**ISSN** 1 746-7233, England, UK
**World Journal of Modelling and Simulation**
Vol. 10 (2014) No. 2, pp. 126-135

# Design and analysis of a spherical pressure vessel using finite element method

Amir Afkar[1,2], Majid Nouri Camari[1], Amin Paykani[3]*

[1] Faculty of Electrical, Mechanical and Construction Engineering, Department of Automotive Engineering, Standard Research Institute (SRI), Karaj P. O. Box 31745-139, Iran
[2] School of Automotive Engineering, Iran University of Science and Technology, Tehran, Iran
[3] Department of Mechanical Engineering, Parand Branch, Islamic Azad University, Parand, Iran

(*Received November 18 2012, Accepted November 7 2013*)

**Abstract.** This paper studies finite element analysis of a spherical pressure vessel under simultaneous thermal and pressure loadings in transient state along with numeric analytical method. A new numerical-analytical method for calculating the transient stress and displacement is introduced. In this study, the design and analysis of spherical pressure vessel are performed using finite element commercial code ANSYS. The Von-Mises yield criterion has been used to determine the distribution of stress intensity. The obtained FEM results are compared with analytical ones and a good agreement between them is noticed.

**Keywords:** spherical pressure vessel, finite element method, stress, temperature, pressure, Von-mises stress

## 1 Introduction

Pressure vessels have been in wide use for many years in chemical, petroleum, military industries as well as in nuclear power plants. They are usually subjected to high pressures and temperatures which may be constant or cycling. Factors such as vessel material, the shape, chemical composition and physical substances used in it, the environment of vessels and etc. all are factors which each can have different effects on performance of pressure vessels. The fluid being stored may undergo a change of state inside the pressure vessel as in case of steam boilers or it may combine with other reagents as in a chemical plant. The pressure vessels are designed with great care because rupture of pressure vessels means an explosion which may cause loss of life and property. The material of pressure vessels may be brittle such that cast iron or ductile such as mild steel. Cylindrical or spherical pressure vessels (e.g., hydraulic cylinders, gun barrels, pipes, boilers and tanks) are commonly used in industry to carry both liquids and gases under pressure. When the pressure vessel is exposed to this pressure, the material comprising the vessel is subjected to pressure loading, and hence stresses, from all directions. The normal stresses resulting from this pressure are functions of the radius of the element under consideration, the shape of the pressure vessel as well as the applied pressure[7].

The basic requirements for design of storage vessels are safety, reliability and economy. However, the pressure vessels may work under the high-pressure and high-temperature environment. Two types of analyses are commonly applied to pressure vessels. The most common method is based on a simple mechanics approach and is applicable to thin-walled pressure vessels which by definition have a ratio of inner radius, r, to wall thickness, t, of r/t  10. The second method is based on elasticity solution and is always applicable regardless of the r/t ratio and can be referred to as the solution for thick-walled pressure vessels. Finite Element Analysis (FEA) is a practical tool in the study of pressure vessels, especially in determining stresses in local areas such as cavities, O-ring grooves and other areas difficult to analyze manually. In the literature, numerous research works have been carried out on design and analysis of pressure vessels. Heckman[2] studied the application of different finite element methods in pressure vessel analysis. He tested three dimensional, symmetric

* Corresponding author. *E-mail address*: a.paykani@gmail.com.

and axi-symmetric models, and concluded that finite element analysis is an extremely powerful tool in analysis of pressure vessels when employed correctly. Chang et al.[1] presented application of ANSYS in stress analysis and optimization design of pressure vessels. They introduced new approach for stress analysis and optimization design of pressure vessels. Nath[4] studied stress analysis of thick-walled cylinders with variable internal pressure states using both theory (lame's formulae) and finite element method (ANSYS). Studies in the literature mostly involve analysis of temperature, stress and displacement of vessel's wall with transient boundary conditions and usually implement purely numerical methods that require complex calculations with powerful computers; on the other hand, if the boundary conditions are constant (the static condition), the temperature, stress and etc. can be calculated using analytical methods. In this paper, a new numeric-analytical method for calculating the transient stress and displacement is introduced, by which the calculation time is reduced compared to other numerical methods. For this purpose, the temperature variation with the time is calculated using finite difference method, and then by using stress and displacement equations as a function of temperature and radius of the wall, the wall static displacements and stresses are defined. In order to validate the results, those variables are also calculated by numerical "finite difference" method and MATLAB commercial software. Furthermore, finite element analysis of a thin-walled pressure vessel under simultaneous thermal and pressure loading is investigated using simulation-based method by FE-based computer code ANSYS. The Von-Mises yield criterion has been used to determine the distribution of stress intensity. The results are obtained and compared by both methods and a good agreement between them is noticed.

