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Adaptive hybrid synchronization of the complex dynamical network with non-derivative and derivative coupling*

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Abstract. First of all, we impose matrix transform on the state vector of the master and slave systems, the master and slave systems could be synchronized under the transform, this case would be called hybrid synchronization in this paper. Secondly, we investigate adaptive hybrid synchronization of the complex dynamical network with non- derivative and derivative coupling. Based on Lasalle's invariance principle, adaptive synchronization criteria is obtained. Analytical result shows that the complex dynamical network with non-derivative and derivative coupling can asymptotically hybrid synchronize to a given trajectory under the designed adaptive controllers. What is more, the coupling matrix is not assumed to be symmetric or irreducible. Finally, simulations results show that the method proposed in this paper is effective.

Keywords: complex networks, hybrid synchronization, non-derivative and derivative coupling

1 Introduction

As we know, the problem of control and synchronization of complex dynamical networks has been extensively investigated in various fields of science and engineering due to its many potential practical applications[1, 5, 9, 13, 15, 16, 25]. During the last decades, control and synchronization of complex dynamical networks has become one of the most interesting subjects in complex dynamical networks theory, most synchronizing techniques, such as linear and nonlinear feedback synchronization[14, 20], impulsive synchronization[22, 26], adaptive synchronization[21, 24], lag synchronization[6, 23], cluster synchronization[2, 18] and so on, have been designed for complex dynamical networks.

In recent years, projective synchronization in complex dynamical networks has attracted a great deal of attention and some results have been reported on it[4, 17]. In projective synchronization of the complex dynamical networks, the scaling factor of the state vector of the drive system is taken as a scaling function. So projective synchronization is a more general type of complex network synchronization. It is the complexity and the unpredictability of the general scaling function to which the master and slave systems could be synchronized, that could be used to get secure communication in applications of the communication engineering. To get more secure communication, We can impose matrix transform on the state vector of the master and slave systems, the master and slave systems could be synchronized under the transform, this case would be

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called hybrid synchronization in this paper. In practice, as the hybrid of the measurable states, so the study of the hybrid synchronization is worth doing. In this paper, we investigate hybrid synchronization of the complex network model proposed in [19, 21].

This work is presented as follows: Section 2 describes model and preliminaries. Section 3 introduces investigates hybrid synchronization of the complex dynamical network with non-derivative and derivative coupling. Section 4 presents example and related simulation results. Section 5 gives the conclusion of the paper.

2 Model and preliminaries

In this section, we introduce the network model considered in this paper and give some useful mathematical preliminaries.

Consider a dynamical network consisting of identical coupled nodes, with each node being an n-dimensional dynamical system. The state equations of complex network with non-derivative and derivative coupling can be described by

$$\frac{\mathrm{d}x^i(t)}{\mathrm{d}t} = f(x^i(t),t) + \sum_{j=1}^{N} c_{ij}x^j(t) + \sum_{j=1}^{N} d_{ij}\dot{x}^j(t), \quad i = 1,2,\ldots,N, \tag{1}$$

where $x^i = (x_1^i, x_2^i, \ldots, x_n^i)^T \in R^n$ are the state variables of node i, $f : R^n \to R^n$ standing for the activity of an individual subsystem is a vector value function. $C = (c_{ij})_{N\times N} \in R^{N\times N}$ and $D = (d_{ij})_{N\times N} \in R^{N\times N}$ are the coupling matrix, c_{ij} and d_{ij} are defined as follows: $\sum\limits_{j=1}^{N} c_{ij} = 0$, $\sum\limits_{j=1}^{N} d_{ij} = 0$. We assume that the uncoupled system defined by $\dot{s}(t) = f(s(t),t)$ exists stable equilibrium point, stable periodic orbit, or chaotic attractor.

Remark 1. Taking into account the complexity of the network, a derivative coupling consists in providing more information about the dynamics in nodes to the other nodes in the network. Therefore, the linkage between the nodes in the network are composed by non-derivative and time derivative coupling.

We can impose matrix transform on the state vector of the master and slave systems, the master and slave systems could be synchronized under the transform, this case would be called hybrid synchronization in this paper.

Definition 1. *Let $H(x^i) = \Phi x^i$, the complex dynamical network Eq. (1) is said to achieve hybrid synchronization with respect to hybrid states $H(x^i(t))$ and $H(s(t))$, if*

$$\lim_{t\to\infty} \left\| H(x^i(t)) - H(s(t)) \right\| = 0, \quad i = 1,2,\ldots,N,$$

Remark 2. $\lim\limits_{t\to\infty} \left\| H(x^i(t)) - H(s(t)) \right\| = 0$is not equivalent to $\lim\limits_{t\to\infty} \left\| x^i(t) - s(t) \right\| = 0$. For example, let $\Phi = \begin{pmatrix} 1 & -1 & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & \ldots & 0 & 0 & 0 \\ & \ldots & & & \ldots & & \ldots & \\ 0 & 0 & 0 & 0 & \ldots & 0 & 1 & -1 \end{pmatrix}_{n\times n}$, when $x^i(t) - s(t) \to 1$, obviously, $x^i(t) - s(t)$ do not achieve synchronization, but the hybrid state $H(x^i(t)) - H(s(t)) \to 0$ achieve synchronization. When Φ is an identity matrix, then $\lim\limits_{t\to\infty} \left\| H(x^i(t)) - H(s(t)) \right\| = 0$ is equivalent to $\lim\limits_{t\to\infty} \left\| x^i(t) - s(t) \right\| = 0$, which is the kind of synchronization case discussed in Ref. [14, 21, 24].

