Optimal decision in fuzzy NLP EOQ model for a single item and dynamic setup cost with space constraint

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Abstract. Various types of uncertainties and imprecision are inherent in real inventory problems. They are classically modeled using the approaches from the probability theory. However, there are uncertainties that cannot be appropriately treated by usual probabilistic models. The questions how to define inventory optimization tasks in such environment how to interpret optimal solutions arise. This paper allows the modification of the Single item EOQ model in presence of fuzzy decision making process where demand is related to the unit price and the setup cost varies with the quantity produced/Purchased. This paper considers the modification of objective function and storage area in the presence of imprecisely estimated parameters. The model is developed for the problem by employing different modeling approaches over an infinite planning horizon. It incorporates all concepts of a fuzzy arithmetic approach, the quantity ordered and the demand per unit compares both fuzzy non linear and other models. Investigation of the properties of an optimal solution allows developing an algorithm whose validity is illustrated through an example problem. Sensitivity analysis of the optimal solution is also studied with respect to changes in different parameter values and to draw managerial insights.

Keywords: fuzzy, NLP, EOQ, setup cost, storage capacity

1 Introduction

Since its formulation in 1915, the square root formula for the economic order quantity (EOQ) was used in the inventory literature for a pretty long time. Ever since its introduction in the second decade of the past century, the EOQ model has been the subject of extensive investigations and extensions by academicians. Although the EOQ formula has been widely used and accepted by many industries, some practitioners have questioned its practical application. For several years, classical EOQ problems with different variations were solved by many researchers and had be separated in reference books and survey papers e.g. Taha [12], Urgeletti [13]. Recently, for a single product with demand related to unit price[2] and for multi products with several constraints. His treatments are fully analytical and much computational efforts were needed there to get the optimal solution.

Operations Research (OR) was first coined in 1940 by Mcclosky and Trefther in a small town, Bowdsey, in the UK. During the Second World War, this OR mathematics was used in a wider sense to solve the complex executive strategic and tactical problems of military teams. Since then the subject has been enlarged in importance in the field of Economics, Management Sciences, Public Administration, Behavioral Science, Social Work Commerce Engineering and different branches of Mathematics etc. But various Paradigmatic changes in science and mathematics concern the concept of uncertainty. In Science, this change has been manifested by a gradual transition from the traditional view, which insists that uncertainty is undesirable and should be avoided by all possible means. According to the traditional view, science should strive for certainty in all its manifestations; hence uncertainty is regarded as unscientific. According to the modern

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view, uncertainty is considered essential to science; it is not any an unavoidable plague but has; in fact, a great utility. But to tackle non-random uncertainty no other mathematics was developed other than fuzzy set theory and showed the intention to accommodate uncertainty in the presence of random variables. Following Zadeh [19], significant contributions in this direction have been applied in many fields including production related areas. Consequently investment in introducing fuzzy is the key to avoid uncertain decision space. Many studies have modified inventory policies by considering the issues of nonrandom uncertain and fuzzy based EOQ models. Vujosevic et al. [18] presented a theoretical EOQ formula when inventory cost is fuzzy. Lee et al. [7] studied an inventory model for fuzzy demand quantity and fuzzy production quantity. Tripathy et al. [15–17] introduced the concept and developed the framework for investing fuzzy in holding cost and setup cost in EOQ model. Tripathy et al. [14] suggested improvements to production systems by employing entropy in the fuzzy model.

Sommer [11] applied fuzzy dynamic programming to an inventory and production scheduling problem in which the management wishes to fulfill a contract for providing a product and then withdraw from the market. Kacprzyk et al. [5] introduced the determination of optimal of firms from a global view point of top management in a fuzzy environment with fuzzy constraints improved on reappointments and a fuzzy goal for preferable inventory levels to be attained. Park [8] examined the EOQ formula in the fuzzy set theoretic perspective associating the fuzziness with the cost data. Here, inventory costs were represented by trapezoidal fuzzy numbers (TrFN) and the EOQ model was transformed to a fuzzy optimization problem.

But Roy et al. [9], Roy et al. [10] have considered the space constraint with the objective goal in fuzzy environment and attacked the fuzzy optimization problem directly using either fuzzy non-linear or fuzzy geometric programming technique similarly Lee et al. [7] and Vujosevic et al. [18] have applied fuzzy arithmetic approach in EOQ model without constraints.