## 2 Assumptions and boundary conditions

In the present study, a spherical pressure vessel is taken into account. The inner radius and wall thickness of the vessel are assumed as 0.19 m and 10 mm, respectively. In the case of simultaneous thermal and pressure loadings, at $t = 0$, temperature of the inner wall is increased from 300 K to 500 K within 5 seconds. This temperature increase is linear with time as illustrated in Eq. (1). While the outer wall of the vessel at $t \geq 0$, exposing to convection with the environment, the environment temperature is constant at 300 K. In five seconds, the pressure inside the vessel is increased from 0 Mpa to 1 Mpa. It is also assumed to increase linearly with time.

$$T_{in} = 40t + 300, \tag{1}$$

$$p = 0.2t. \tag{2}$$

## 3 Finite element analysis of thin-walled pressure vessel

### 3.1 Thermal loading

#### 3.1.1 Finite difference method to obtain the temperature distribution with time

In the present study, we directly implemented finite element method to determine the approximate temperature distribution. For this purpose, we divided the wall into cylindrical elements and considered nodes between the elements. Then, the energy exchange equation is used for each element as shown in Fig. 1. The black and red numbers represent node number and element number, respectively. Firstly, nodes 1 to 7 are selected to write the energy exchange equation. If we call these nodes as m, according to Fig. 1 the energy exchange equation for the nodes will be as following:

The $\delta V$ on each node of distance of $R_a$ from center is defined as:

$$\Delta V = 4\pi \left(R_a + m\Delta r\right)^2 \Delta r, \tag{3}$$

$$-kA\frac{\Delta T}{\Delta r} + kA\frac{\Delta T}{\Delta r} = \Delta V\frac{\Delta T}{\Delta t}. \tag{4}$$

