, computer worm attacks

1 Introduction

Applications based on computer networks are becoming more and more popular in our daily life. While bringing convenience to us, computer networks are exposed to various threats. Computer viruses, which are programs developed to attempt to attach themselves to a host and spread to other computers mainly through the Internet, can damage network resources. Consequently, understanding the law governing the spread of computer virus is of considerable interest. Mathematical modeling has been playing an ever more important role in the study of epidemiology. Various epidemic models have been proposed and explored extensively and great progress has been achieved in the studies of disease control and prevention [1–3] and the references therein. Many authors have studied the autonomous epidemic models. The basic and important research subjects for these models are the calculation of the reproductive number of disease or threshold values, the local and global stability of disease-free equilibrium and endemic equilibrium, the existence, uniqueness and stability of periodic oscillation of disease, Hopf bifurcation, the persistence, permanence and extinction of disease, etc. Many important results with regard to these subjects can be found in many articles, for example, see [4–6] and the references cited therein. Particularly, the works in allusion to various types of SIRS epidemic models can be found in [4, 7–13] and the references cited therein.

The similarity between the spread of a biological virus and malicious worm propagation encourages researchers to adopt an epidemic model to the network environment^[14]. Research in modeling computer viruses and worms implement epidemic models like SIR (Susceptible-Infected- Recovered)^[15, 16], SIS (Susceptible-Infected-Susceptible)^[14], SEIR (susceptible-exposed-infectious-recovered)^[18–21, 24], SIRS (SusceptibleInfectedRecoveredSusceptible)^[22–24], and SEIQV (susceptible, exposed, infected, quarantined, and vaccinated)^[25–27], VEISV (vulnerable exposed infectious secured vulnerable)^[28]. The purpose is to study worm propagation by developing different transaction states based on the behavior of the virus or the worm. This section summarizes research topics in the areas of modeling malicious worms, modeling benign worms, quarantine defense mechanism, and stability analysis.

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Worm attacks are considered by network experts the highest security risk on computer network security, functionality and assets. Attackers use a malicious worm as a primary tool to target software vulnerabilities. Computer worms are built to propagate without warning or user interaction, causing an increase in network traffic service requests that will eventually lead to distributed denial-of-service (DDoS). However, recent worm assaults exceeded common impacts such as DDoS or backdoor listener and caused financial losses and threatened the security of classified information. In [29], based on the SEIR biological model, the new VEISV worm model is proposed. The derivation of the reproduction rate shows a worm-free equilibrium global stability and unique worm epidemic equilibrium local stability. Furthermore, simulation results show the positive impact of increasing security countermeasures in the V-state, and the equilibrium points. The authors of [30], proposes a computer worm model considering countermeasures and analyzes the stability of the model. The proposed VEISV multi-malware worm model is appropriate for realistic up-to-date security countermeasure implementation, and the model takes into consideration the accurate positions for hosts replacements and hosts out-of-service in state transactions. Furthermore, initial simulation results show the positive impact of increasing security measures on a worm propagation wave. Additionally, confirmation of stability points is under development by using phase plot. In many epidemic models, bilinear incidence rate βSI (where S , and I denote the number of susceptible individuals and infectious individuals) is frequently used (see [1, 31, 34] and the references therein). Capasso and Serio^[35] introduced the saturated incidence rate $\frac{\beta SI}{1+\alpha I}$, where $\frac{\beta I}{1+\alpha I}$ tends to a saturation level when I gets large, and βI measures the infection force when the disease is entering a fully susceptible population and $\frac{1}{1+\alpha I}$ measures the inhibition effect from the behavioral change of susceptible individuals when their number increases or from the crowding effect of the infective individuals. This incidence rate is more reasonable than the bilinear incidence rate because it includes the behavioral change and crowding effect of the infective individuals and prevents the unboundedness of the contact rate by choosing suitable parameters. and it was used in many epidemic models afterwards^[36, 37].

To develop the VIESV model we took into account information terminology and security counter measures that have been used to prevent and defend against worm attacks. Thus, we use the state name vulnerable instead of susceptible and secured instead of recovered. The state transitions of hosts in the VEISV model is summarized as follows: vulnerable \rightarrow exposed \rightarrow infectious \rightarrow secured \rightarrow vulnerable. The vulnerable state includes all hosts which are vulnerable to worm attack. Exposed state includes all hosts which are exposed to attack but not actively infectious due to the latent time requirement. Infectious state includes all hosts which were attacked and actively scanning and targeting new victims. Secured state includes all hosts which gained one or more security counter measures, providing the host with a temporary or permanent immunity against the malicious worm. In this paper, we consider a novel VEISV^[28] worm model with a saturated recovery function as the following form

