ISSN 1 746-7233, England, UK
World Journal of Modelling and Simulation
Vol. 9 (2013) No. 3, pp. 223-234

Optimized PID tracking controller for piezoelectric hysteretic actuator model

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(*Received August 28 2012, Revised May 10 2013 Accepted June 3 2013*)

Abstract. World Smart materials such as piezoelectric actuators possess useful properties especially in the field of micropositioning. Tracking control accuracy of these systems is limited due to its inherent hysteresis nonlinearity. Designing a precise tracking controller for a piezoelectric actuator with hysteresis nonlinearity is our purpose in this paper. A Bouc-Wen model is used to describe the hysteresis behavior of the actuator. Multi-parameter-optimization techniques are used twice in this paper; first for obtaining a linear model equivalent to the overall nonlinear actuator model, and second for tuning the gains of the PID controller for suppressing the tracking errors and hence getting precision tracking. Several advantages are gained on solving parameter optimization problems using computers; neither reliance on intuition nor experience in control is required, in addition to producing a satisfactory system response with optimal use of effort and time, high convergence speed, high approximation accuracy, high efficiency, and suitability for many kinds of goal functions. Simulation results performed on the nonlinear system verify the efficiency of the proposed method.

Keywords: piezoelectric actuator, hysteresis, Bouc-Wen model, parameter optimization, parameter estimation

1 Introduction

Piezoelectricity is a fundamental process in electromechanical energy conversion. It relates electric polarization to mechanical stress/strain in piezoelectric materials. Piezoelectric materials when subjected to an electric field they produce mechanical strain and alternately when subjected to a mechanical strain they generate an electric charge. This property gives piezoelectric materials the ability to act as actuators or sensors.

Piezoelectric actuators have many advantages; in addition to their fast response time, possibility of motion in sub-nanometer, and bigger driving force, they have no moving parts in contact to each other to limit the resolution and consequently no wear and tear which causes a decrease in life time and precision. Due to these advantages, piezoelectric actuators have been widely used in many electromechanical applications such as in active vibration control, ultrasonic motors, micro-positioners, micro-sensors, etc.

For the purpose of designing a precise tracking controller in a piezoelectric actuator system, an appropriate model describing the behavior of the piezoelectric actuator is required. One of the critical fields in the study of piezoelectric actuator modeling is the hysteresis phenomenon, which occurs via applied voltage and induced displacement.

The hysteresis is not a differentiable and nor a one-to-one nonlinear mapping. But it is a nonlinear operator with local memory and, as a result, the hysteresis exhibited at a given time instant depends on not only the input at the present time but also the operational history of the system considered.

Fundamental study of piezoelectric actuator depicts that the hysteresis effect deteriorate the tracking performance of the piezoelectric actuator. It severely limits systems' performance such as increasing the time of feedback control compensation, giving rise to undesirable accuracy or oscillations, even leading to instability.

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Many models have been proposed to capture the hysteretic characteristics for analyzing the hysteresis behavior of piezoelectric systems. These hysteretic models can be classified into two categories; physics-based models and phenomenology-based models. In physics-based models, the basic processes involved are simulated in order to be able to describe the basic magnetizing modes. Examples of physics-based models are Jiles-Atherton model and homogenized energy model. In phenomenology-based models, the gross behavior of the material is described mathematically by generating curves, following predefined rules, for the material properties. Among these models; Preisach model, Prandtl-Ishlinkii model, Duhem model, and Bouc-Wen model[16].

Many strategies have been made to compensate the hysteretic error of piezoelectric actuators by using either charge control or voltage control. Charge control approach requires measurement of the induced charge and a specially designed charge drive amplifier. The need for additional electric circuits and increased complexity of control hardware make this approach costly, uncommon, and offers limited sensitivity. Also charge control may lead to drift and saturation problems, and reduces the responsiveness, the operating range and life of the actuator. Compensation of hysteretic error by voltage control is accomplished by means of two control principles: 1) the feedback voltage control which utilizes various sensors to measure the output of the actuator, and 2) the forward voltage control scheme which includes the actuator model. This model can be either a direct or an inverse model. Their usage differs by model type. The direct actuator model is used to obtain feedback for the controller, while the inverse actuator model estimates the input for the desired output to be obtained[18]. Voltage control strategies for piezoelectric actuator proved to be more promising, economical and commercially acceptable control method than those of charge control[9].

A system model may consist of a set of cross-linked sub-models that depend on a large number of parameters. Optimization of model performance involves the selection of the 'best' set of values for the parameters. Parameter optimization problems present themselves in a variety of important domains ranging from engineering to AI, from mathematics to biological sciences, in areas such as function optimization, system identification, control, machine learning, design and many others[11].