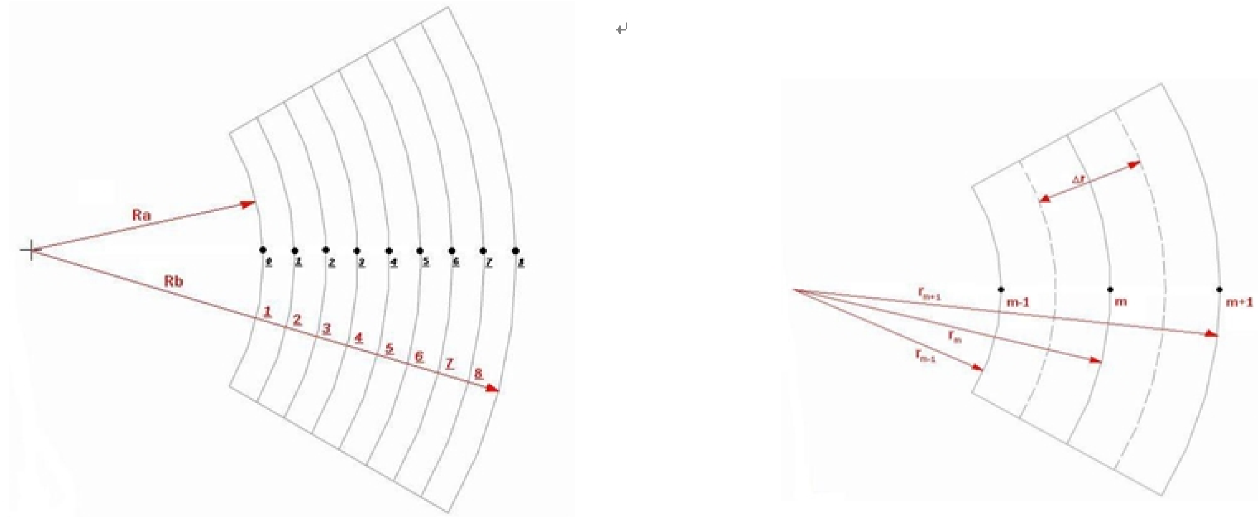


**Fig. 1.** Configuration of the FVM study

$$\frac{4\pi k}{\Delta r}\left(R_a+m\Delta r-\frac{\Delta r}{2}\right)^2\left[T_{m-1}^i-T_m^i\right]+\frac{4\pi k}{\Delta r}\left(R_a+m\Delta r+\frac{\Delta r}{2}\right)^2\left[T_{m+1}^i-T_m^i\right]=$$
$$\frac{4\pi}{\Delta t}\left(R_a+m\Delta r\right)^2\Delta r\rho c_p\left[T_m^{i+1}-T_m^i\right]. \tag{5}$$

The temperature of node $m$ at $T_m^{i+1}$ time is obtained according to other terms:

$$\begin{aligned} T_m^{i+1} &= \frac{\frac{4\pi k}{\Delta r}\left(R_a+m\Delta r-\frac{\Delta r}{2}\right)^2}{\frac{4\pi}{\Delta t}\left(R_a+m\Delta r\right)^2\Delta r\rho c_p}T_{m-1}^i+\frac{\frac{4\pi k}{\Delta r}\left(R_a+m\Delta r+\frac{\Delta r}{2}\right)^2}{\frac{4\pi}{\Delta t}\left(R_a+m\Delta r\right)^2\Delta r\rho c_p}T_{m+1}^i \\ &+\frac{\frac{4\pi(R_a+m\Delta r)^2\Delta r\rho c_p}{\Delta t}-\frac{4\pi k}{\Delta r}\left(R_a+m\Delta r-\frac{\Delta r}{2}\right)^2-\frac{4\pi k}{\Delta r}\left(R_a+m\Delta r+\frac{\Delta r}{2}\right)^2}{\frac{4\pi(R_a+m\Delta r)^2\Delta r\rho c_p}{\Delta t}}T_m^i. \end{aligned} \tag{6}$$

The above equation is used for nodes 1 to 7, whereas the convection boundary condition should be applied at the outer wall. For node 8 the heat exchange equation will be as following[5]:

$$4\pi R_b^2h_0\left(T_\infty-T_8^i\right)+4\pi k\left(R_b-\frac{\Delta r}{2}\right)^2\frac{\partial T}{\partial r}=4\pi R_b^2\frac{\Delta r}{2}\rho c_p\frac{\partial T}{\partial t}, \tag{7}$$

$$h_0R_b^2\left(T_\infty-T_8^i\right)+k\left(R_b-\frac{\Delta r}{2}\right)^2\frac{T_7^i-T_8^i}{\Delta r}=R_b^2\frac{\Delta r}{2}\rho c_p\frac{T_8^{i+1}-T_8^i}{\Delta t}. \tag{8}$$

The value of $T_8^{i+1}$ is obtained based on other terms:

$$T_8^{i+1}=\frac{h_0R_b^2}{R_b^2\frac{\Delta r}{2\Delta t}\rho c_p}T_\infty+\frac{\frac{k}{\Delta r}\left(R_b-\frac{\Delta r}{2}\right)^2}{R_b^2\frac{\Delta r}{2\Delta t}\rho c_p}T_7^i+\frac{R_b^2\frac{\Delta r}{2\Delta t}\rho c_p-h_0R_b^2-\frac{k}{\Delta r}\left(R_b-\frac{\Delta r}{2}\right)^2}{R_b^2\frac{\Delta r}{2\Delta t}\rho c_p}T_8^i. \tag{9}$$

Where $\Delta r=0.00125m$, $R_a=0.19m$, $R_b=0.2m$, $h_0=90\frac{W}{m^2K}$, $T_\infty=300K$, $\Delta t=0.05\,\text{sec}$.
With the initial condition of uniform temperature of 300 K, nine equations will be listed as following:

$$\begin{cases} T_0^{i+1} = T_0^i + 2 \\ T_1^{i+1} = 0.303858459T_0^i + 0.307856555T_2^i + 0.388284985T_1^i \\ T_2^{i+1} = 0.303871339T_1^i + 0.307843532T_3^i + 0.388285065T_2^i \\ T_3^{i+1} = 0.303884417T_2^i + 0.307830677T_4^i + 0.388285153T_3^i \\ T_4^{i+1} = 0.303896779T_3^i + 0.307817987T_5^i + 0.388285233T_4^i \\ T_5^{i+1} = 0.303909227T_4^i + 0.307805459T_6^i + 0.388285314T_5^i \\ T_6^{i+1} = 0.303921517T_5^i + 0.307793309T_7^i + 0.388285393T_6^i \\ T_7^{i+1} = 0.303933654T_6^i + 0.307780877T_8^i + 0.388285469T_7^i \\ T_8^{i+1} = 0.607891278T_7^i + 0.390197132T_8^i + 0.573476702 \end{cases} \tag{10}$$

### 3.1.2 Calculation of stress distribution

The Von-mises yield criterion is mainly used in the engineering analysis to study stress distribution and stress concentration. It is obtained through the absolute difference between the radial and tangential stresses[3]:

$$\text{Vonmises} = |\sigma_r - \sigma_\theta| = \frac{2\alpha E}{1-\upsilon}\left[\frac{-3a^3}{(b^3-a^3)\,r^3}\int_a^b Tr^2dr - \frac{1}{2r^3}\int_a^r Tr^2dr - \frac{T}{2}\right]. \tag{11}$$

If we write the above equation for nodes at time $i+1$:

$$\text{Vonmises}_m^{i+1} = \frac{2\alpha E}{1-\upsilon}\left[\frac{-3a^3}{(b^3-a^3)\,r_m^3}\int_a^b T_m^{i+1}r_m^2dr - \frac{1}{2r_m^3}\int_a^r T_m^{i+1}r_m^2dr - \frac{T_m^{i+1}}{2}\right]. \tag{12}$$

Since we have temperature distribution only for nodes, therefore we cannot get the integrals exactly and the integral must get approximately by the following method:

$$\int_a^b T_m^{i+1}r_m^2dr$$
$$=0.00125\left[\begin{array}{l} 0.190625^2\left(\frac{T_0^{i+1}+T_1^{i+1}}{2}\right) + 0.191875^2\left(\frac{T_1^{i+1}+T_2^{i+1}}{2}\right) + 0.193125^2\left(\frac{T_2^{i+1}+T_3^{i1}}{2}\right) + \\ 0.194375^2\left(\frac{T_3^{i+1}+T_4^{i+1}}{2}\right) + 0.195625^2\left(\frac{T_4^{i+1}+T_5^{i+1}}{2}\right) + 0.196875^2\left(\frac{T_5^{i+1}+T_6^{i+1}}{2}\right) + \\ 0.198125^2\left(\frac{T_6^{i+1}+T_7^{i+1}}{2}\right) + 0.199375^2\left(\frac{T_7^{i+1}+T_8^{i+1}}{2}\right) \end{array}\right]. \tag{13}$$

$$\int_a^r T_m^{i+1}r_m^2dr = \sum_{m=1}^{m}\left(\frac{T_m^{i+1}+T_{m-1}^{i+1}}{2}\right)\left(\frac{r_m+r_{m-1}}{2}\right)^2. \tag{14}$$

The amount of the displacement of outer wall of the vessel can be defined from the radial and tangential stresses along with static equilibrium equation of the vessel wall:

$$u = \left(\frac{1+\upsilon}{1-\upsilon}\right)\frac{\alpha}{r^2}\int_r^a Tr^2dr + \left(\frac{2\alpha}{1-\upsilon}\left(\frac{1-2\upsilon}{b^3-a^3}\right)\int_a^b Tr^2dr\right)r + \left(\frac{\alpha\,(1+\upsilon)\,a^3}{(1-\upsilon)\,(b^3-a^3)}\int_a^b Tr^2dr\right)\frac{1}{r^2}. \tag{15}$$

If we write the above equation for elements we have:

$$u_m^{i+1} = \left(\frac{1+\upsilon}{1-\upsilon}\right)\frac{\alpha}{r_m^2}\int_a^r T_m^{i+1}r_m^2dr + \left(\frac{2\alpha}{1-\upsilon}\left(\frac{1-2\upsilon}{b^3-a^3}\right)\int_a^b T_m^{i+1}r_m^2dr\right)r_m + \left(\frac{\alpha\,(1+\upsilon)\,a^3}{(1-\upsilon)\,(b^3-a^3)}\int_a^b T_m^{i+1}r_m^2dr\right)\frac{1}{r_m^2}. \tag{16}$$