$$\begin{cases} \frac{dV}{dt} = -\frac{fEV}{1+\alpha_1 V} - \psi_1 V + \varphi S, \\ \frac{dE}{dt} = \frac{fEV}{1+\alpha_1 V} - (\alpha + \psi_2)E, \\ \frac{dI}{dt} = \alpha E - (\gamma + \theta)I, \\ \frac{dS}{dt} = \mu N + \psi_1 V + \psi_2 E + \gamma I - \varphi S. \end{cases} \quad (1)$$

with the following initial conditions:

$$V(0) = V_0, E(0) = E_0, I(0) = I_0, S(0) = S_0. \quad (2)$$

where $V(t), E(t), I(t)$ and $S(t)$ denote the number of vulnerable, exposed, infectious and secured respectively and β is Contact rate, α is state transition rate from E to I , ψ_1 is state transition rate from V to S , ψ_2 is state transition rate from E to S , γ is state transition rate from I to S , φ is state transition rate from S to V , θ is dysfunctional rate, μ is replacement rate and $f = \frac{\alpha\beta}{N}$.

We built the VEISV model based on the following assumptions: (1) The total number of hosts N is fixed and defined in Eq. (1):

$$N = V(t) + E(t) + I(t) + S(t).$$

(2) Initially, all hosts are vulnerable to attack. The total number of quarantined hosts, without considering the quarantine time, will move to the secure state after installing the required security patches or updates. In this case, the basic infection reproductive number is

$$R_0 = \frac{f\varphi N}{(\alpha + \psi_2)(\psi_1 + \varphi + \alpha_1\varphi N)}.$$

Since, $S(t) = N - V(t) - E(t) - I(t)$, we can use the reduction method by considering only the first three equations of (1) to analyze our model

$$\begin{cases} \frac{dV}{dt} = \varphi N - \frac{fEV}{1+\alpha_1V} - (\psi_1 + \varphi)V - \varphi E - \varphi I, \\ \frac{dE}{dt} = \frac{fEV}{1+\alpha_1V} - (\alpha + \psi_2)E, \\ \frac{dI}{dt} = \alpha E - (\gamma + \theta)I. \end{cases} \quad (3)$$

Let $W(t) = V(t) + E(t) + I(t)$. Summing up the equations in (3) gives

$$\frac{dW}{dt} = \varphi N - (\psi_1 + \varphi)V - (\psi_2 + \varphi)E - (\varphi + \gamma + \theta)I. \quad (4)$$

Define $\zeta = \max\{\psi_1 + \varphi, \psi_2 + \varphi, \varphi + \gamma + \theta\}$. From Eq. (4) we get

$$\frac{dW}{dt} \leq \varphi N - \zeta W.$$

Then, one can get

$$W(t) \leq e^{-\zeta t}(V(0) + E(0) + I(0)) + \frac{\varphi N}{\zeta}(1 - e^{-\zeta t}).$$

Therefore we study (3) in the closed set

$$\Omega = \left\{ (V, E, I) \in \mathbb{R}_+^3 \mid 0 \leq I + E + V \leq \frac{\varphi N}{\zeta} \right\}.$$

2 Stability analysis VEISV model

2.1 Equilibria

To obtain its equilibria, (3) can be written as

$$\begin{cases} \varphi N - \frac{fEV}{1+\alpha_1V} - (\psi_1 + \varphi)V - \varphi E - \varphi I = 0, \\ \frac{fEV}{1+\alpha_1V} - (\alpha + \psi_2)E = 0, \\ \alpha E - (\gamma + \theta)I = 0. \end{cases} \quad (5)$$

For $\frac{dE}{dt} = 0$, the equilibrium occurs at: $E = 0$ or $E > 0$ and $V = \frac{\varphi N}{\psi_1 + \varphi}$. For $E = 0$, the worm-free equilibrium occurs at:

$$P_0 = (V^0, E^0, I^0) = \left(\frac{\varphi N}{\psi_1 + \varphi}, 0, 0 \right).$$

For $E > 0$, the worm-epidemic equilibrium is:

$$P_1 = (V^1, E^1, I^1),$$

where

$$\begin{aligned}
 V^1 &= \frac{\alpha + \psi_2}{f - \alpha_1(\alpha + \psi_2)}, \\
 I^1 &= \frac{\varphi N \alpha (f - \alpha_1(\alpha + \psi_2)) - \alpha(\psi_1 + \varphi)(\alpha + \psi_2)}{\varphi(\alpha + \gamma + \theta) + (\alpha + \psi_2)(\gamma + \theta)(f - \alpha_1(\alpha + \psi_2))}, \\
 E^1 &= \frac{\gamma + \theta}{\alpha} I^1.
 \end{aligned}$$

If we set $f > f_1$ then one can get $V^1 > 0, I^1 > 0, E^1 > 0$, where $f_1 = \alpha_1(\alpha + \psi_2) + \frac{(\psi_1 + \varphi)(\alpha + \psi_2)}{\varphi N}$.

