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Inverse optimal control of hyperchaotic finance system*

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Abstract. This paper aims to control a new 4D hyperchaotic finance system by means of inverse optimal control scheme. For this purpose, the inverse optimal controller is designed and added to the new hyperchaotic finance system. Based on Lyapunov stability theory, the stability of the hyperchaotic finance system at its zero equilibrium point is guaranteed by applying appropriate inverse optimal control signal. Numerical simulations are demonstrated to verify the effectiveness of the proposed inverse optimal control method.

Keywords: hyperchaotic finance system, inverse optimal control, Lyapunov stability theory

1 Introduction

Chaos control contains two aspects, namely, chaos control and synchronization. Control of two chaotic systems to be synchronized has been the subject of intense research for over a decade[6, 7, 9, 11, 16, 23]. Chaos control attempts to eliminate chaotic behaviors while synchronization means to control a chaotic system (called response system) to follow another chaotic system (called drive system).

One of the main characteristics of chaotic system is the high sensitivity to initial conditions. So this system is difficult to synchronization, stability and control[20]. In the field of control engineering area, the task of chaotic nonlinear systems is one of the major problems due to the complex behavior, coupling, controlling and stability requirements. In 1989, Chaos control has attracted a great quantity of attention from various fields since Hüber published the first paper on chaos control[19], so many great efforts have been devoted towards chaos control in the past decades, especially in the stabilization of unstable equilibrium points and unstable periodic solutions[19]. Particularly, some useful methods have been developed in case of chaos suppression of any known chaotic systems, these include sliding mode control, time-delayed feedback control, double delayed feedback control, robust control, optimal control, intelligent control, etc[3, 12–15, 17–20, 27].

Finance is the dedication of costs and responsibilities over run the allocated time under conditions of certainty and uncertainty. Financial systems purpose to charge assets further to their risk level and anticipated rate of return. In the domain of finance, they have nonlinear factors which all types of increasingly sophisticated and the multiplicity and ambiguity have evinced themselves in the inward structure of the system. Also, there

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are really convoluted case and outer features in such system. Therefore, it has become significant to inquire the control of the complicated constant financial system and balance of the unsteady periodic or stationary solutions.

Chaos in a finance system was shown firstly in 2001[21, 22]. Then, a new modified chaotic finance attractor was proposed in 2007[1]. Afterwards, a hyperchaotic finance system from the modified chaotic finance system was presented in 2009[5]. Recently, Yu, Cai and Li[26] have introduced a new 4D chaotic finance system and achieved its control with the linear feedback and speed feedback controllers[26]. Control of chaos in chaotic finance systems was implemented with several methods. Linear feedback, speed feedback, selection of gain matrix, revision of gain matrix controllers[25], time-delayed feedback controllers[4], and a passive controller[8] were used for the control of chaotic finance system. The control of modified chaotic finance system was applied by means of linear feedback, speed feedback and adaptive control methods[2]. The control of the former hyperchaotic finance system was realized with the speed feedback[5], linear feedback[24], and time-delayed feedback controllers[10]. The control of the latter hyperchaotic finance system was achieved with the speed feedback and linear feedback controllers[26].

To the knowledge of the authors, no study on the control of a hyperchaotic finance system has been carried out with the inverse optimal control approach. In this paper, further investigation on the control of the new hyperchaotic finance system is explored. In Section 2, the related chaotic and hyperchaotic finance systems are described. In Section 3, the inverse optimal controller have been employed for achieving the control of the new hyperchaotic finance system. In Section 4, numerical simulations have been presented in figures to confirm the effectiveness of the inverse optimal control method. Finally, conclusions are given in Section 5.

2 Hyperchaotic finance system

The 3D chaotic finance system is described as in the following equations:

$$\begin{cases} \dot{x}_1 = x_3 + (x_2 - a)x_1 \\ \dot{x}_2 = 1 - bx_2 - x_1^2 \\ \dot{x}_3 = -x_1 - cx_3, \end{cases} \tag{1}$$

where $x = [x_1, x_2, x_3]^T$ is state variables and a, b, c are positive real constants, they represent the interest rate, investment demand, price exponent, saving, per investment cost, and elasticity of demands of commercials, respectively [21, 22].

When the parameter values are chosen as follows: $a = \frac{9}{10}, b = \frac{1}{5}, c = \frac{6}{5}$ the chaotic finance system exhibits chaotic behaviour[25]. The 3D phase plane of chaotic finance system under the initial conditions $x(0) = [2, 1, -1]^T$ is demonstrated in Fig. 1.