It is obvious that the radial displacement is maximum on outer wall, so the displacement at outer wall is calculated. By substituting numerical values for parameters, the following correlation is obtained for radial displacement of outer wall:

$$u_8 = 5.100788782 \times 10^{-3} \times \int_a^b Tr^2 dr. \tag{17}$$

It is written in finite difference form as:

$$u_8^{i+1} = 5.100788782 \times 10^{-3} \times \int_a^b T_m^{i+1} r_m^2 dr_m. \tag{18}$$

Finally, the values of temperatures and stresses at nodes are obtained and the diagrams based on time are plotted.

## 3.2 Pressure loading

### 3.2.1 Calculation of stress and displacement distribution

The Von-mises stress is expressed as difference between radial and tangential stresses[6].

$$\text{Vonmises} = |\sigma_r - \sigma_\theta| = \left| \frac{Pa^3\left(2r^3 + b^3\right)}{2r^3\left(b^3 - a^3\right)} - \frac{Pa^3\left(b^3 - r^3\right)}{r^3\left(a^3 - b^3\right)} \right| = \left| \frac{3}{2} \frac{Pa^3}{\left(a^3 - b^3\right)} \left(\frac{b}{r}\right)^3 \right|. \tag{19}$$

It is clear that the Von-mises stress is inversely proportional to radius and directly proportional to pressure. Therefore, the value of it at inner and outer nodes is studied to obtain minimum and maximum values as following:

$$\text{vonmises}_8 = \left| \frac{3}{2} \frac{P\,(0.19)^3}{\left(0.19^3 - 0.2^3\right)} \left(\frac{0.2}{0.2}\right)^3 \right| = 9.017090272P, \tag{20}$$

$$\text{vonmises}_0 = \left| \frac{3}{2} \frac{P\,(0.19)^3}{\left(0.19^3 - 0.2^3\right)} \left(\frac{0.2}{0.19}\right)^3 \right| = 10.51709027P. \tag{21}$$

Hence, the value of displacement is maximum at outer wall compared to inner wall. Because, firstly, pressure inside of vessel is moving outwards and this displacement is along the radius and secondly, displacement in the outer wall equals to the relative displacement of the outer wall to each node plus displacement of the presumed node: $u_8 = u_m + u_{8/m} \Rightarrow u_8 > u_m$.

Also, the displacement can be extracted from the equations of stress-strain and displacement[6]:

$$u = re_\theta = \left[\frac{Pa^3}{\left(b^3 - a^3\right)E}\right](1 - 2\upsilon)\, r + \left[\frac{Pa^3}{\left(b^3 - a^3\right)E}\right](1 + \upsilon)\frac{b^3}{2r^2}. \tag{22}$$

Its value at outer wall will be:

$$\begin{aligned} u_8 &= \left[\frac{Pa^3}{\left(b^3 - a^3\right)E}\right](1 - 2\upsilon)\, b + \left[\frac{Pa^3}{\left(b^3 - a^3\right)E}\right](1 + \upsilon)\frac{b^3}{2b^2} \\ &= \left[\frac{Pa^3}{\left(b^3 - a^3\right)E}\right]\frac{3}{2}(1 - \upsilon)\, b = 6.596713409 \times 10^{-12} P. \end{aligned} \tag{23}$$

## 3.3 Simultaneous thermal and pressure loadings

Calculation of displacement and stresses in a vessel under simultaneous thermal and pressure loading is easily possible by using the principle of superposition. It can be done through summation of thermal and pressure loading effects.

## 4 Finite element analysis of thin-walled pressure vessel using ansys

The ANSYS CAE (Computer-Aided Engineering) software program was used in conjunction with 3D CAD (Computer-Aided Design) solid geometry to simulate the behavior of pressure vessel under thermal and pressure loading conditions. The schematic of the vessel is depicted in Fig. 2. Since the shape of vessel and the applied forces are symmetric about the vertical axis and horizontal plane passing from center of the vessel, it is enough to model 1/4 part of the vessel. Due to the nature of the vessel, the model was meshed with 125864 nodes three-dimensional shell structural elements (ANSYS Reference SHELL63). Each node has three translational and three rotational degrees of freedom. The material is assumed to be isotropic and linear elastic. The vessel is made of stainless steel and its characteristics are presented in Tab. 1.