In order to solve parameter optimization problems, one has to specify a representation for the parameters of a system and then an algorithm that will perform the optimization of these parameters. Classical methods for such optimization include the Monte Carlo and the full or fractional factorial methods. More recently several new methods have emerged including the genetic algorithms, the particle swarm optimization and ant colony optimization. These methods for parameter optimization have been devised and implemented in both free and commercial software. Computational time for parameter optimization has been greatly reduced due to improvements in calculation algorithms and the advent of high performance computers. Solving parameter optimization problems using computers have several advantages including high convergence speed, high approximation accuracy, high efficiency, robustness against stochastic inaccuracies, and suitability for many kinds of goal functions.

In this paper, compensation of hysteretic error is accomplished by means of the forward voltage control scheme which includes a direct model of the actuator. Multi-parameter optimization techniques are used twice in this work; first for obtaining a linear model equivalent to the overall nonlinear actuator model, and second for tuning the gains of the PID controller for suppressing the tracking errors and hence getting precision tracking.

Traditionally, ad-hoc methods have been used to choose PID parameters that can serve as acceptable starting points for the tuning optimization process. These classical methods, while valuable in terms of providing some insight into the process of control design, can often be surpassed in effectiveness by more modern methods. In this paper, for obtaining the initial guess for the PID parameters, an automated controller design method packaged in graphical software is used. This decreases the time and effort for tuning parameters considerably and proves that the day of ad-hoc methods for tuning PID controllers is ending.

The paper is organized as follows: in Section 2, the hysteresis behavior of piezoelectric actuator is described using Bouc-Wen model. This hysteresis model is integrated into the actuator mechanical motion dynamics to completely represent the overall dynamics of a piezoelectric actuator with a well-known hysteresis. In Section 3, a brief mathematical background behind the used multi-parameter optimization techniques; the nonlinear least squares algorithm and the gradient-descent parameter optimization technique, along with

Simulink Parameter Estimation and Simulink Response Optimization software will be given. Section 4 is devoted to the linearization of the nonlinear hysteretic dynamics using nonlinear least squares parameter estimation technique. Simulation result is also given to show the matching between the nonlinear and linearized models. Then gradient descent based optimization is adopted in Section 5 to the design of a precise tracking controller. The parameters chosen for optimization are the gains of the PID controller. The simulation results of this section prove that using Simulink multi-parameter-optimization technique makes achieving precision tracking an intuitive and easy process. Finally, this paper concludes with a brief summary in Section 6.

2 Modeling of piezoelectric hysteretic actuator

For the purpose of designing a precise tracking controller for a piezoelectric actuator system, an appropriate model describing the behavior of the piezoelectric actuator is required. Consider a piezoelectric actuator subject to a hysteresis nonlinearities described by Bouc-Wen model. It can be identified as a second-order linear model preceded by hysteretic nonlinearity as follows[19]:

$$m\ddot{x} + b\dot{x} + kx = k(du - h), \tag{1}$$

$$\dot{h} = \alpha d\dot{u} - \beta|\dot{u}|h - \gamma\dot{u}|h|, \tag{2}$$

where u is the input voltage to the piezoelectric actuator, x denotes the displacement of the piezoelectric actuator, m, b, k, d denote the effective mass, damping, mechanical stiffness and effective piezoelectric coefficients, respectively. h indicates the variable obtained from the hysteretic nonlinear dynamics. α, β and γ are parameters of the hysteretic loop's magnitude and shape. For the actuator system considered in this paper, the parameters values for the above coefficients are adopted from [10] and are given in Tab. 1. The control input signal was limited to 150 V due to the physical limitation of the piezoelectric materials. The Simulink block diagram for the Bouc-Wen model, Eq. (2), is shown in Fig. 1. The complete representation of the overall dynamics of the piezoelectric actuator is modeled in Fig. 2 by appending Eq. (2) to the linear model of the actuator, Eq. (1).

Table 1. The parameters values for the underlying actuator

Parameter	Value	Unit
m	5	kg
b	20144	N s/m
k	5×10^6	N/m
d	1.1452×10^{-7}	m/V
α	0.365	—
β	0.0485	—
γ	-0.0221	—

Fig. 3 shows the Bouc-Wen model responses to sinusoidal waveform test signals of frequency 15Hz associated with various amplitudes. Fig. 4 shows the ellipsoid-shaped trajectory for the overall nonlinear actuator model with an applied voltage of $85\sin(30\pi t)V$.

3 Parameter estimation and optimization

3.1 Basic optimization concepts

A system model may consist of a set of cross-linked sub-models that depend on a large number of parameters. Improving of model performance involves the selection of the 'best' set of values for the parameters. The approach that aims at finding the best solution to a problem among the existing available alternatives involves optimization. Optimization is a process which simply searches for any existing feasible and optimum