2.2 Local stability of equilibria

In this section, we deal with the local dynamics of (3). First, we examine the local stability of the equilibria by analyzing the eigenvalues of the Jacobian matrices of (3) at the equilibria.

The Jacobian matrix of (3) at the equilibrium $P(V, E, I)$ is

$$J(P) = \begin{pmatrix} -\frac{fE}{(1+\alpha_1V)^2} - (\psi_1 + \varphi) & -\frac{fV}{1+\alpha_1V} - \varphi & -\varphi \\ \frac{fE}{(1+\alpha_1V)^2} & \frac{fV}{1+\alpha_1V} - (\alpha + \psi_2) & 0 \\ 0 & \alpha & -(\gamma + \theta) \end{pmatrix}. \quad (6)$$

The Jacobian matrix of (3) at the worm-free equilibrium $P_0 = (\frac{\varphi N}{\psi_1 + \varphi}, 0, 0)$ is

$$J(P_0) = \begin{pmatrix} -(\psi_1 + \varphi) & -\frac{fV^0}{1+\alpha_1V^0} - \varphi & -\varphi \\ 0 & \frac{fV^0}{1+\alpha_1V^0} - (\alpha + \psi_2) & 0 \\ 0 & \alpha & -(\gamma + \theta) \end{pmatrix}.$$

The corresponding eigenvalues of $J(P_0)$ are:

$$\lambda_1 = -(\psi_1 + \varphi), \quad \lambda_2 = \frac{fV^0}{1 + \alpha_1V^0} - (\alpha + \psi_2), \quad \lambda_3 = -(\gamma + \theta).$$

Because all parameters of the model have positive real values, $\lambda_1 < 0, \lambda_3 < 0$, for a worm-free equilibrium to be locally asymptotically stable, the following condition has to be satisfied:

$$\lambda_2 = \frac{fV^0}{1 + \alpha_1V^0} - (\alpha + \psi_2) < 0. \quad (7)$$

But (7) hold if $R_0 < 1$. Thus we have the following theorem:

Theorem 1. Consider model (3). If $0 < R_0 < 1$, then the worm-free equilibrium P_0 is locally asymptotically stable while it is unstable if $R_0 > 1$.

The Jacobian matrix of (3) at the equilibrium $P_1(V^1, E^1, I^1)$ is

$$\begin{aligned}
 J(P_1) &= \begin{pmatrix} -fE^1 \frac{1}{(1+\alpha_1V^1)^2} - (\psi_1 + \varphi) & -\frac{fV^1}{1+\alpha_1V^1} - \varphi & -\varphi \\ fE^1 \frac{1}{(1+\alpha_1V^1)^2} & \frac{fV^1}{1+\alpha_1V^1} - (\alpha + \psi_2) & 0 \\ 0 & \alpha & -(\gamma + \theta) \end{pmatrix}, \\
 &= \begin{pmatrix} -\frac{E^1(\alpha + \psi_2)^2}{f(V^1)^2} - (\psi_1 + \varphi) & -\varphi - \alpha - \psi_2 & -\varphi \\ \frac{E^1(\alpha + \psi_2)^2}{f(V^1)^2} & \frac{fV^1}{1+\alpha_1V^1} - (\alpha + \psi_2) & 0 \\ 0 & \alpha & -(\gamma + \theta) \end{pmatrix}.
 \end{aligned}$$

The corresponding characteristic equation is given by

$$\lambda^3 + A_1\lambda^2 + A_2\lambda + A_3 = 0, \quad (8)$$

where

$$\begin{aligned} A_1 &= \gamma + \theta + \psi_1 + \varphi + \frac{E^1(\alpha + \psi_2)}{f(V^1)^2}, \\ A_2 &= (\varphi + \alpha + \psi_2 + \gamma + \theta) \frac{E^1(\alpha + \psi_2)^2}{f(V^1)^2} + (\gamma + \theta)(\psi_1 + \varphi), \\ A_3 &= [(\gamma + \theta)(\psi_2 + \varphi + \alpha) + \alpha\varphi] \frac{E^1(\alpha + \psi_2)^2}{f(V^1)^2}. \end{aligned}$$

If $R_0 > 1$, then A_1, A_2, A_3 and $A_1A_2 - A_3 > 0$. Therefore, by the Routh-Hurwitz criterion, if $R_0 > 1$ hold, then P is locally asymptotically stable.