**Fig. 2.** Schematic of the spherical pressure vessel

**Table 1.** Characteristics of stainless steel

| | |
|---|---|
| Density | $\rho = 7750 \frac{kg}{m^3}$ |
| Young's Modulus | $190 \times 10^9 pa$ |
| Poisson's ratio | 0.305 |
| Thermal expansion | $9.7 \times 10_{-6} \frac{1}{k}$ |
| Specific heat | $486 \frac{J}{kg \cdot K}$ |

## 5 Results and discussion

Fig. 3 shows the temperature distribution at $t = 5$ sec at the sphere wall obtained by analytical and numerical methods. As can be seen in Fig. 3, the temperature is 500 K and 325 K at the inner and outer surfaces, respectively. It is obvious that there is a good agreement between both results. The comparison of Von-mises stress at $t = 5$ sec for analytical and numerical methods is depicted in Fig. 4. It is evident from Fig. 4 that its value is 248 MPa and 141 Mpa at the inner and outer surfaces, respectively. In addition, there is a good agreement between both results.

Fig. 5and Fig. 6 illustrate the variations of hoop stress and radial stress at the sphere wall at $t = 5$ sec (the time when the inner wall temperature reaches 500 K) and after removing thermal load (i.e. $t = 6$ and $t = 7$ sec) for both methods. Fig. 7 shows the cooling of the inner wall with time. After removing the thermal load, due to the convection at the outer wall, stresses are rapidly reduced. Fig. 8 demonstrates the comparison for sphere deformation under thermal and pressure loadings at the sphere wall. The results obtained indicate good agreement between analytical and numerical results.