The new hyperchaotic finance system [26] is defined by a set of four-order differential equations as follows:

$$\begin{cases} \dot{x}_1 = x_3 + (x_2 - a)x_1 + x_4 \\ \dot{x}_2 = 1 - bx_2 - x_1^2 \\ \dot{x}_3 = -x_1 - cx_3 \\ \dot{x}_4 = -dx_1x_2 - ex_4, \end{cases} \tag{2}$$

if the the parameters a, b, c, d, e meet

$$\frac{eb + dc + abce - ce}{c(d - e)} > 0,$$

the hyperchaotic finance system (2) has three equilibrium points:

$$p_1 = \left(0, \frac{1}{b}, 0, 0\right), p_{2,3} = \left(\pm\beta, \frac{e + ace}{c(e - d)}, \mp\frac{\beta}{c}, \frac{d\beta(1 + ac)}{cd - ce}\right),$$

where

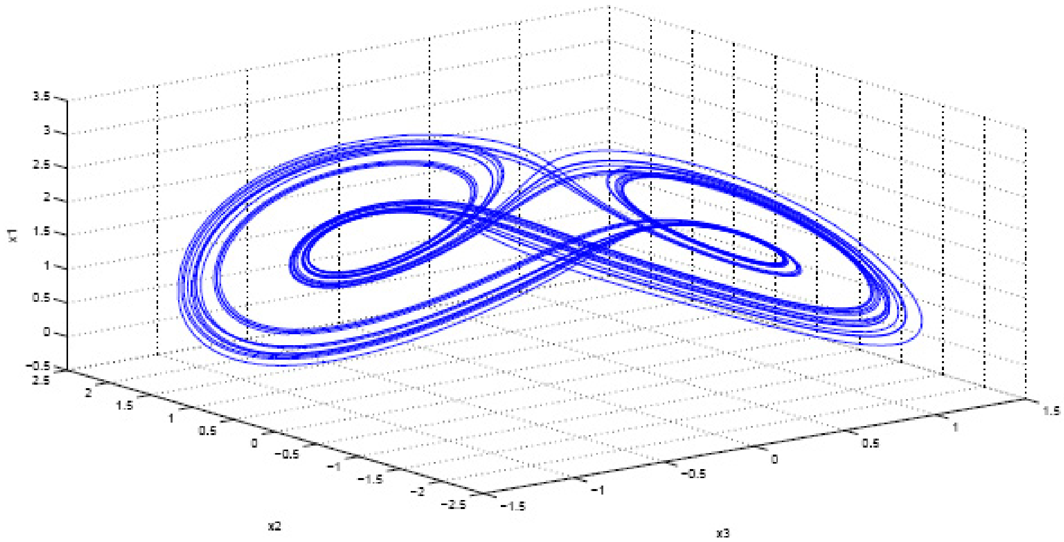

Fig. 1. 3D phase plane of chaotic finance system (1)

$$\beta = \sqrt{\frac{eb + abce}{c(d-e)} + 1}.$$

when the parameter values $a = \frac{9}{10}, b = \frac{1}{5}, c = \frac{3}{2}, d = \frac{1}{5}, e = \frac{17}{100}$ and the initial conditions $x = [1, 2, -1, -2]^T$ it shows chaotic behaviour. The time series of the hyperchaotic finance system are shown in Fig. 2, the phase planes of the hyperchaotic finance system are shown in Fig. 3.