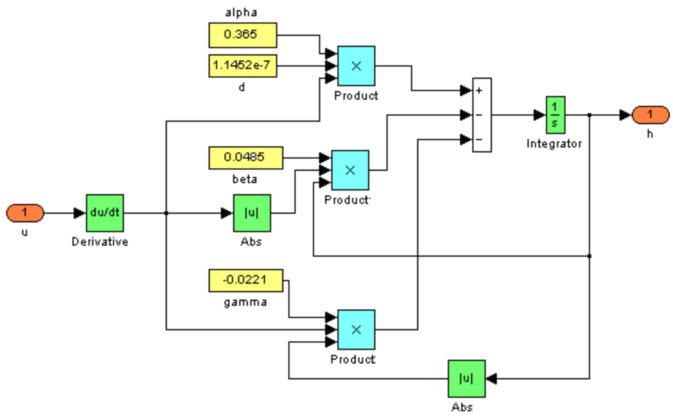
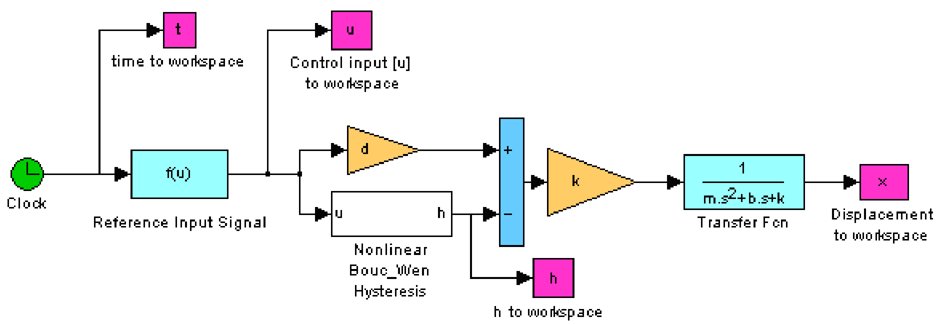


Fig. 1. Simulink block diagram for nonlinear Bouc-Wen model



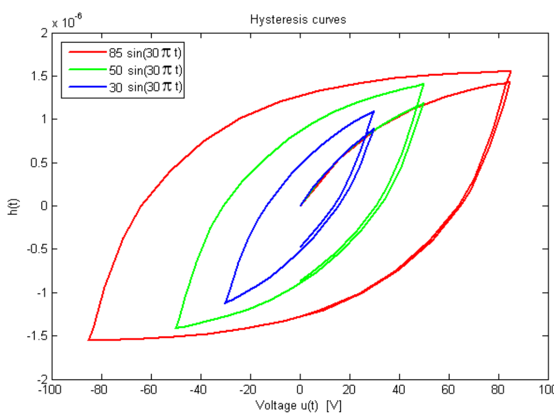


Fig. 3. Simulated trajectory results of Bouc-Wen model with different input amplitudes

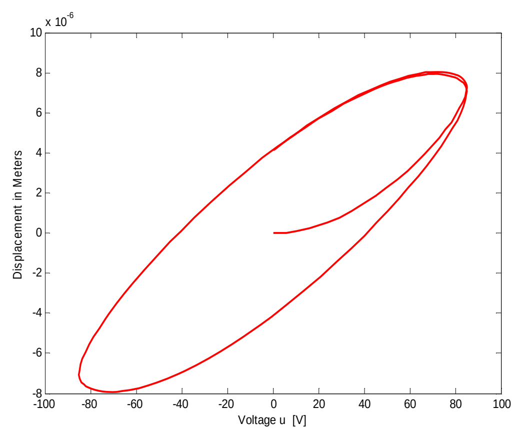


Fig. 4. Nonlinear actuator model response for $u(t) = 85\sin(30\pi t)$

solutions under specific circumstances. Here, the main goal is to minimize the objective function, which is a mathematical expression describing a relationship of the optimization parameters or the result of an operation (such as simulation) that uses the optimization parameters as inputs. The objective function, $f(z)$, to be minimized, might be subject to constraints in the form of equality constraints, inequality constraints, and parameter bounds, as follows:

$$\min f(z), \tag{3}$$

subject to

$$G_i(z) = 0 (i = 1, \ldots, m_e), \tag{4}$$

$$G_i(z) \leq 0 (i = m_e + 1, \ldots, j), \tag{5}$$

$$z_l \leq z \leq z_u, \tag{6}$$

where z is the vector of length n design parameters, and the vector function $G(z)$ returns a vector of length j containing the values of the equality and inequality constraints evaluated at z.

3.2 Nonlinear least-squares optimization method

Nonlinear least squares is the form of least squares analysis which is prevalent in a large number of practical applications, especially when fitting model functions to data, i.e., nonlinear parameter estimation, where the output, $y(z,t)$, is required to follow some continuous model trajectory, $\phi(t)$, for vector z and scalar t.

Here the function $f(z)$ to be minimized is a sum of squares as given:

$$\min_{z \in \Re^n} f(z) = \frac{1}{2}\|F(z)\|_2^2. \tag{7}$$

This problem can be expressed as

$$\min_{z \in \Re^n} \int_{t_2}^{t_1} (y(z,t) - \phi(t))^2 \mathrm{d}t, \tag{8}$$

where $y(z,t)$ and $\phi(t)$ are scalar functions.