Table 1. Summary of the related research

Authors	Demand	Setup cost	Holding cost	Unit cost of production	Constraint	Planning horizon	Structure of the Model	Model class
Vujosevic et al. (1996)	Constant	Constant	$\frac{C_h C_p Q}{2 \times 100}$	Constant	No	Finite	Fuzzy	Defuzzi-fication
Tripathy et al. (2009)	Constant	Constant	$\frac{H r^2 q^2}{2\lambda}$	Reliability and demand	Reliability	Infinite	Fuzzy	NLP
Tripathy et al. (2011)	Constant	Constant	$\frac{H \lambda q^2}{2 r^2}$	Reliability and demand	Reliability	Infinite	Fuzzy	NLP
Tripathy et al. (2011)	Constant	Constant	$\frac{H q^2}{2 r^2 \lambda}$	Reliability and demand	Reliability	Infinite	Fuzzy	NLP
Roy et al. (1995)	Constant	Variable	$\frac{C_1 q}{2}$	No	Space	Infinite	Fuzzy	NLP
Roy et al. (1997)	Constant	Variable	$\frac{C_1^2 q}{2}$	Demand	Space	Infinite	Fuzzy	NLP, GPP
Present paper (2011)	Constant	Variable	$\frac{C_1 K D^{-\beta} q}{2 \times 100}$	Demand	Space	Infinite	Fuzzy	NLP

In this paper a single item EOQ model is developed where unit price varies inversely with demand and setup cost increases with the increase of production. In company or industry, total expenditure for production and storage area are normally limited but imprecise, uncertain, non-specificity, inconsistency vagueness and flexible. These are defined within some ranges. However, the no stochastic and ill formed inventory models can be realistically represented in the fuzzy environment. The problem is reduced to a fuzzy optimization problem associating fuzziness with the storage area and total expenditure. The optimum order quantity is evaluated by both fuzzy non linear programming (FNLP) method and the results are obtained for linear membership functions. The model is illustrated with numerical example and with the variation in tolerance limits for both shortage area and total expenditure. A sensitivity analysis is presented. The numerical results for fuzzy and crisp models are compared. The remainder of this paper is organized as follows. In Section 2, assumptions and notations are provided for the development of the model and the mathematical formulation is developed. In Section 3, mathematical analysis of fuzzy non linear programming (FNLP) is formulated. The solution of

the FNLP inventory is derived in Section 4. The numerical example is presented to illustrate the development of the model in Section 5. The sensitivity analysis is carried out in Section 6 to observe the changes in the optimal solution. Finally Section 7 deals with the summary and the concluding remarks.

2   Mathematical model

A single item inventory model with demand dependent unit price and variable setup cost under storage constraint is formulated as

$$\min \ C(D,q) = C_{03}q^{\nu-1}D + KD^{1-\beta} + \frac{1}{2 \times 100}C_1KD^{-\beta}q,$$
$$\text{s.t. } Aq \leq B, \forall D, q > 0, \tag{1}$$

where, q = number of order quantity, D = demand per unit time C_1 = holding cost per item per unit time. C_3 = Setup cost = $C03q^{\nu}$, ($C_{03}(> 0)$ and $\nu(0 < \nu < 1)$ are constants). P = Unit production cost =$KD^{-\beta}$, $K(> 0)$ and $\beta(> 1)$ are constants. Here lead time is zero, no back order is permitted and replenishment rate is infinite. A and B are nonnegative real numbers, B is the space constraint goal. The above model in a fuzzy environment is

$$\widetilde{\min} \ C(D,q) = C_{03}q^{\nu-1}D + KD^{1-\beta} + \frac{1}{2 \times 100}C_1KD^{-\beta}q,$$
$$\text{s.t. } Aq \leq \widetilde{B}, \forall D, q > 0. \tag{2}$$

(A wavy bar ( ) represents fuzzification of the parameters).