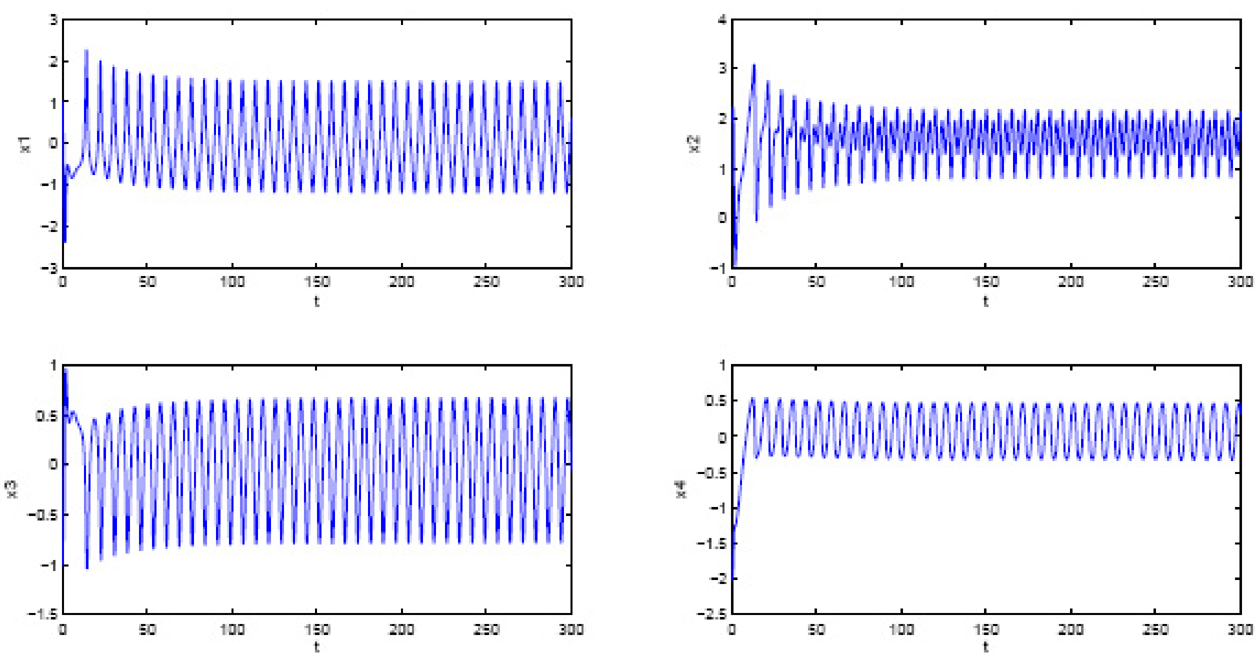

Fig. 2. Time series of hyperchaotic finance system (2)

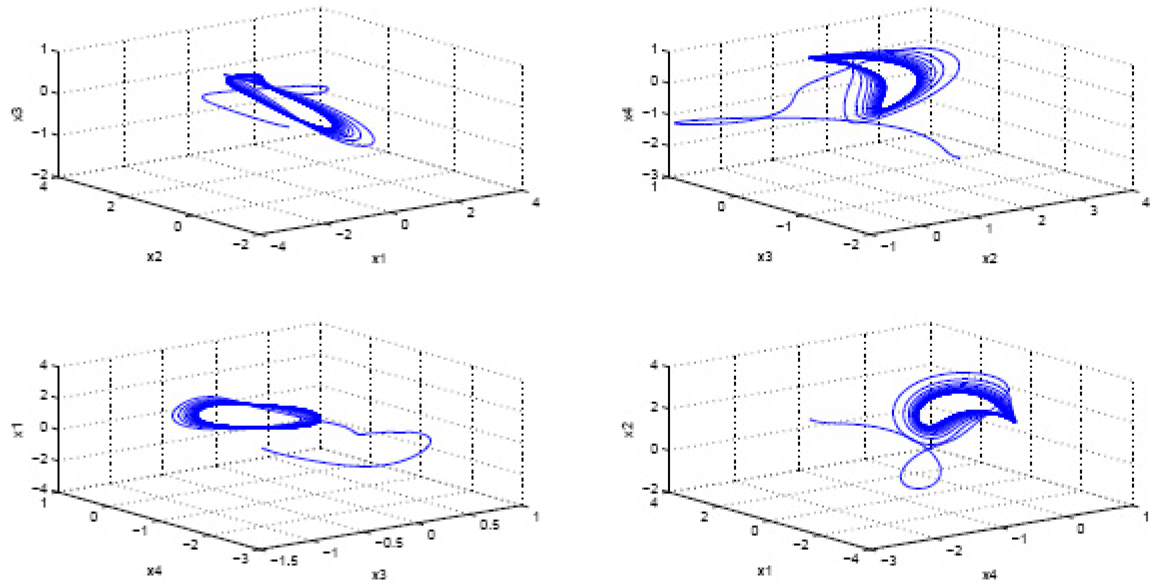

Fig. 3. Phase planes of hyperchaotic finance system (2)

3 Main result

In this section, the inverse optimal control method is applied for the control of the new hyperchaotic finance system (2). We suppose that $\tilde{x} \in D$ is an equilibrium point of the controlled hyperchaotic system (2), and D is the domain of it, that is, $f(\tilde{x}) = 0$.