When the integral is discretized, the above can be formulated as a least-squares problem:

$$\min_{z \in \Re^n} f(z) = \sum_{i=1}^{j} (\bar{y}(z,t_i) - \bar{\phi}(t_i))^2, \tag{9}$$

where $\bar{y}$ and $\bar{\phi}$ include the weights of the quadrature scheme.

In problems of this kind, the residual $\|F(z)\|$ is likely to be small at the optimum. Many optimization methods to ensure that some subsequence of iterations converges to an optimal solution can be used. Among them, an increasingly popular method for ensuring convergence uses trust regions called **trust-region reflective nonlinear least squares** method is used in this paper.

Trust region is a term used in mathematical optimization to denote the subset of the region of the objective function to be optimized. To understand the trust-region approach to optimization, suppose we are at a point z in n-space and we want to improve, i.e., move to a point with a lower function value. The basic idea is to approximate f with a simpler function q, which reasonably reflects the behavior of function f in a neighborhood N around the point z. This neighborhood is the trust region. A trial step s is computed by minimizing over N. This is the trust-region subproblem,

$$\min\{q(s), s \in N\}. \tag{10}$$

If an adequate model of the objective function is found within the trust region, i.e. $f(z+s) < f(z)$, then the current point is updated to be $z+s$, and the region is expanded; otherwise, the current point remains unchanged and the region of trust, N, is shrunk and the trial step computation is repeated[3, 6].

3.3 Estimate Simulink model parameters

Simulink Parameter Estimation software (SPES) provides a graphical user interface (GUI) for estimating the parameters of a Simulink model using empirical input and output data pairs. Using optimization techniques, the software estimates the parameter such that a user-selected objective function is minimized. The objective function typically calculates a least-square error between the empirical and model data signals. Among several optimization techniques offered by SPES, **trust-region reflective nonlinear least squares** method is selected for this paper.

3.4 Gradient descent optimization method

The gradient direction is defined as:

$$\nabla f(z) = \begin{Bmatrix} \frac{\partial f(z)}{\partial z_1} \\ \frac{\partial f(z)}{\partial z_2} \\ \vdots \\ \frac{\partial f(z)}{\partial z_n} \end{Bmatrix}, \tag{11}$$

where the symbol ∇ is called the gradient operator.

It is well known fact that the function value increases at the fastest rate when moving along the gradient direction. The gradient direction is called the direction of steepest ascent. The negative of the gradient vector denotes the direction of steepest descent. Thus the gradient descent method is expected to give minimum point faster than other non-gradient based optimization method[4, 17].

3.5 Tuning within Simulink models

Simulink Response Optimization Software (SROS) provides a GUI to assist in design of control and physical systems. With this product, parameters can be tuned within a nonlinear Simulink model to meet time-domain performance requirements by graphically placing constraints within a time-domain window or tracking and closely matching a reference signal. No limitations are put on the number of Simulink variables to be tuned including scalars, vectors, and matrices.

Placing some kind of constraint on a signal in the Simulink model is performed by connecting a special block; the Signal Constraint block, to that signal. SROS automatically converts time-domain constraints into a constrained optimization problem and then solves the problem using optimization techniques. The constrained optimization problem formulated iteratively simulates the Simulink model, compares the results of the simulations with the constraint objectives, and uses optimization methods to adjust tuned parameters to better meet the objectives[1].

Experimental response is commonly called the actual value. A group of actual values constitutes an actual vector. The parameter optimization objective function to be minimized is the difference between the target vector and the actual vector. Minimizing the objective function is carried out by optimizing the target function parameters so that the target function fits the experimental data. That is why parameter optimization is sometimes referred to as curve fitting or model fitting[5].

In the next sections, obtaining an approximate linear model for the nonlinear Bouc-Wen is performed using SPES with trust-region reflective nonlinear least squares method, while tuning the gains of the PID controller is performed using SROS with gradient descent optimization technique.

4 Linearization of the nonlinear hysteretic dynamics

In this section, the nonlinear Bouc-Wen hysteretic dynamics, Eq. (2), are linearized to produce a static system, which is desirable for control design. Basically there are three methods available in the literature to linearize the hysteretic type of nonlinear systems; 1) the Fokker-Planck equation approach; 2) the perturbation techniques; and 3) the stochastic linearization approach. All of them have certain advantages and limitations[8]. In this paper, the nonlinear dynamics of the actuator are linearized using SPES with trust-region reflective nonlinear least squares method. The equivalent linear equation will be looked for in the form:

$$\dot{h} = k_1\dot{u} + k_2 h, \tag{12}$$

where k_1 and k_2 are the linearization coefficients which are calculated by minimizing the difference function between the original nonlinear system and the equivalent linearized system.

$$\tilde{e} = \alpha d\dot{u} - \beta|\dot{u}|h - \gamma\dot{u}|h| - (k_1\dot{u} + k_2 h). \tag{13}$$

SPES provides a “Control and Estimation Tools Manager” GUI that makes the process of estimating the parameters quick and easy. For setting up the estimation data, first the Simulink block diagram for the overall dynamics of the piezoelectric actuator, Fig. 2, is run for a sinusoidal excitation u with frequency of 30 Hz and peak magnitude of 5V. The variables u, h, and t are saved in MATLAB workspace as mat-files to be imported as input and output data. Second, the linearized system, Eq. (12), is modeled in Simulink as shown in Fig. 5. Finally, in the Control and Estimation Tools Manager, k_1 and k_2 are selected to be the parameters for estimation and the nonlinear least squares is selected to be the optimization method.

