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Traffic network design problem with uncertain demand: A multi-stage bi-level programming approach*

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Abstract. By exploring of the multi-stage network design problem of comprehensive traffic, the paper proposes a bi-level programming model with uncertain demand. The upper level problem is to maximize the consumer surplus of all demand scenarios with budget constraint. While under the network investment decisions of the upper level problem, the lower level problem is to maximize the consumer surplus of every demand scenario with consideration of the collaborations of transportation modes, traffic load balancing and capacity constraints. Then, a numerical example is given to verify the effectiveness of the multi-stage be-level programming model. Compared with the existing research, the proposed model can simultaneously optimize the final form of the comprehensive traffic network and construction timing. The model considers all the annual coordination between transportation infrastructure construction and demand. It reflects the cooperation and balanced development of transportation modes, and makes the decision support more effective for the gradual improvement of the comprehensive traffic network.

Keywords: bi-level programming, multi-stage, uncertain demand, traffic network design problem

1 Introduction

The network design problem (NDP) is one of optimizing the improvement of a transportation network with respect to a system-wide objective while considering the route choice behavior of network users[9]. It has been extensively studied by engineers, mathematicians, operations research analysts, and planners. In recent years, with the rapid growth in total scale of transportation infrastructure and the continuous expansibility of transportation networks, the transportation network in China has been improved continually. However, on the whole, the transportation infrastructure still cannot meet the transportation needs of national economic and social development[1, 5]. The structural problems are still outstanding and every transportation mode that is developed in dispersion, which lacks coordination, cooperation, organic linking and integrated operation. And these problems resulted in poor connections and services, which seriously reduced the overall integrated transport efficiency and increased transportation costs. Constructing an integrated transport infrastructure network with reasonable division of transport tasks, complementary advantages and organic linking is a basis for building a perfect comprehensive transportation system, that is, a system that is environmentally friendly and has an efficient operation. With increasingly urgent constraints of resources, environment and ecology, how to make the passenger and freight transport of each transportation develop in phase under limited funds has been becoming increasingly prominent.

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Integrated transport network design is the key content of regional transportation system strategy management, whose decision-making process needs to consider the interaction between government departments and the public. Thus the bi-level programming model has become an ideal tool for transport network design problems, and made great progress in urban transport network design[5]. In recent years, bi-level programming model has began to be applied in the studies of integrated transport and multimodal transport network design[8, 14, 16]. These studies have laid a good foundation for this paper, but they mostly took network design under mid long term fixed demand as the research object.

The construction of an integrated transport network is a gradual process under budget constraints. Because of huge investment, long investment recovery period and strong public interest in transportation infrastructure, financing difficulties have become a prominent realistic problem. In this case, considering the constraints of capital budget of each period and achieving effective use of limited funds to meet the transport demand as much as possible is closer to the decision-makers need in the practice of investment decision-making. The existing literature on transportation projects timing does not fully consider the impacts of construction time sequence on the integrated transport network. Some of the existing literature in the road transport network design model has considered the multi-period problem[7, 10]. Using these ideas for references, this paper analyzes the multi-period problem in integrated transport network design, studies investment allocation among transport modes under network environment, comprehensively considers the dynamic relationship between passenger and freight traffic, and reflects the random nature of OD demand and the impact of elastic demand, which can provide a decision support for coordinated development of various modes of transport and transport infrastructure orderly building in the integrated transport network.

2 Problem statement

The objective of bi-level programming model in transport network design generally includes[15]: (1) to minimize total system travel time with fixed demand; (2) to maximize network reserve capacity with fixed demand; (3) multi-objective optimization, such as comprehensively considering total system travel time, construction cost, environmental impact, and so on; (4) to maximize consumer surplus with elastic demand. Consumer surplus is the difference between the willingness to pay and actual travel cost, which can be used to evaluate the social benefits of transportation systems bringing to consumers of transport. With consumer surplus as the objective of network design, the impact of transport network improvement measures on commuter behavior can be examined. Therefore, this study takes the consumer surplus with elastic demand as the objective of programming.

In reality, the transport demand per originCdestination (OD) pair and its transport cost interact with each other. The changes in transport cost result in an increase or decrease in travel demand per OD pair. The paper allows for the assumption of elastic demand for the overall trip demand. Compared with fixed demand, the traffic assignment model with elastic demand gets feedback from the transport cost. The potential demand level between origin r and destination s for user type k can be written as[10]:

$$d_{\text{rsk}}^{t,\omega} = D_{\text{rsk}}^{t,\omega} exp(-\theta_k \pi_{\text{rsk}}^{t,\omega}),$$

where $D_{\text{rsk}}^{t,\omega}$ denotes the forecast value of demand between origin r and destination s for user type k at time t under the demand scenario ω, which is a constant value t, $d_{rsk}^{t,\omega}$ is the simulation value of demand between origin r and destination s for user type k at time t under the demand scenario ω, which is a stochastic variable. The user type k can be used to separate passenger transport demand (while $k = 1$) and freight transport demand (while $k = 2$). $\pi_{\text{rsk}}^{t,\omega}$ is the minimum travel cost between origin r and destination s for user type k at time t under the demand scenario ω. θ_k is a constant value reflecting the sensitivity level of elastic demand to transport cost changes.