In order to control the hyperchaotic finance system (2) to zero, the inverse optimal controllers of u_1, u_2 and u_3 are added to the hyperchaotic finance system (2). Then the controlled hyperchaotic system is given by

$$\begin{cases} \dot{x}_1 = x_3 + (x_2 - a)x_1 + x_4 + u_1 \\ \dot{x}_2 = 1 - bx_2 - x_1^2 + u_2 \\ \dot{x}_3 = -x_1 - cx_3 \\ \dot{x}_4 = -dx_1x_2 - ex_4 + u_3, \end{cases} \tag{3}$$

where u_1, u_2 and u_3 are the control inputs. We have the following theorem.

Theorem 1. *If the non-linear state feedback controllers are designed as*

$$\begin{cases} u_1 = -x_4 \\ u_2 = -1 - q(x) \\ u_3 = dx_1x_2, \end{cases} \tag{4}$$

where $q(x) = -kx_2, k > 0$ *is a constant to be determined later. Then the zero solution of the controlled hyperchaotic finance system (3) is globally asymptotically stable.*

Proof: Construct a Lyapunov function as

$$V = \frac{1}{2}(x_1^2 + x_2^2 + x_3^2 + x_4^2). \tag{5}$$

Then, the time derivative of Lyapunov function (5) along the trajectory of the controlled hyperchaotic system (3) is given by

$$\begin{aligned}\frac{dV}{dt} =& x_1(x_3 + x_1x_2 - ax_1 + x_4 + u_1) + x_2(1 - bx_2 - x_1^2 + u_2)\\ &+ x_3(-x_1 - cx_3) + x_4(-dx_1x_2 - ex_4 + u_3)\\ =& - ax_1^2 + (x_4 + u_1)x_1 - bx_2^2 + x_2(1 + u_2)\\ &- cx_3^2 - ex_4^2 + (u_3 - dx_1x_2)x_4,\end{aligned} \tag{6}$$

substituting the controller (4) from Eq. (6), yield

$$\begin{aligned}\frac{dV}{dt} =& - ax_1^2 - bx_2^2 - cx_3^2 - ex_4^2 + x_2q(x)\\ =& L_fV + (L_gV)q(x),\end{aligned} \tag{7}$$

where

$$\begin{cases} L_fV = -ax_1^2 - bx_2^2 - cx_3^2 - ex_4^2 \\ L_gV = x_2. \end{cases} \tag{8}$$

From Eq. (7), we can easily obtain $L_fV \leq 0$ and $\frac{dV}{dt} \leq 0$ if $L_gV = 0$.

Next, we design a simple linear state feedback controller $q(x)$ as follows:

$$q(x) = -\beta R^{-1}(x)(L_gV) = -kx_2. \tag{9}$$

where $\beta > 0$ is a constant, $R^{-1}(x) = \frac{k}{\beta}$, and $k > 0$ is a constant to be determined later. Substituting controller (9) into Eq. (7), we have

$$\tfrac{dV}{dt} = -ax_1^2 - (b + k)x_2^2 - cx_3^2 - ex_4^2, \tag{10}$$

which implies that

$$\tfrac{dV}{dt} < 0, \forall x \neq 0. \tag{11}$$

That is, the controller (4) can globally asymptotically stabilize the closed-loop hyperchaotic finance system (3).

Next, consider a optimal performance index as follows. Based on the idea of inverse optimal control, in order to determine the value of k, we define the following performance index:

$$\mathcal{J}(u) = \lim_{t\to\infty}\left\{2\beta V + \int_0^t [l(x(\tau)) + q(x)^T(x(\tau))R(x(\tau))q(x(\tau))]d\tau\right\}, \tag{12}$$

where

$$l(x) = -2\beta L_fV + \beta^2R^{-1}(x)(L_gV)^2 > 0. \tag{13}$$

Substituting Eq. (8) into Eq. (13), we can have

$$l(x) = 2\beta ax_1^2 + 2\beta bx_2^2 + 2\beta cx_3^2 + 2\beta ex_4^2 + \beta kx_2^2.$$