2.3 Global stability of equilibria

In this section, we study the global stability of the equilibria of system (3).

Theorem 2. *The worm-free state P_0 is globally asymptotically stable if $R_0 \leq 1$. If $R_0 > 1$, the worm-free equilibrium P_0 is an unstable point.*

Proof. Learn from the first equation of system (3)

$$V'(t) \leq \varphi N - (\psi_1 + \varphi)V.$$

Thus

$$V(t) \leq \frac{\varphi N}{\psi_1 + \varphi} + \left(V(0) - \frac{\varphi N}{\psi_1 + \varphi} \right) e^{-(\psi_1 + \varphi)t}.$$

When $t \rightarrow \infty$, we obtain

$$V(t) \leq \frac{\varphi N}{\psi_1 + \varphi}. \quad (9)$$

Let us consider the following Lyapunov function defined by $H(E) = E$, The time derivative of $H(E)$ is

$$\begin{aligned} H'(E) &= \frac{fEV}{1 + \alpha_1 V} - (\alpha + \psi_2)E \\ &= \frac{fE}{\frac{1}{V} + \alpha_1} - (\alpha + \psi_2)E. \end{aligned} \quad (10)$$

So, by (9) and (10), one can get

$$\begin{aligned} H'(E) &\leq fE \left(\frac{\varphi N}{\psi_1 + \varphi + \alpha_1 \varphi N} \right) - (\alpha + \psi_2)E \\ &= (\alpha + \psi_2) \left(\frac{f\varphi N}{(\alpha + \psi_2)(\psi_1 + \varphi + \alpha\varphi N)} - 1 \right) E \\ &= (\alpha + \psi_2)(R_0 - 1)E. \end{aligned} \quad (11)$$

Furthermore $H'(E) = 0$ if and only if $E = 0$. Therefore, the largest compact invariant set in $(V, E, I) \mid H'(E) = 0$, when $R_0 \leq 1$, is the singleton P_0 . LaSalle's invariance principle implies that P_0 is globally asymptotically stable in Ω .

Now, by definition of Ω , we can define

$$\Sigma = \{(V, E, I) \in \mathbb{R}_+^3 \mid 0 \leq I + E + V \leq 1\}.$$

Definition 1 The system (3) is said to be uniformly persistent if there is $c > 0$ such that any solution $(V(t), E(t), I(t))$ of system (3) which initial value $(V(0), E(0), I(0)) \in \text{int } \Sigma$ satisfies

$$\min\{\liminf_{t \rightarrow \infty} V(t), \liminf_{t \rightarrow \infty} E(t), \liminf_{t \rightarrow \infty} I(t)\} \geq c.$$

Theorem 3. If $R_0 > 1$, then system (3) is uniformly persistent in $\text{int } \Sigma$.

Proof. The necessity of $R_0 > 1$ follows from Theorem 2 and the fact that the asymptotical stability of P_0 precludes any kinds of persistence. The sufficiency of the condition $R_0 > 1$ follows from a uniform persistence result, Theorem 4.3 in [38]. To demonstrate that (3) satisfies all the conditions of Theorem 4.3 in [38] when $R_0 > 1$, choose $X = R^3$ and $E = \Sigma$. The maximal invariant set M on the boundary $\partial \Sigma$ is the singleton P_0 and is isolated. Thus, the hypothesis (H) of [38] holds for system (3). The theorem is proved by observing that, in the setting of (3), the necessary and sufficient condition for uniform persistence in Theorem 4.3^[38] is equivalent to P_0 being unstable.

In the following, using the geometrical approach of Li and Muldowney^[39] to global stability problems in R^n , we discuss the global stability of P .

For $R_0 > 1$, we can choose $f, \alpha, \varphi, \psi_2$ such that $\max\{1, \frac{\alpha c}{\alpha c - \alpha - \psi_2}\} < \frac{f c^2}{\varphi}$, where c is a uniform persistence constant.

Theorem 4. If $R_0 > 1$, then P is globally asymptotically stable of system (5) in Σ .