3   Mathematical analysis of fuzzy non linear programming (FNLP)

A fuzzy non linear programming problem with fuzzy resources and objective are defined as

$$\widetilde{\min} \ g_0(x),$$
$$\text{s.t. } g_i(x \leq \widetilde{b}_i, i = 1, 2, 3, \cdots m.) \tag{3}$$

In fuzzy set theory, the fuzzy objective and fuzzy resources are obtained by their membership functions, which may be linear or nonlinear. Here μ_0 and $\mu_i(i = 1, 2, \cdots, m)$ are assumed to be non increasing continuous linear membership functions for objective and resources respectively such as

$$\mu_i(g_i(x)) = \begin{cases} 1, & \text{if } g_i(x) < b_i. \\ 1 - \frac{g_i(x)-b_i}{P_i}, & \text{if } b_i \leq g_i(x) < b_i + P_i, \quad i = 0, 1, 2, \cdots, m. \\ 0, & \text{if } g_i > b_i + p_i. \end{cases}$$

In this formulation, the fuzzy objective goal is b_0 and its corresponding tolerance is P_0 and for the fuzzy constraints, the goals are $b_i's$ and their corresponding tolerances are $P_i's(i = 1, 2, \cdots, m)$. To solve the problem (3), the max-min operator of Bellman et al. [1] and the approach of Zimmermann [20] are implemented.

The membership function of the decision set, $\mu_D(x)$, is $\mu_D(x) = \min\{\mu(x), \mu_1(x), \cdots, \mu_m(x)\}, \forall x \in X$

The min operator is used here to model the intersection of the fuzzy sets of objective and constraints. Since the decision maker wants to have a crisp decision proposal, the maximizing decision will correspond to the value of x, $x_{\max}$ that has the highest degree of membership in the decision set.

$\mu_D(x_{\max}) = \max_{x \geq 0}[\min \mu_0(x), \mu_1(x), \cdots, \mu_m(x)]$. It is equivalent to solving the following crisp non linear programming problem.

$$\max \ \alpha,$$
$$\text{s.t. } \mu_0(x) \geq \alpha, \mu_i(x) \geq \alpha(i = 1, 2, \cdots, m), \forall x \geq 0, \alpha \in (0, 1). \tag{4}$$

A new function, i.e the lagrangian function $L(\alpha, x\lambda)$ is formed by introducing $(m+1)$ lagrangian multipliers $\lambda = (\lambda_0, \lambda_1, \cdots, \lambda_m)$.

$L(\alpha, x, \lambda) = \alpha - \Sigma_{i=o}^{m} \lambda_i (g_i(x) - b_i - (1-\alpha)P_i)$. The necessary condition of Kuhn et al. [6] for the optimal solution to this problem implies that optimal values $x_1^*, x_2^*, x_3^*, \cdots x_n^*$, and $\lambda_1^*, \lambda_2^*, \lambda_3^*, \cdots \lambda_n^*$ should satisfy

$$\frac{\partial L}{\partial \alpha} = 0, \frac{\partial L}{\partial x_j} = 0, j = 1, 2, \cdots, n.$$

$$\lambda_i(g_i(x) - b_i - (1-\alpha)P_i) = 0, g_i(x) \le b_i + (1-\alpha)P_i, \lambda_i \le 0, i = 0, 1, \cdots, m. \quad (5)$$

Moreover, Kuhn-Tucker's sufficient condition demands that the objective function for maximization and the constraints should be respectively colane and convex. In this formulation, it can be shown that both objective function and constraints satisfy the required sufficient conditions. Now, solving Eq. (5), the optimal solution for the FNLP problem is obtained.

4   Solution of the proposed inventory model

The proposed inventory model depicted by Eq. (2)

$$\widetilde{\min} \, C(D, q) = C_{03}q^{\nu-1} + KD^{1-\beta} + \frac{1}{2 \times 100}C_1KD^{-\beta}q,$$

$$\text{s.t. } Aq \le \widetilde{B}, \forall D, q > 0.$$

reduces to following Eq. (4),

$$\max \, \alpha$$

$$\text{s.t. } C_{03}q^{\nu-1} + KD^{1-\beta} + \frac{1}{2 \times 100}C_1KD^{-\beta}q \le C_0 + (1-\alpha)P, \quad (6)$$

$$Aq \le B + (1-\alpha)p, \forall D, q > 0 \text{ and } \alpha \in (0, 1).$$

Here, the objective goal is C_0 with tolerance P_0 and the space constraint goal is B with tolerance P. So, the corresponding Lagrangian function is