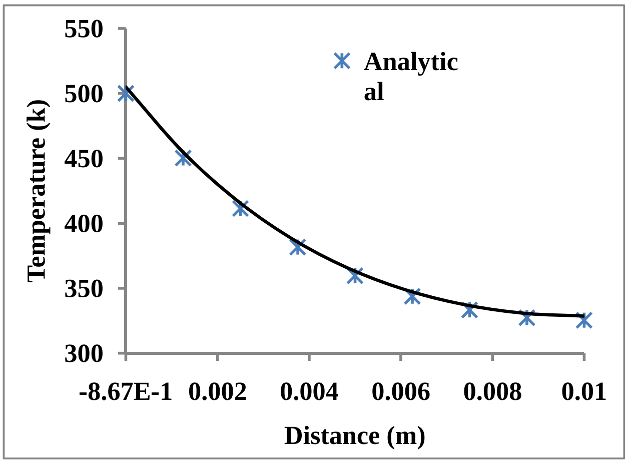


**Fig. 3.** Comparison of temperature distribution at the sphere wall for two methods at $t = 5$ sec

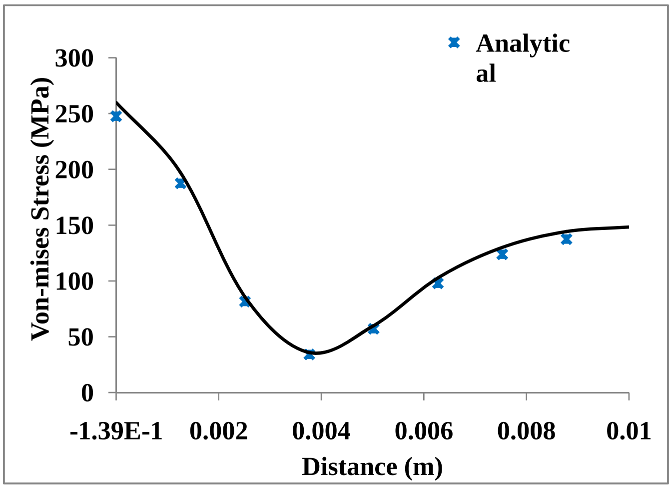


**Fig. 4.** Comparison of Von-mises stress distribution at the sphere wall for two methods at $t = 5$ sec

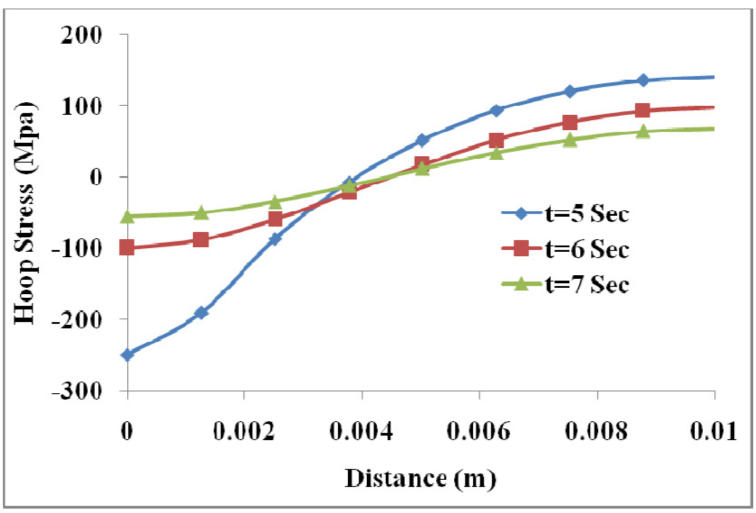


**Fig. 5.** Comparison of Hoop stress distribution at the sphere wall at $t = 5, 6, 7$ sec using analytical method

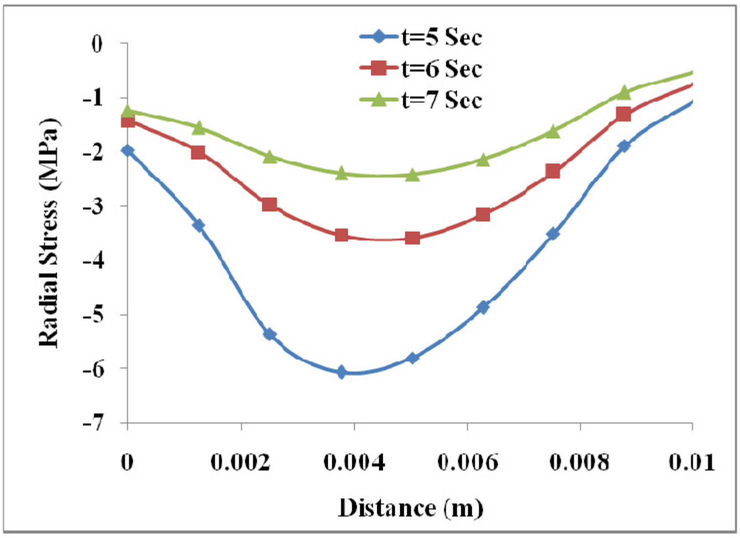


**Fig. 6.** Comparison of Radial stress distribution at the sphere wall at $t = 5, 6, 7$ sec using analytical method

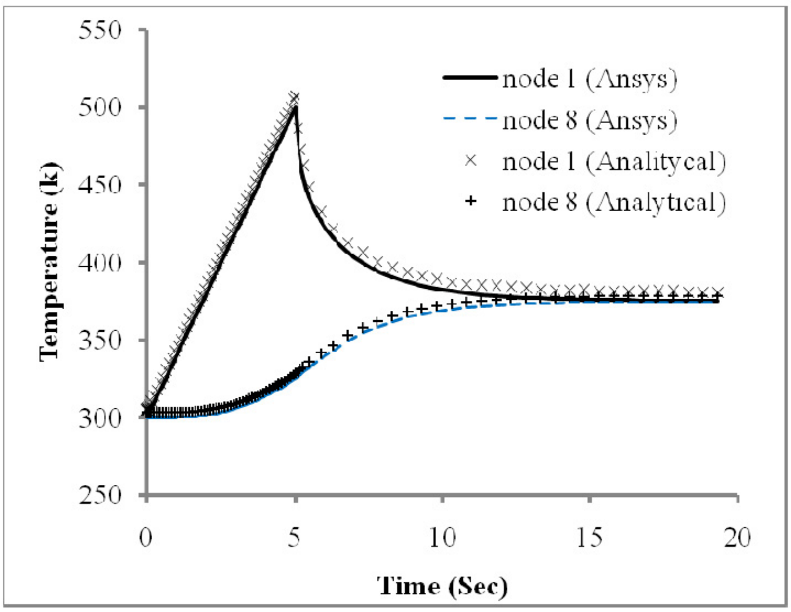


**Fig. 7.** Comparison of temperature distribution at the sphere inner wall using analytical and numerical methods

## 6 Conclusions

In this paper, finite element analysis of a pressure vessel under simultaneous thermal and pressure loading is investigated using both analytical and simulation-based methods with ANSYS package. The Von-Mises yield criterion has been used to determine the distribution of stress intensity. A new numeric-analytical method for calculation of transient temperature, stress and displacement of the vessel's wall is introduced. The variation of the wall temperature with time is calculated by numerical method to estimate the stress and displacement variations. The static equations are applied as functions of the temperature and radius of the wall. The results obtained by proposed method are compared by that of purely numerical method calculated by ANSYS and showed a good agreement.

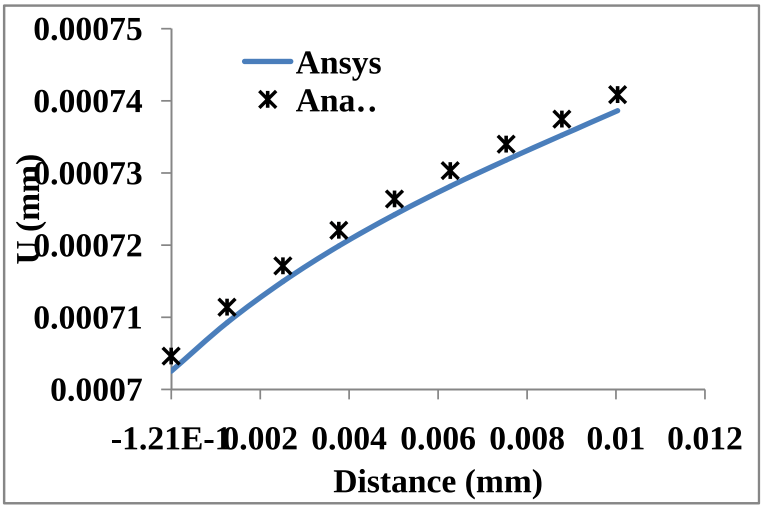