Proof. Note that the Jacobian matrix J of (3) is given by (6) and its second additive compound matrix $J^{[2]}$ is

$$J^{[2]} = \begin{pmatrix} -\frac{fE}{(1+\alpha_1V)^2} + \frac{fV}{1+\alpha_1V} - m & 0 & \varphi \\ \alpha & -\frac{fE}{(1+\alpha_1V)^2} - n & -\frac{fV}{1+\alpha_1V} - \varphi \\ 0 & \frac{fE}{(1+\alpha_1V)^2} & \frac{fV}{1+\alpha_1V} - k \end{pmatrix},$$

where

$$m = \psi_1 + \varphi + \alpha + \psi_2,$$

$$n = \psi_1 + \varphi + \gamma + \theta,$$

$$k = \alpha + \psi_2 + \gamma + \theta.$$

Set the function $P(x) = P(V, E, I) = \text{diag}\{a_1, \frac{E}{I}, \frac{E}{I}\}$, where $\max\{1, \frac{\alpha c}{\alpha c - \alpha - \psi_2}\} < a_1 < \frac{f c^2}{\varphi}$, c is a uniform persistence constant. Then,

$$P_f P^{-1} = \text{diag}\left\{0, \frac{E'}{E} - \frac{I'}{I}, \frac{E'}{E} - \frac{I'}{I}\right\},$$

where matrix P_f is obtained by replacing each entry P_{ij} of P by its derivative in the direction of solution of (3). Furthermore, the matrix $B = P_f P^{-1} + P J^{[2]} P^{-1}$ can be written in block form,

$$B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix},$$

where

$$\begin{aligned}
 B_{11} &= -\frac{fE^2}{1+\alpha_1V} + \frac{fV}{1+\alpha_1V} - m, \\
 B_{12} &= a_1 \left[0 \quad \frac{\varphi I}{E} \right], \quad B_{21} = \frac{1}{a_1} \begin{bmatrix} \frac{\alpha E}{I} \\ 0 \end{bmatrix}, \\
 B_{22} &= \begin{bmatrix} \frac{E'}{E} - \frac{I'}{I} - \frac{fE^2}{1+\alpha_1V} - n & -\frac{fV}{1+\alpha_1V} - \varphi \\ \frac{fE}{1+\alpha_1V} & \frac{E'}{E} - \frac{I'}{I} + \frac{fV}{1+\alpha_1V} - k \end{bmatrix}.
 \end{aligned}$$

Let $(\omega_1, \omega_2, \omega_3)$ denote the vectors in $\mathbb{R}^3 \cong \mathbb{R}(n2)$, we select a normal in \mathbb{R}^3 as $(\omega_1, \omega_2, \omega_3) = \max\{|\omega_1| + |\omega_2| + |\omega_3|\}$ and let μ denote the Lozinskiĭ measure with respect to this norm. Following the method in [40], we have the estimated $\mu(B) \leq \sup\{g_1, g_2\}$, where

$$g_1 = \mu_1(B_{11}) + |B_{12}|, \quad g_2 = \mu_1(B_{22}) + |B_{21}|,$$

$|B_{12}|, |B_{21}|$ are matrix norms with respect to l_1 vector norm, and μ_1 denotes the Lozinskiĭ measure with respect to l_1 norm (see [41]). More specifically

$$\mu_1(B_{11}) = -\frac{fE^2}{1+\alpha_1V} + \frac{fV}{1+\alpha_1V} - m.$$

Furthermore, using the second and third equations in (2), we can obtain

$$|B_{12}| = a_1 \frac{\varphi I}{E},$$

and

$$|B_{21}| = \frac{\alpha E}{a_1 I},$$

respectively. To calculate $\mu_1(B_{22})$, add the absolute value of the off-diagonal elements to the diagonal one in each column of B_{22} , and then take the maximum of two sums (see [41]). We thus obtain

$$\begin{aligned}
 \mu_2(B_{22}) &= \max \left\{ \frac{E'}{E} - \frac{I'}{I} - \frac{fE^2}{1+\alpha_1V} - n, \frac{E'}{E} - \frac{I'}{I} + \frac{fV}{1+\alpha_1V} - k \right\} \\
 &= \frac{E'}{E} - \frac{I'}{I} + \frac{fV}{1+\alpha_1V} - \min\{n, k\}.
 \end{aligned}$$

Thus,

$$g_1 = -\frac{fE^2}{1+\alpha_1V} + \frac{fV}{1+\alpha_1V} - m + a_1 \frac{\varphi I}{E}, \tag{12}$$

$$g_2 = \frac{\alpha E}{a_1 I} + \frac{E'}{E} - \frac{I'}{I} + \frac{fV}{1+\alpha_1V} - \min\{n, k\}. \tag{13}$$