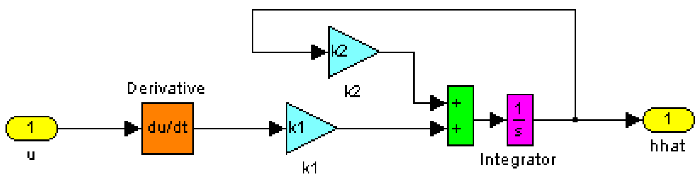


Fig. 5. Simulink block diagram for linearized model

After few iterations, the obtained linearization coefficients were $k_1 = 3.996 \times 10^{-8}$, and $k_2 = -2.008$, and as result the non-linear equation governing the internal variable h is replaced with the followed linear one:

$$\dot{\hat{h}} = k_1 \dot{u} + k_2 \hat{h} = 3.996 \times 10^{-8} \dot{u} - 2.008 \hat{h}. \tag{14}$$

Fig. 6 shows the output signals of the nonlinear hysteresis model h, its linearized model, and the difference function “residual” $\bar{e}$ which has a peak of 2×10^{-8}. For model validation, the responses of nonlinear and linearized models are plotted in Fig. 7.

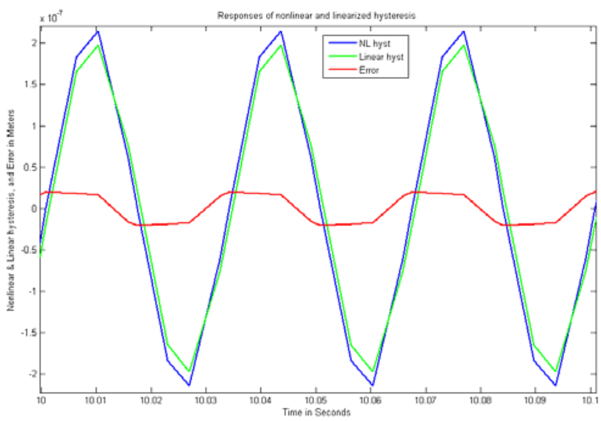


Fig. 6. Output signals of nonlinear and linearized hysteresis models

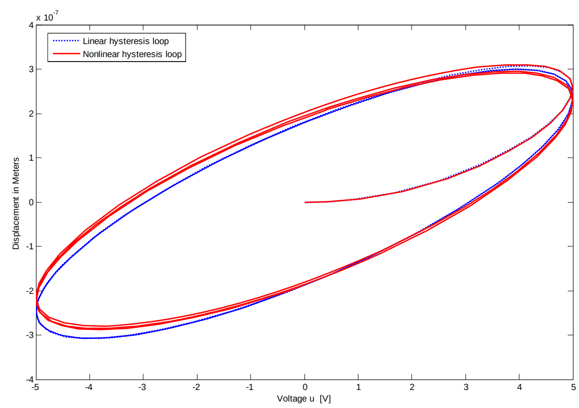


Fig. 7. Nonlinear and linearized models responses for input $u = 5V/30Hz$

To augment Eq. (1) and Eq. (14) in a state-space form. Let us first define a new state variable:

$$v = \hat{h} - k_1 u. \tag{15}$$

Then from Eq. (14), we get:

$$\dot{v} = \dot{\hat{h}} - k_1 \dot{u} = k_2 \hat{h} = k_2 v + k_1 k_2 u. \tag{16}$$

Substituting Eq. (15) into Eq. (1), we obtain:

$$\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x + \frac{k}{m}v = \frac{k(d - k_1)}{m}u. \tag{17}$$

With an integrated consideration of Eq. (16) and Eq. (17) along with the selection of a state vector $X = [x\ \dot{x}\ v]^T$, the entire dynamic system including the hysteresis effect can be expressed in linear state space as follows:

$$\dot{X} = AX + Bu, \tag{18}$$

$$y = CX, \tag{19}$$

where the system, input, and output matrices are given as:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{k}{m} & -\frac{b}{m} & -\frac{k}{m} \\ 0 & 0 & k_2 \end{bmatrix}, B = \left[\begin{pmatrix} 0 \\ \frac{k(d-k_1)}{m} \\ k_1 k_2 \end{pmatrix}\right], C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \tag{20}$$

respectively. By substituting the values of piezoelectric actuator parameters into Eq. (20), and converting the state-space model into its transfer function form, we get:

$$\frac{x}{u} = \frac{1.364 \times 10^{-12}s^2 + 0.07456s + 0.23}{s^3 + 4031s^2 + 1.008 \times 10^6 s + 2.008 \times 10^6}, \tag{21}$$

with two zeros at -3.0848 and -5.4663×10^{10}, and three poles at -2.0082, -2.657×10^2 and -3.7633×10^3.