If let $k = 1$, there is $l(x) > 0$. Substituting Eq. (9) into Eq. (7), we have

$$\tfrac{dV}{dt} = L_fV - \beta R^{-1}(x)(L_gV)^2. \tag{14}$$

Multiplying -2β on both sides of (14), and nothing (13), we have

$$-2\beta\tfrac{dV}{dt} = l(x) + \beta^2R^{-1}(x)(L_gV)^2. \tag{15}$$

From Eq. (9), we have

$$q(x)^TR(x)q(x) = \beta^2R^{-1}(x)(L_gV)^2. \tag{16}$$

Substituting Eq. (16) into Eq. (15), we have

$$l(x) + q(x)^TR(x)q(x) = -2\beta\tfrac{dV}{dt}. \tag{17}$$

Substituting Eq. (17) into Eq. (12), we have

$$\mathcal{J}(u) = \lim_{t\to\infty}\left[2\beta V(x(t)) + \int_0^t (-2\beta\frac{dV}{dt})d\tau\right] = 2\beta V(x(0)). \tag{18}$$

Therefor, the controller (4) is optimal with performance index (12). Substituting $k = 1$ into the controller (4), we get the optimal controller as fellow:

$$\begin{cases} u_1 = -x_4, \\ u_2 = -1 + x_2, \\ u_3 = dx_1x_2. \end{cases} \tag{19}$$

The proof is completed.

Remark 1. We want to study and characterize the stability of $\tilde{x}$. For convenience, when the equilibrium point of hyperchaotic system (2) is at the origin of R^n, we state all theorems and definitions for the case; that is, $\tilde{x} = 0$. Because any equilibrium point can be shift to the origin via change of variables, there is no loss of generality in doing like this.

Remark 2. Suppose $\tilde{x} \neq 0$ is an equilibrium point of the hyperchaotic system (2), we consider the change of variables $y = x - \tilde{x}$. The following is the derivative of y. $\dot{y} = \dot{x} = f(x) = f(y + \tilde{x}) = s(y)$, where $s(0) = 0$. There is equilibrium at the origin in the new variable y of the system. So, we should always assume that $f(x)$ satisfies $f(0) = 0$ and study the stability of the origin $x = 0$ without loss of generality.

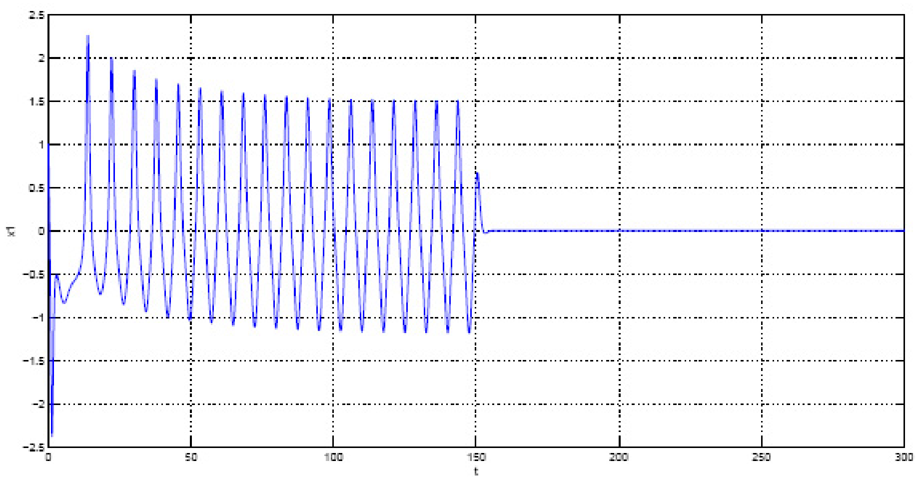

Fig. 4. State response of hyperchaotic finance system with controller (4)

4 Numerical simulations

In this section, the following illustrative example is used to demonstrate the effectiveness of the above method.

The related parameters are shown in the Section 2. The simulation time is set $300s$, the inverse optimal controller is activated in $150s$. Simulation results are shown in Fig. 4-7. As shown in Fig. 4-7, the chaotic trajectory of the hyperchaotic finance system (2) with controller (4) can be controlled rapidly to its zero point under different initial conditions, too. The above numerical simulation results show the effectiveness of the controller presented by Section 3.