From (5), we get

$$\frac{E'}{E} = \frac{fV}{1+\alpha_1V} - (\alpha + \psi_2), \tag{14}$$

$$\frac{I'}{I} = \frac{\alpha E}{I} - (\gamma + \theta). \tag{15}$$

The uniform persistence constant c can be adjusted so that there exists $T > 0$ independent of $(V(0), E(0), I(0)) \in \text{int } \Sigma$, the compact absorbing set, such that

$$E(t) > c \text{ and } I(t) > c \text{ for } t > T. \tag{16}$$

Substituting (14), (15) into (12) and (13) and using (16), we obtain, for $t > T$,

$$\begin{aligned} g_1 &\leq -\frac{fE}{1+\alpha_1V} + \frac{E'}{E} - (\psi_1 + \varphi) + a_1 \frac{\varphi I}{E} \\ &\leq -fc + \frac{E'}{E} - (\psi_1 + \varphi) + \frac{a_1\varphi}{c} \\ &\leq \frac{E'}{E} - (\psi_1 + \varphi) \leq 2\frac{E'}{E} - (\psi_1 + \varphi) \end{aligned} \quad (17)$$

and

$$\begin{aligned} g_2 &= \frac{\alpha E}{a_1 I} + \frac{E'}{E} - \frac{I'}{I} + \frac{fV}{1+\alpha_1V} - \min\{n, k\} \\ &= 2\frac{E'}{E} - \frac{a_1 - 1}{a_1} \frac{\alpha E}{I} + (\alpha + \psi_2) - (\psi_1 + \varphi) \\ &\leq 2\frac{E'}{E} - (\psi_1 + \varphi). \end{aligned} \quad (18)$$

Therefore, by (17) and (18), we obtain

$$\mu(B) \leq \sup\{g_1, g_2\} \leq 2\frac{E'}{E} - (\psi_1 + \varphi).$$

for $t > T$. Here $\psi_1 + \varphi$ is a positive constant. Along each solution $(V(t), E(t), I(t))$ of (5) with $(V(0), E(0), I(0)) \in K$, where K is the compact absorbing set, we thus have

$$\frac{1}{t} \int_0^t \mu(B) ds \leq \frac{2}{t} \log \frac{E(t)}{E(0)} - (\psi_1 + \varphi),$$

and since (3) is uniformly persistent, then the quantity $\bar{q} \leq -\frac{\psi_1 + \varphi}{2} \leq 0$, which is defined as the indicator of global stability of the unique positive equilibrium in [39]. This completes the proof.

3 Numerical simulation

In Figs. 1-2, we display the numerical solution of system (3) at $N = 1000$, $\varphi = 0.005$, $\alpha = 0.2866$, $\beta = 50$, $\psi_1 = 0.003$, $\psi_2 = 6.08$, $\theta = 0.1$, $\mu = 1$, $\gamma = 0.5$; $\alpha_1 = 0.0001$; and $R_0 = 1.80, 1.87, 1.94, 1.95$ and $R_0 = 2.01, 2.02, 2.03, 2.03$ respectively.

In Fig. 3, we display the numerical solution of system (3) at $N = 1000$, $\alpha = 0.2866$, $\beta = 100$, $\psi_1 = 0.003$, $\psi_2 = 6.08$, $\theta = 0.01$, $\mu = 0.001$, $\gamma = 0.5$; $\alpha_1 = 0.0001$; for $R_0 = 4.03, 4.05, 4.07$ and $R_0 = 4.0739$.

In Fig. 4, we display the numerical solution of system (3) at $N = 1000$, $\alpha = 0.2866$, $\beta = 150$, $\psi_1 = 0.01$, $\psi_2 = 20.08$, $\theta = 0.01$, $\mu = 0.001$, $\gamma = 0.5$; $\alpha_1 = 0.001$; for $R_0 = 0.84, 0.90, 0.95$ and $R_0 = 0.97$.

4 Conclusion

Worms can spread throughout the Internet very quickly and are a great security threat. Constant quarantine strategy is a defensive measure against worms, but its reliability in current imperfect intrusion detection systems is poor. In this paper, we have investigated the global dynamics of a VEISV model (1). The basic reproductive rate, R_0 , is derived. Using the reproduction rate, the worm-free equilibrium is globally asymptotically stable if $R_0 \leq 1$. If $R_0 > 1$, then VEISV model is uniformly persistent. If $R_0 > 1$, then worm-epidemic state is globally asymptotically stable. Numerical simulations are carried out to illustrate the feasibility of the obtained results, especially the positive impact of increasing security counter measures in the vulnerable state on worm-exposed and infectious propagation waves.