$$L(\alpha, D, q, \lambda_1, \lambda_2) = \alpha - \lambda_1 \Big(C_{03}q^{\nu-1} + KD^{1-\beta} + \frac{1}{2 \times 100}C_1KSD^{-\beta}q$$

$$- C_0 - (1-\alpha)P_0 \Big) - \lambda_2(Aq - B - (1-\alpha)P).$$

From Kuhn-Tucker's necessary conditions,

$$\frac{\partial L}{\partial \alpha} = 1 - \lambda_1 P_0 - \lambda_2 P \ge 0,$$

$$\frac{\partial L}{\partial D} = \lambda_1(C_{03}q^{\nu-1} + (1-\beta)KD^{-\beta} + \frac{1}{2 \times 100}C_1Kq\beta^{-(\beta+1)}D) \le 0,$$

$$\frac{\partial L}{\partial q} = \lambda_1(C_{03}(\nu-1)q^{\nu-2}D + \frac{1}{2 \times 100}C_1KD^{-\beta}) - A\lambda_2 \le 0,$$

$$\frac{\partial L}{\partial \lambda_1} = C_{03}q^{\nu-1}D + KD^{1-\beta} + \frac{1}{200}C_1KD^{-\beta} - C_0 - (1-\alpha)P_0 \le 0,$$

$$\frac{\partial L}{\partial \lambda_2} = Aq - B - (1-\alpha)P \le 0 \quad and \quad \alpha(1 - \lambda_1 P_0 - \lambda_2 P) = 0,$$

$$\lambda_1 D \left(C_{03}q^{\nu-1} + (1-\beta)KD^{-\beta} + \frac{1}{2 \times 100}C_1Kq\beta^{-(\beta+1)}D \right) = 0,$$

$$\lambda_1 q \left(C_{03}(\nu-1)q^{\nu-2}D + \frac{1}{2 \times 100}C_1KD^{-\beta} \right) - A\lambda_2 = 0,$$

$$\lambda_1 \left(C_{03}q^{\nu-1}D + KD^{1-\beta} + \frac{1}{200}C_1KD^{-\beta} - C_0 - (1-\alpha)P_0 \right) = 0,$$

$$\lambda_2(Aq - B - (1-\alpha)P) = 0, \alpha, D, q \geq 0 \quad and \quad \lambda_1, \lambda_2 \leq 0,$$

solving these equations, optimum quantities are

$$q^* = \frac{B + (1-\alpha^*)P}{A}, \quad D^* = \left[\frac{C_{03}q^{\nu-1} \pm C_{03}^2 q^{2(\nu-1)} - \frac{4K^2C_1q\beta(\beta-1)-1}{2\times100}}{2(1-\beta)K}\right]^{\frac{-1}{\beta}},$$

$q = f(a)$ and $D = f(q)$ where α^* is a root of

$$K\beta D^{*1-\beta} + \frac{1}{200}C_1 K q^* D^{*-\beta}(1+\beta) - C_0 - (1-\alpha)P_0 = 0,$$

$$C^*(D^*, q^*) = C_{03}q^{*\nu-1}D^* + KD^{*1-\beta} + \frac{1}{2\times100}C_1 K D^{*-\beta}q^*.$$

So, by both FNLP and NLP techniques, the optimal values of q^* and D^* and the corresponding minimum cost are evaluated for the known values of other parameters.

5 Numerical example

For a particular EOQ problem, let $C_{03} = Rs.200$, $K = 100$, $C_1 = Rs.100$, $\nu = 0.5$, $\beta = 1.5$, $A = 10$ units, $B = 50$ units, $C_0 = Rs.2000$ and $P = 20$ and $P = 15$ units. For these values the optimal value of productions batch quantity q^*, optimal demand rate D^*, minimum average total cost $C^*(D^*, q^*)$ and Aq^* obtained by FNLP are given in Tab. 2.