**Fig. 8.** Comparison of sphere deformation under thermal and pressure loadings at the sphere wall using analytical and numerical methods

## 7 Nomenclature

| | |
|---|---|
| $A$ | Area |
| $E$ | Young's Modulus |
| $F$ | Force |
| $P$ | Pressure |
| $T$ | Temperature |
| $V$ | Volume |
| $R_a$ | Inner radius |
| $R_b$ | Outer radius |
| $h$ | Convection heat transfer coefficient |
| $i$ | Time moment |
| $k$ | Conduction heat transfer coefficient |
| $m$ | node |
| $q$ | Heat flux per area unit |
| $r$ | radius |
| $t$ | time |
| $u$ | Radial displacement |
| $\Delta V$ | Volume element |
| $\Delta A$ | Area element |
| $\Delta t$ | Time element |
| $\Delta\theta$ | Angle element |
| $\Delta r$ | Radius element |
| $T_\infty$ | Environment temperature |
| $a$ | Coefficient of thermal expansion |
| $v$ | Poisson's ratio |
| $c_p$ | Specific heat capacity |

## References

[1] D. Chang, R. Wang. Application of ansys in stress analysis and optimization design of cryogenic pressure vessls. *Cryogenics and Superconductivity*, 2007, **6**.
[2] R. Davis, H. Keith. Finite-element analysis of pressure vessels. *Journal of Basic Engineering*, 1972, **94**(2): 401–405.
[3] W. Johnson, P. Mellor. *Engineering plasticity*. E. Horwood, 1983.

[4] R. Nath. *Design and Analysis of Thick walled cylinders with holes*. Ph.D. Thesis, 2011.
[5] M. Özisik. Heat transfer: a basic approach, 1985. *MacGraw-Hill, New York, USA*.
[6] J. Shigley, C. Mischke, R. Budynas, et al. *Mechanical engineering design*, vol. 89. McGraw-Hill New York, 1989.
[7] J. Vyas, M. Solanki. Design and analysis of pressure vessel. 2010.