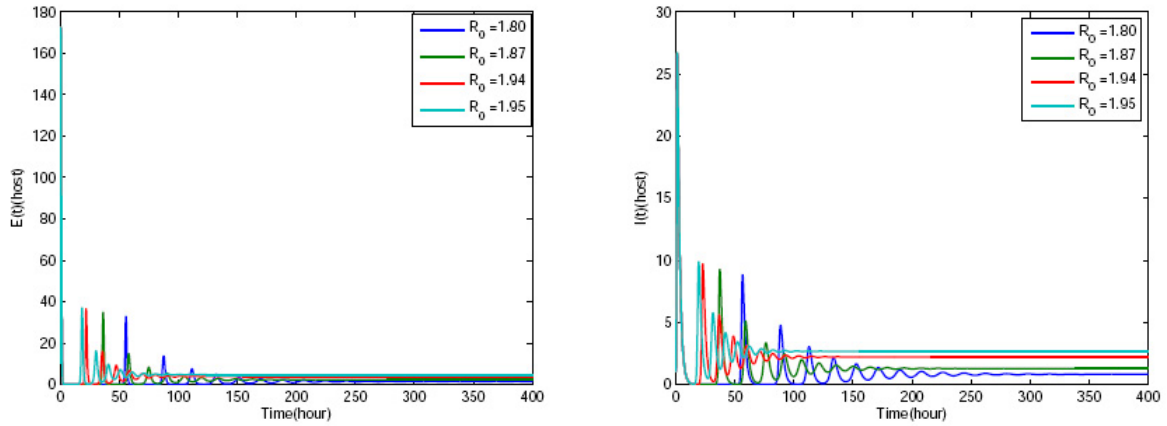


Fig. 1. Uniformly persistent of system (3) for $R_0 = 1.80, 1.87, 1.94$ and $R_0 = 1.95$

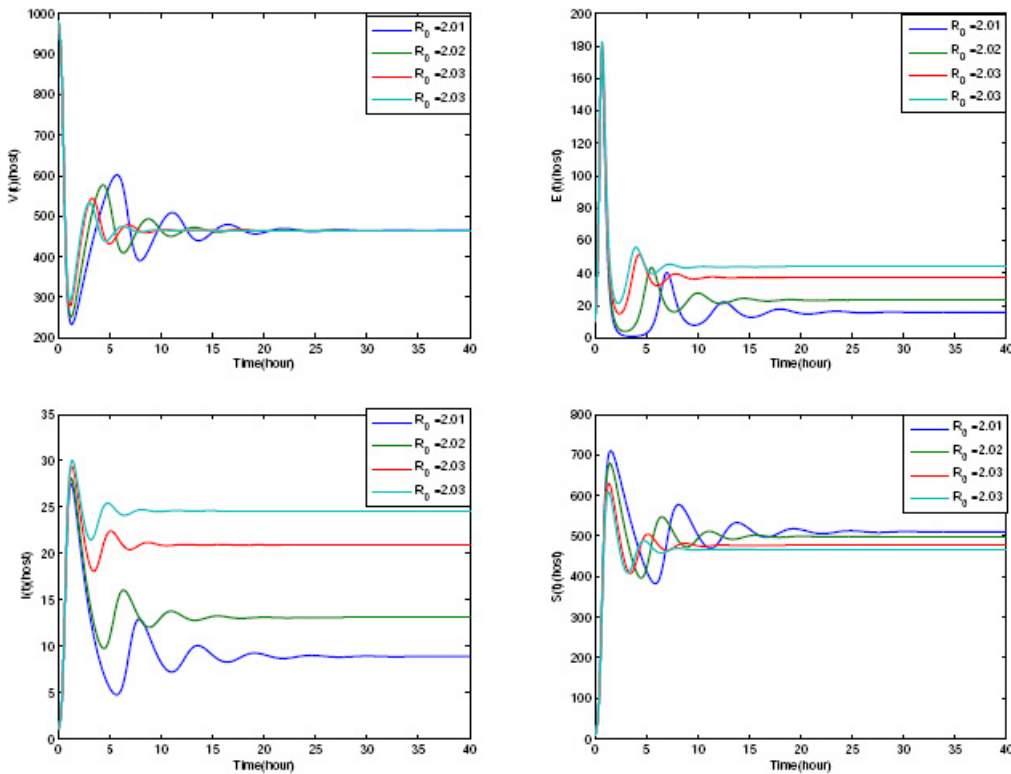


Fig. 2. Numerical solution of system (3) for $R_0 = 2.01, 2.02, 2.03$ and $R_0 = 2.03$