5 Controller design and simulation results

Controller design has centered mainly on simple, linear, proportional-integral-derivative (PID) controller. Although a PID controller is one of the earlier control strategies, it still has a wide range of applications in industrial control due to its easily implementation in the field environment.

A mathematical description of the PID controller is:

$$u(t) = K_p e(t) + K_i \int_o^t e(\tau)d\tau + K_d \frac{de(t)}{t}, \tag{22}$$

where $u(t)$ is the input signal to the plant model, the error signal $e(t)$ is defined as $e(t) = r(t) - y(t)$, $y(t)$ is the output signal, and $r(t)$ is the reference input signal. K_p, K_i, and K_d are the proportional, integral, and derivative gains, respectively.

Gains for PID controllers have traditionally been tuned using classical methods such as Ziegler-Nichols and hand-tuning methods. Ziegler and Nichols described in [12] two methods for tuning the parameters of PID controllers. These two methods are the Ziegler-Nichols' closed loop method, and the Ziegler-Nichols' open loop method. Ziegler-Nichols' closed loop method is applied on the system with feedback, when the system appears to involve some pure integration and/or dominant complex-conjugate poles; i.e. the response is similar to an underdamped $2^{(nd)}$ order response.The second method; Ziegler-Nichols' open loop method or reaction curve method, is applied if the system's response to a unit-step is S-shaped, indicating that the plant involves no pure integration and the system response is not dominated by a pair of complex-conjugate poles. This method is applied on the plant itself, without feedback[15]. The gains found by either Ziegler-Nichols' method are regarded as starting point for a hand-tuning[13]. Hand-tuning method is a trial and error process that takes long time and effort to find the acceptable values relying on the experience and intuition of the engineers.

In this section, instead of using hand-tuning method, parameters of PID controller are tuned using SROS with gradient descent optimization technique. For setting up the tuning optimization process, initial values of PID gains are required. Since our plant does not involve any integrators nor dominant complex conjugate poles, these initial values are obtained from the Ziegler-Nichols' open loop method. Briefly this method is considered as a way of relating the process parameters; delay time, process gain and time constant, to the controller gains. It has been developed for use on delay-followed-by-first-order-lag processes but can also be adapted to real processes[7].

Table 2. PID controller parameters obtained for Ziegler-Nichols' open loop method

Controller type	From step response		
	K_p	$T_i = K_p/K_i$	$T_d = K_d/K_p$
P	$1/\alpha$		
PI	$0.9/\alpha$	3L	
PID	$1.2/\alpha$	2L	L/2

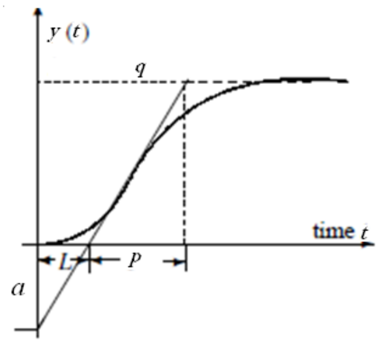


Fig. 8. Characterization of a step response in Ziegler-Nichols' open loop method

The step response is characterized by only two parameters L and $a(a = qL/p)$, as shown in Fig. 8. The point where the slope of the step response has its maximum is first determined, and the tangent at this point is drawn. The intersections between the tangent and the coordinate axes give the parameters L and a[14]. The controller parameters are then obtained from Tab. 2.

The MATLAB SISO Design Tool facilitates the controller design process by providing interactive and automated tools to tune controllers for a feedback control system[2]. Applying the automated Ziegler-Nichols' open loop controller design method in the SISO Design Tool to the linearized model, initial values of the PID controller were computed and given as:

$$K_p = -5.307 \times 10^6, K_i = 9.4784 \times 10^6, K_d = 0.7431 \times 10^6. \quad (23)$$

The Simulink block diagram for optimizing the PID gains for minimum tracking error is shown in Fig. 9. A sine signal with a frequency of 30 Hz and peak magnitude of 1 μm is entered as reference input and simulated using the Runge-Kutta method with a minimum step size of 1 μs and a maximum step size of 1 ms as well as a tolerance of 10^{-5}.