The system consumer surplus (SCS) for user type k at time t under the demand scenario ω is $\text{SCS}_k^{t,\omega}$, and the system consumer surplus (SCS) for user type k under the demand scenario ω in the planning period is denoted by SCS_k^{ω}[10].

In this research, we have the following pretreatment and assumptions.

(1) Regional integrated transport network design problem is a hybrid network design problem. By taking

discrete network design as the research object, we make choices in a given set of integrated transport infrastructure projects.
(2) It is assumed that the investment on common highway and expressway between nodes is used to upgrade and renovate the existing road.
(3) It is assumed that passengers and cargo owners always tend to select the most effective traffic paths and transportation modes, therefore the model is based on the principle of utility maximization to achieve the distribution of traffic flow in integrated transport network.
(4) It is assumed that the integrated transport physical network structure, station layout, scale, and multimode transportation linking outside the scope of planning do not change in the entire decision-making cycle.

3 Bi-level programming model

3.1 Upper level model

Transport demand is influenced by parameters such as economic development, population scale and fuel price. The uncertainty in transport demand has been widely recognized as a fundamental construct in the traffic network design. This paper uses discrete demand scenarios to describe the uncertainty in transport demand. The objective is to maximize the expectation of system consumer surplus for all user types in order to increase the reliability of the planning model.
(1) Objective of the upper level problem

$$\max U = \sum_{k\in K}\sum_{\omega\in W} p^{\omega} SCS_k^{\omega},$$

where p^{ω} is the probability of demand scenario ω occurring.
(2) Budget of payment of construction cost constraint

The planners are interested in evaluating a certain set of candidate transport infrastructure projects. The total payment of construction cost at time t is subject to the available payment budget B^t.

$$\sum_{i\in \mathrm{A}_1} I_i^t y_i^t \le B^t,$$

where A_1 denotes the set of candidate transport infrastructure projects on link i, I_i^t denotes the payment of construction cost of the project on link i at time t, and y_i^t is the action implementation indicator with a binary value of 0 if the project on link i is completed at time t and 0 if it is otherwise.
(3) Construction timing constraint

$$y_i^{t_m} \le y_i^{t_m+1},$$

This constraint means that if the project on link i is open to traffic at time t_m, the project will be open to traffic at time $t_m + 1$.

3.2 Lower level problem

The lower level problem represents the user's decision-making process in travel routing by selecting the optimal transport mode choice from among a set of available transport infrastructure projects.
(1) Objective of the lower level problem

The objective function at the lower level is to maximize the system consumer surplus for all user types under a certain demand scenario in the planning period.

$$\max SCS^{\omega} = \sum_{t\in T}\sum_{k\in K}\sum_{rs\in A} \frac{d_{\mathrm{rsk}}^{t,\omega}}{\theta_k(1+\rho)^t},$$

where ρ is the social discount rate.

(2) Conservation of flow constraints

$$\sum_{i\in L_{D(j)}} x_{ik}^{t,\omega} - \sum_{i\in L_{O(j)}} x_{ik}^{t,\omega} = d_{\text{rsk}}^{t,\omega},$$

where $x_{ik}^{t,\omega}$ represents the flow of passengers and commodities on link i for user type k at time t under the demand scenario ω, $x_{ik}^{t,\omega}$ is a decision variable. $L_{D(j)}$ denotes the set arcs leaving node j. $L_{o(j)}$ denotes the set arcs terminating at node j. $d_{\text{rsk}}^{t,\omega} > 0$ if j is a supply node. $d_{\text{rsk}}^{t,\omega} < 0$ if j is a demand node. t, $d_{\text{rsk}}^{t,\omega} = 0$ if j is a transshipment node.

(3) Traffic load constraints

Transport infrastructure needs large-scale investment and high operation cost. Low utilization rates of transport infrastructure projects are unreasonable from an economic perspective and increase the pressures of paying loan and operation subsidy. However, a transport infrastructure project cannot meet the demands of future developments in the future if the traffic load is too heavy in a short time after its completion. The traffic load constraints are as follows

$$\sum_{i\in A_0} (x_{ik}^{t,\omega}/\lambda_{ik}^t) \leq R_i(M_i + M_i' y_i^t),$$

where M_i and M_i' denote the existing capacity of link i and the capacity of the project on link i. λ_{ik}^t denotes the average load of vehicles of using link i for user type k at time t. R_i is the maximum load of link i.

$$\sum_{i\in A_0} (x_{ik}^{t,\omega}/\lambda_{ik}^t) \geq r_i(M_i + M_i' y_i^t),$$

where r_i is the minimum load of link i.