After 26 iterations Tab. 2 reveals the optimal replenishment policy for single item with demand dependent unit cost and dynamic setup cost. In this table the optimal numerical results of fuzzy model are compared with the results of crisp model and fuzzy model of Roy et al. [10]. The optimum replenishment quantity q^* and Aq^* are both 12.93% more and -6.59% less than that of other crisp model and fuzzy model respectively, the optimum quantity demand D^* is 9.70 but 9.21 and 9.81 for comparing models, hence 5.35% more from the other crisp model and -1.11% less from the fuzzy model. The minimum total average cost $C^*(D^*, q^*)$ is 48.62 but 54.43 and 53.93 comparing models, hence -10.67% and -9.84% less from other crisp and fuzzy model respectively. It permits the better use of present fuzzy model as compared to the crisp model and other fuzzy model. The results are justified and agree with the present model. It indicates the consistency of the fuzzy space of EOQ model from other models.

Table 2. Optimal values for the proposed inventory model

Model	Method	Iteration	q^*	D^*	$C^*(D^*, q^*)$	α^*	Aq^*
Fuzzy model	FNLP	26	5.646724	9.702500	48.62299	0.5688503	56.46724
Crisp model, Roy et al. (1997)	NLP	-	5	9.21	54.43	1	50
Fuzzy model, Roy et al. (1997)	FNLP	-	6.0449	9.8115	53.9324	0.3033	60.449

Table 3. Sensitivity analysis of the parameters P_0

P_0	Iteration	α^*	q^*	D^*	T_0	T	$C^*(D^*, q^*)$	Aq^*
25	24	0.6482626	5.527606	9.632251	8.793435	5.276061	48.79343	55.27606
50	25	0.8166107	5.275084	9.480031	9.16947	2.7508395	49.16946	52.75084
100	27	0.9062166	5.140675	9.397084	9.37834	1.406751	49.37834	51.40675
150	26	0.9369897	5.094516	9.368275	9.451545	0.9451545	49.45155	50.94516
200	27	0.9525556	5.071167	9.353638	9.48888	0.711666	49.48888	50.71167
1000	32	0.9904195	5.014371	9.317883	9.5805	0.1437075	49.58054	50.14371

6 Sensitivity analysis

Now the effect of changes in the system parameters on the optimal values of q, D, $C(D,q)$ and Aq when only one parameter changes and others remain unchanged the computational results are described in Tabs. 3 and 4. As a result

$\alpha^*, q^*, D^*, C^*(D^*, q^*)$ and Aq^* are less sensitive to the parameters P_0 and P. Following Dutta et al. [3] and Hamacher et al. [4] it is observed that the effect of tolerance in the said EOQ model with the earlier numerical values and construct Tabs. 3 and 4 for the degrees of violation and $T_0 (= (1 - \alpha)P_0)$ and $T (= (1 - \alpha)P$ for two constraints given by Eq. (6).

From Tab. 3, it is seen that: (i) For higher tolerances P_0, the value of $\alpha_{\max}$ does not achieve 1, (ii) For higher acceptable variations P_0, the optimal solutions remain invariant and the optimal solutions are very close to the solutions ($q^* = 5.646724$, $D^* = 9.7025$, $C^*(D^*, q^*) = 48.62299$ and $Aq^* = 56.46724$) of fuzzy model and ($q^* = 5$, $D^* = 9.308755$, $C^*(D^*, q^*) = 49.60392$ and $Aq^* = 50$) of the crisp model without tolerance ($\alpha = 1$) respectively.

From Tab. 4 it is shown that: (i) For different values of P, degrees of violations T_0 and T are never zero, i.e. different optimal solutions are obtained. (ii) As P increases from 16, the minimum average cost $C^*(D^*, q^*)$ decreases, q^* and D^* increase.

Table 4. Sensitivity analysis of the parameter P

P	Iteration	α^*	q^*	D^*	T_0	T	$C^*(D^*, q^*)$	Aq^*
16	26	0.5715773	5.685476	9.725147	8.568454	6.854763	48.56845	56.85476
20	25	0.5819704	5.836059	9.812218	8.360592	8.360592	48.36059	58.36059
23	25	0.5892732	5.944672	9.874125	8.214536	9.446716	48.21454	59.44672
36	25	0.6169771	6.378882	10.11459	7.660458	13.78882	47.66046	63.78882
38	27	0.6267645	6.441095	10.14817	7.58471	14.41095	47.58471	64.41095
40	26	0.6244443	6.502223	10.18096	7.511114	15.02223	47.51111	65.02223