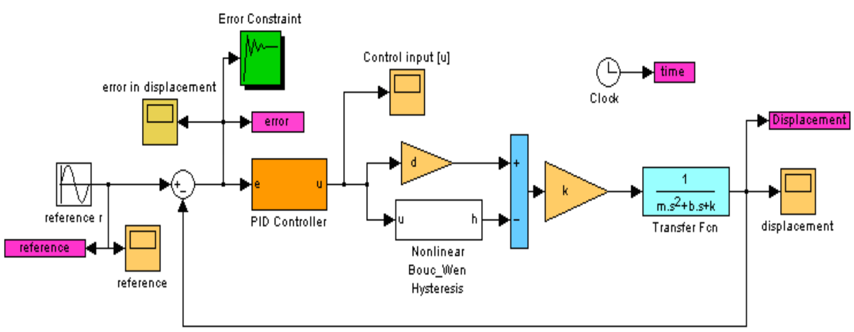
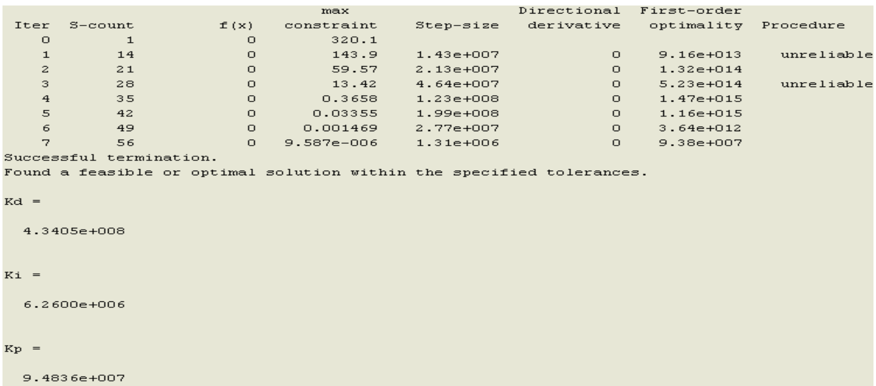


Fig. 9. Optimizing PID gains for minimum tracking error

SROS provides a GUI to assist in design of control systems. Prior to the beginning of the optimization using SROS for the nonlinear system, the Signal Constraint block was connected to the error signal as in Fig.

9, where time domain constraints were placed graphically. The upper and lower constraint bounds in the Signal Constraint window were adjusted to $\pm 10^{-8}$; i.e., the displacement signal was required to track the reference signal within maximum absolute error of 10^{-8}. For optimization method, the algorithm used was the gradient descent.

By starting the optimization, the SROS automatically converts the time domain constraints into a constrained optimization problem with initial values; Eq. (23), and then solves the problem using the gradient descend method. The constrained optimization problem formulated by SROS iteratively calls for simulations of the nonlinear system; Fig. 9, compares the results of the simulations with the constraint objectives as shown in Fig. 11, and uses gradient methods to adjust tuned PID gains to better meet the objectives.



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                                   max                      Directional   First-order
 Iter   S-count         f(x)    constraint    Step-size     derivative    optimality    Procedure
    0         1            0         320.1
    1        14            0         143.9    1.43e+007              0     9.16e+013     unreliable
    2        21            0         59.57    2.13e+007              0     1.32e+014
    3        28            0         13.42    4.64e+007              0     5.23e+014     unreliable
    4        35            0        0.3658    1.23e+008              0     1.47e+015
    5        42            0       0.03355    1.99e+008              0     1.16e+015
    6        49            0      0.001469    2.77e+007              0     3.64e+012
    7        56            0    9.587e-006    1.31e+006              0     9.38e+007
Successful termination.
Found a feasible or optimal solution within the specified tolerances.