3.3 Solution method

The bi-level programming model in this paper includes a nonlinear objective with binary variables in the upper level problem and a nonlinear objective with nonlinear constraints in the lower level problem and considers multi-period transport network design under different scenarios, which challenges the efficiency of algorithms. The problem in this paper can be efficiently solved by the Knitro 7.0 software package for Matlab which is provided by Ziena Optimization, Inc. The active set algorithm and the interior point algorithm are implemented within the Knitro 7.0 software package [2, 3, 11].

4 Case study

The paper presents an integrated transport network to verify the effectiveness of the proposed model and algorithm in this section. The network and data related are based on the research report of the X region comprehensive transportation planning. The programming problem is to figure out the construction time of integrated transportation projects from No.1 to No.7, given the transport demand from node 1 to node 4, while ignoring the demand of other nodes and node 3 having transfer stations for different transportation modes. The paper sets time $t = 1, 2, 3, 4, 5$, demand scenario $w = 1, 2, 3$, each of which has the same occurring probability ($p^1 = p^2 = p^3 = 0.333$), and annual budget and the transport demand from node 1 to node 4 of the X region in the future five years with three demand scenarios are shown in Tab. 1.

Tab. 2 shows part of parameters of our model, which are based on data related to the research report of the X region comprehensive transportation planning. Among them, the paper chooses passenger car unit (PCU) to indicate road capacity; because of good planned rail transportation, the ton is selected to indicate rail freight capacity and person for passenger capacity. It is supposed that unit hour time value of passenger and freight are 16 Yuan per person and 16.4 Yuan per ton, respectively, the maximum load and the minimum

Table 1. Annual budget and transport demand from node 1 to node 4 in X region

t	$\omega=1$		$\omega=2$		$\omega=3$		
	$k=1$	$k=2$	$k=1$	$k=2$	$k=1$	$k=2$	B^t
	million person	million ton	million person	million ton	million person	million ton	million yuan
1	9.79	18.03	12.24	22.53	14.69	27.05	48
2	10.96	20.37	13.7	25.46	16.44	30.55	60
3	12.01	21.46	15.01	26.82	18.01	32.19	75
4	13.36	22.91	16.7	28.64	20.04	34.36	95
5	14.51	24.28	18.14	30.35	21.76	36.42	125

Table 2. Parameters of transport cost and capacity

i	Project	$\bar{\tau_{ik}}$		φ_{ik}		M_i million PCU	M'_i million PCU
		$k=1$(h)	$k=2$(h)	$k=1$ 1000 Yuan per person	$k=2$ 1000 Yuan per ton		
1	Expressway from node 1 to node 2	4.3	5.38	0.22	1.08	1.8	14.6
2	Common highway from node 2 to node 4	9.73	11.1	0.27	1.56	1.8	7.3
3	Expressway from node 2 to node 3	3.13	3.91	0.16	0.78	1.8	14.6
4	Railway from node 1 to node 3	7.4	13.3	0.17	0.33	0	12
5	Railway from node 3 to node 4	5.21	9.38	0.12	0.23	0	15
6	Common highway from node 1 to node 3	8.33	9.51	0.23	1.33	1.8	7.3
7	Expressway from node 3 to node 4	4.69	5.86	0.23	1.17	1.8	14.6

load of all projects are 1.0 and 0.15. respectively, and the average load of passenger and freight road vehicles are 4 person and 3 ton per PCU, respectively.

By substituting all the parameters and the demand data shown in Tabs. 1 and 2 into the multi-stage bi-level programming model, a numerical example of the proposed model is given. By running program in Matlab 2010b and calling the Knitro 7.0 software package, the results are calculated. The results obtained in this study are presented in Tab. 3.

Table 3. Construction schedule of 7 transport projects in X region

t	y_i^t						
	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$
1	1	1	0	1	0	0	1
2	1	1	0	1	0	0	1
3	1	1	1	1	0	0	1
4	1	1	1	1	0	0	1
5	1	1	1	1	0	0	1

5 Conclusions

In the gradual improvement of integrated traffic networks, because of limited funds, introduction of time dimension into transport network design to optimize both the ultimate form of network and construction time is very important for making transport construction coordinated with demand and promoting balanced development of transportation modes. This paper proposed a multi-stage bi-level programming model for a comprehensive traffic network design. Among them, the upper level problem is to maximize the consumer surplus of all demand scenarios with budget constraints; while under the network investment decisions of the upper level problem, the lower level problem is to maximize the consumer surplus of every demand scenario with consideration of the collaborations of transportation modes, traffic load balancing and capacity constraints. Using scenarios and elasticity to reflect OD demand uncertainty, the model improves the robustness of planning. Finally, a case study shows that the model is an effective method to apply into decision support for the coordinated development and orderly construction of transportation structures. The proposed model has so many decision variables and nonlinear equality constraints that it makes the case study limited in small-scale networks with several demand scenarios. Algorithms for optimization of large-scale networks need to be further studied.

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