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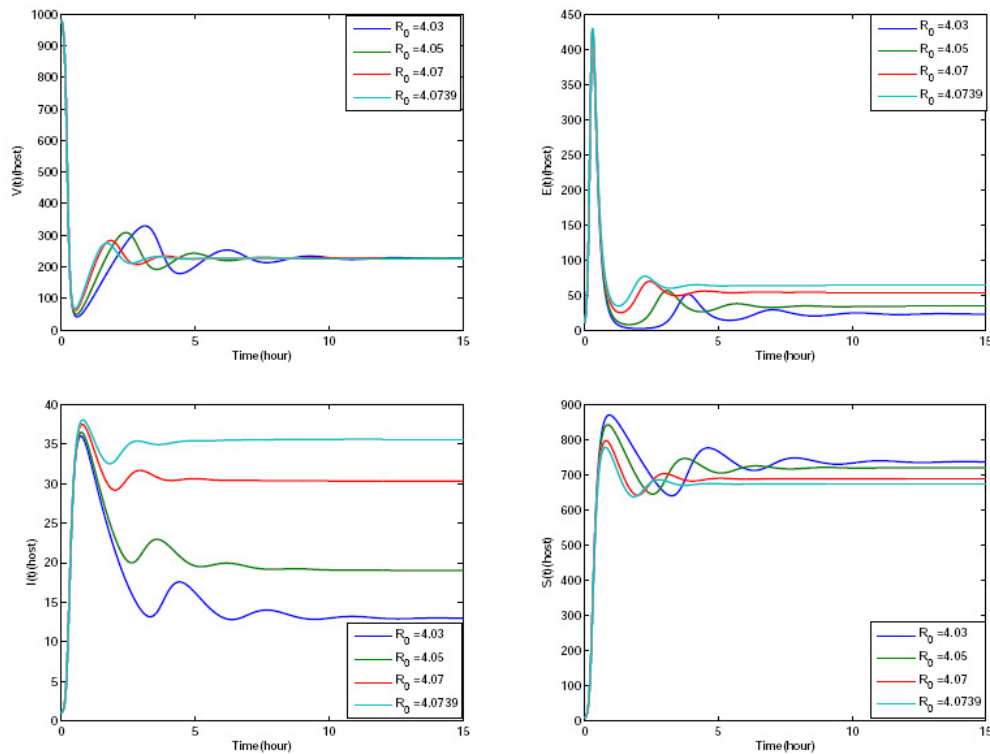


Fig. 3. Numerical solution of system (3) for $R_0 = 4.03, 4.05, 4.07$ and $R_0 = 4.0739$

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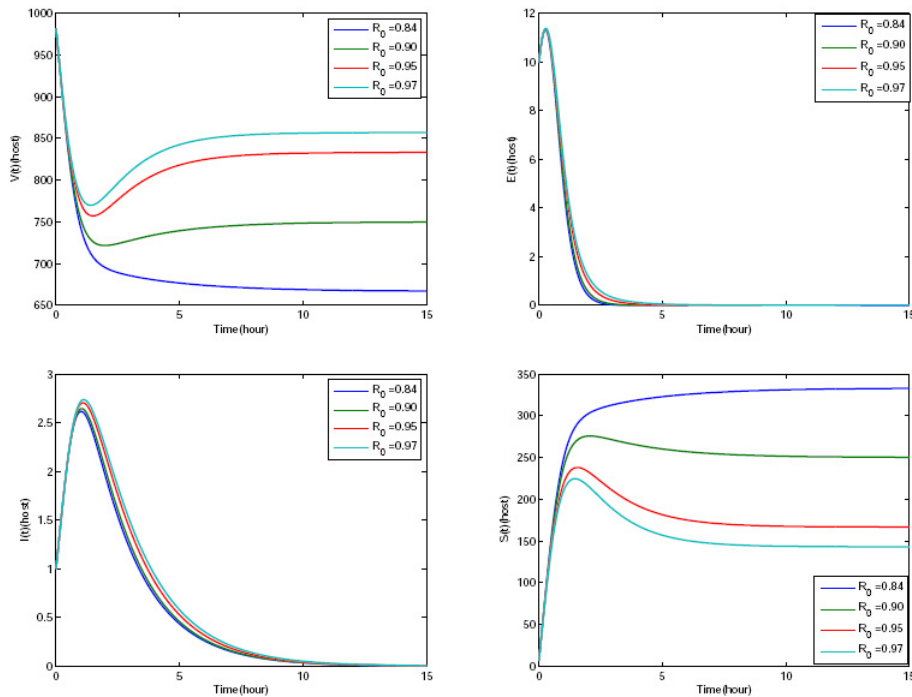


Fig. 4. Numerical solution of system (3) for $R_0 = 0.84, 0.90, 0.95$ and $R_0 = 0.97$

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