Kd =

  4.3405e+008


Ki =

  6.2600e+006


Kp =

  9.4836e+007
```

Fig. 10. Optimization process of PID parameters

After few iterations, an optimal and feasible solution was found as shown in Fig. 10 and Fig. 12. The obtained PID gains were 9.4836×10^7, 6.2610×10^6, and 4.3405×10^8 related to K_p, K_i and K_d, respectively.

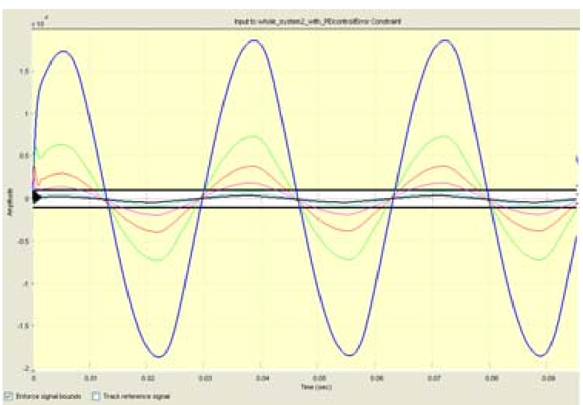

Fig. 11. Optimization progress result

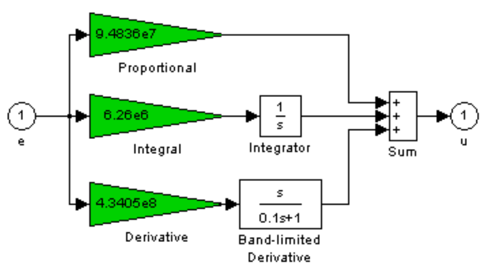

Fig. 12. Optimal and feasible PID gains

The simulation results are given in Figs. 13-15. As shown in Fig. 13, the blue dashed-line curve related to the displacement almost coincides with the red dash-line curve related to the reference, and the green solid-line curve is related to the tracking error. Except at the beginning, the tracking error, magnified in Fig. 14, has the value of less than $\pm 4 \times 10^{-9}$ which demonstrates the ultra-fine position tracking capability of the system. The control signal corresponding to the reference is given in Fig. 15, which shows the worst peak magnitude of the control signal is less than 45 V, which is less than the saturated level, i.e., 150 V.

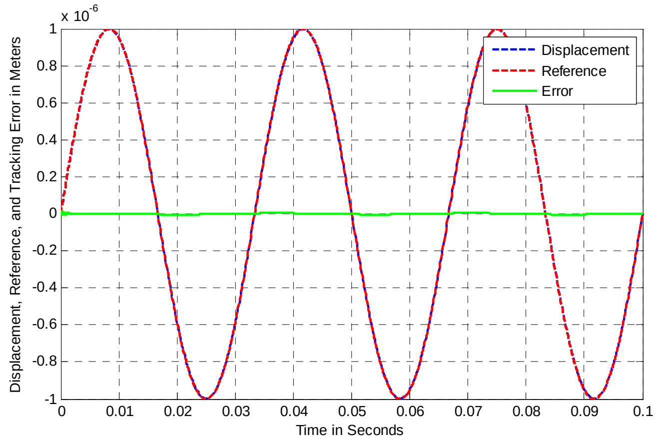

Fig. 13. Responses of displacement and reference signals

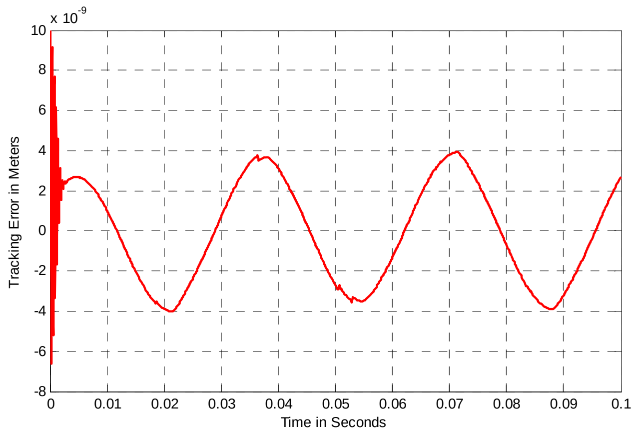

Fig. 14. Tracking error

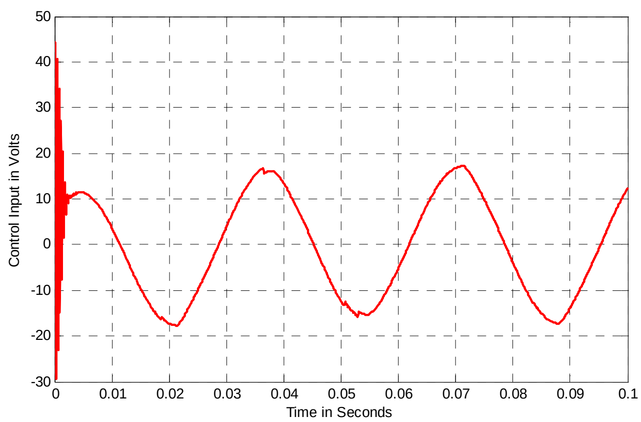

Fig. 15. Actuator control input

6 Conclusions

Designing an ultra-fine position tracking controller for a piezoelectric actuator with hysteresis nonlinearity was our concern in this paper. The hysteresis behavior of the actuator was described by the Bouc-Wen model. Multi-parameter-optimization techniques were used twice in this paper. First the trust-region reflective nonlinear least squares method was used to approximate the nonlinear actuator model by a linearized model.

Second for suppressing the tracking errors, the gradient descend method was used to tune the parameters of PID controller.

The number of iterations necessary for the optimization to converge or terminate depends on the initial guess for the tuned parameters, the specific positioning of the constraints, and the optimization settings. For obtaining the initial guess for the PID parameters, the automated Ziegler-Nichols' open loop controller design method was used, which decreases the time and effort for tuning parameters considerably.

Several advantages are gained on solving parameter optimization problems using computers; neither reliance on intuition nor experience in control is required, in addition to producing a satisfactory system response with optimal use of effort and time, high convergence speed, high approximation accuracy, high efficiency, and suitability for many kinds of goal functions. Simulation results performed on the nonlinear system verify the efficiency of the proposed method.

All work for this paper was done using MATLAB Version 7.6 (R2008a). As of R2009a, Simulink Parameter Estimation and Simulink Response Optimization functionality are merged into a new product; Simulink Design Optimization. Simulink Parameter Estimation and Simulink Response Optimization are no longer available.

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