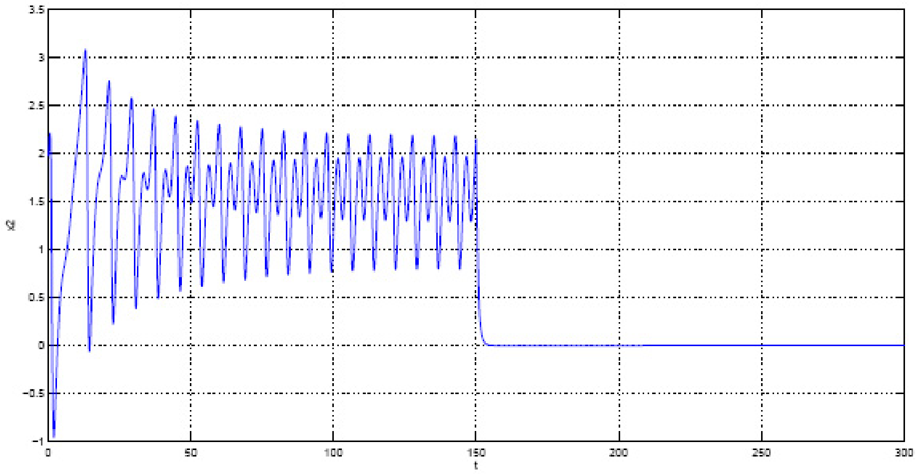

Fig. 5. State response of hyperchaotic finance system with controller (4), x_2 time series

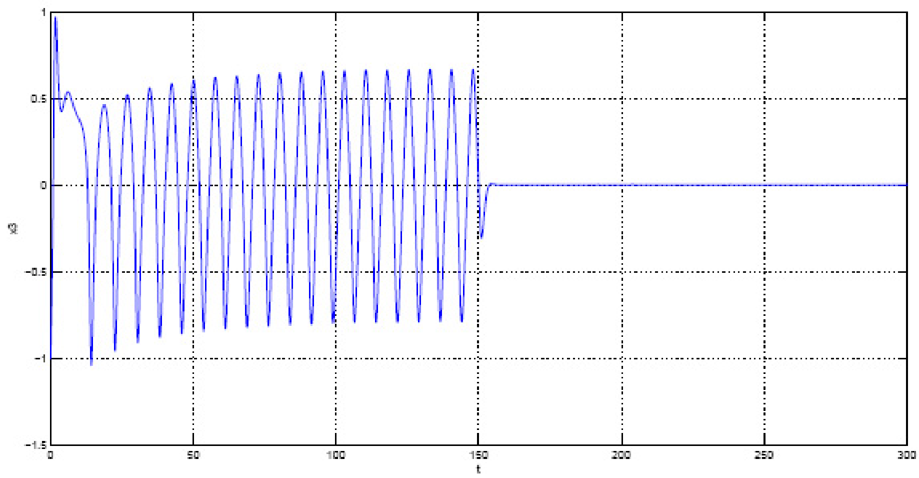

Fig. 6. State response of hyperchaotic finance system with controller (4), x_3 time series

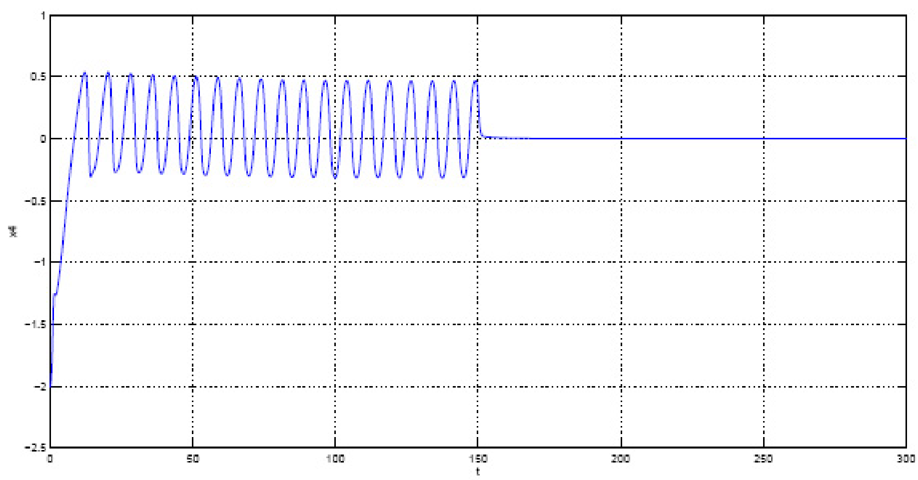

Fig. 7. State response of hyperchaotic finance system with controller (4), x_4 time series

5 Conclusions

In this paper, the stabilization problem of the hyperchaotic finance system has been investigated by inverse optimal control scheme in detail. This method can avoid the difficulty caused by solving the Hamilton-Jacobi-Bellman (HJB) equation, and hence is very efficient in practice when the controller is demanded to be optimal in a certain performance index.

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