7 Conclusion

In constraint to Roy, the approach in this paper provides solutions better than those obtained by using properties and this paper follows real life inventory model for single item in fuzzy environment by FNLP technique. Some sensitivity analyses on the tolerance limits have been presented. The results of the fuzzy model is compared with that of crisp model which reveals that fuzzy model gives better result than the usual crisp model. Inventory modelers have so far considered auto are type of setup cost that is fixed or constant. This is rarely seen to occur in the real market. In the opinion of the author, an alternative (and perhaps more realistic) approach is to consider the setup cost as a function quantity produced/purchased may represent the tractable decision making procedure in fuzzy environment. A new mathematical model is developed and numerical example is provided to illustrate the solution procedure. The new modified EOQ model was numerically compared to the traditional EOQ model. Finally, the effect decision space was demonstrated numerically to have an adverse affect on the total average cost per unit. This method is quite general and can be extended to other similar inventory models including the ones with shortages and deteriorate items.

References

[1] R. Bellman, L. Zadeh. Decision making in a fuzzy environment. *Management Science*, 1970, **17**: B141 – B164.
[2] T. Cheng. An economic order quantity model with demand—dependent unit cost. *European Journal of Operational Research*, 1989, **40**: 252–256.
[3] D. Dutta, J. Rao, R. Tiwari. Effect of tolerance in fuzzy linear fractional programming. *Fuzzy sets and systems*, 1993, **55**: 133–142.

[4] H. Hamacher, H. Leberling, H. Zimmermann. Sensitivity analysis in fuzzy linear programming. *Fuzzy sets and systems*, 1978, (1): 269–281.

[5] J. Kacprazyk, P. Staniewski. Long term inventory policy-making through fuzzy decision making models. *Fuzzy sets and systems*, 1982, **8**: 17–132.

[6] H. Kuhn, A. Tucker. Proceedings second berkely symposium and mathematical statistics and probability. *Nonlinear Programming. In J. Neyman*, 1951, 481–494.

[7] H. Lee, J. Yao. Economic production quantity for fuzzy demand quantity and fuzzy production quantity. *European Journal of operational Research*, 1998, **109**: 203– 11.

[8] K. Park. Fuzzy set theoretic interpretation of economic order quantity. *IEEE Transactions on systems, Man, and Cybernetics*, 1987, **6**(17): 1082–1084.

[9] T. Roy, M. Maiti. A fuzzy inventory model with constraint. *Operational Research Society of India*, 1995, **32**(4): 287–298.

[10] T. Roy, M. Maiti. A fuzzy eoq model with demand dependent unit cost under limited storage capacity. *European Journal of Operational Research*, 1997, **99**: 425–432.

[11] G. Sommer. *Fuzzy inventory scheduling*. Academic press, New York, 1981.

[12] H. Taha. *Operations Research - An introduction*, 2nd edn. Macmilliion, New York, 1976.

[13] G. U. Tinareli. Inventory control models and problems. *European Journal of operational Research*, 1983, (14): 1–12.

[14] P. Tripathy, M. Pattnaik. An entropic order quantity model with fuzzy holding cost and fuzzy disposal cost for perishable items under two component demand and discounted selling price. *Pakistan Journal of Statistics and Operations Research*, 2008, **4**(2): 93–110.

[15] P. Tripathy, M. Pattnaik. Optimal disposal mechanism with fuzzy system cost under flexibility and reliability criteria in non-random optimization environment. *Applied Mathematical Sciences*, 2009, **3**(37): 1823–1847.

[16] P. Tripathy, M. Pattnaik. A non-random optimization approach to a disposal mechanism under flexibility and reliability criteria. *The Open Operational Research Journal*, 2011, **5**: 1–18.

[17] P. Tripathy, P. Tripathy, M. Pattnaik. A fuzzy eoq model with reliability and demand dependent unit cost. *International Journal of Contemporary Mathematical Sciences*, 2011, **6**(30): 1467–1482.

[18] M. Vujosevic, D. Petrovic, R. Petrovic. Eoq formula when inventory cost is fuzzy. *International Journal Production Economics*, 1996, **45**: 499–504.

[19] L. Zadeh. Fuzzy sets. *Information and Control*, 1965, **8**: 338– 353.

[20] H. Zimmermann. Description and optimization of fuzzy systems. *International Journal of General System*, 1976, **2**: 209–215.