Our objective is to design the controller $u^i(t)$ to achieve hybrid synchronization of the network Eq. (1).

In the paper, we have the following hypothesis:

Assumption 1. Suppose that there exist $l_1 > 0$, satisfying

$$\left\| f(x^i(t),t) - f_i(s(t),t) \right\| \leq l_i \left\| x^i(t) - s(t) \right\|,$$

where $x^i(t)$ and $s(t)$ are time varying vectors. In the following, the norm$\|\cdot\|$of vector x is defined as $\|x\| = (x^T x)^{\frac{1}{2}}$.

It has been verified that many typical benchmark chaotic systems such as the Lorenz system, Chen system, Lüsystem and the unified chaotic system satisfy Assumption 1.

3 Adaptive hybrid synchronization with non-derivative and time derivative coupling

In order to achieve hybrid synchronization, we introduce adaptive control strategy to the nodes in the network Eq. (1). So the controlled network is given by

$$\frac{dx^i(t)}{dt} = f(x^i(t),t) + \sum_{j=1}^{N} c_{ij} x^j(t) + \sum_{j=1}^{N} d_{ij} \dot{x}^j(t) + u^i(t). \tag{2}$$

Let

$$e^i(t) = x^i(t) - s(t) = (e_1^i, e_2^i, \ldots, e_n^i)^T, \quad i = 1, 2, \ldots, N.$$

So the error equation is

$$\frac{de^i(t)}{dt} = f(x^i(t),t) - f(s(t),t) + \sum_{j=1}^{N} c_{ij} x^j(t) + \sum_{j=1}^{N} d_{ij} \dot{x}^j(t) + u^i(t).$$

Define

$$E^i(t) = H(x^i(t)) - H(s(t)) = (E_1^i, E_2^i, \ldots, E_n^i)^T = \Phi e^i(t), \quad i = 1, 2, \ldots, N,$$

then

$$\dot{E}^i(t) = \Phi \dot{e}^i(t). \tag{3}$$

Remark 3. $\lim_{t\to\infty} \left\| H(x^i(t)) - H(s(t)) \right\| = 0$is equivalent to $\lim_{t\to\infty} \left\| E^i(t) \right\| = 0$.

Theorem 1. *Under Assumption 1. The controlled complex dynamical network Eq. (2) can be achieved hybrid synchronization by using the following update laws of the coupling strengths:*

$$u^i(t) = -\alpha_i(t) e^i(t) - \beta(t) \dot{e}^i(t), \quad \dot{\alpha}_i(t) = \theta_i (E^i(t))^T E^i(t), \quad \dot{\beta}_i(t) = \varphi_i (E^i(t))^T \dot{E}^i(t), \tag{4}$$

where θ_i, φ_i, $i \in \{1, 2, \ldots, N\}$ are any positive constants.

Proof. Since $\sum\limits_{j=1}^{N} c_{ij} = 0$, $\sum\limits_{j=1}^{N} d_{ij} = 0$, it is clear that

$$\sum_{j=1}^{N} c_{ij} x^j(t) = \sum_{j=1}^{N} c_{ij} x^j(t) - s(t) \sum_{j=1}^{N} c_{ij} = \sum_{j=1}^{N} c_{ij} (x^j(t) - s(t)) = \sum_{j=1}^{N} c_{ij} e^j(t),$$
$$\sum_{j=1}^{N} d_{ij} \dot{x}^j(t) = \sum_{j=1}^{N} d_{ij} \dot{x}^j(t) - \dot{s}(t) \sum_{j=1}^{N} d_{ij} = \sum_{j=1}^{N} d_{ij} (\dot{x}^j(t) - \dot{s}(t)) = \sum_{j=1}^{N} d_{ij} \dot{e}^j(t).$$

We choose a non-negative function as

$$V=\frac{1}{2}\sum_{i=1}^{N}(E^i(t))^TE^i(t)+\frac{1}{2}\sum_{i=1}^{N}\frac{1}{\theta_i}(\alpha_i(t)+L_i)^2+\frac{1}{2}\sum_{i=1}^{N}\frac{1}{\varphi_i}(\beta_i(t)-\sum_{j=1}^{N}d_{ij})^2, \tag{5}$$

where L_i is a sufficiently large positive constant which is to be determined.

Then the differentiation of V along the trajectories of Eq. (5) is that

$$\begin{aligned}
\dot{V}=&\sum_{i=1}^{N}(E^i(t))^T\dot{E}^i(t)+\sum_{i=1}^{N}\frac{1}{\theta_i}(\alpha_i(t)+L_i)\dot{\alpha}_i(t)+\sum_{i=1}^{N}\frac{1}{\varphi_i}(\beta_i(t)-\sum_{j=1}^{N}d_{ij})\dot{\beta}_i(t)\\
=&\sum_{i=1}^{N}(E^i(t))^T\Phi\dot{e}^i(t)+\sum_{i=1}^{N}\frac{1}{\theta_i}(\alpha_i(t)+L_i)\dot{\alpha}_i(t)+\sum_{i=1}^{N}\frac{1}{\varphi_i}(\beta_i(t)-\sum_{j=1}^{N}d_{ij})\dot{\beta}_i(t)\\
=&\sum_{i=1}^{N}(E^i(t))^T\Phi[f(x^i(t),t)-f(s(t),t)+\sum_{j=1}^{N}c_{ij}e^j(t)+\sum_{j=1}^{N}d_{ij}\dot{e}^j(t)+u^i(t)]+\sum_{i=1}^{N}\frac{1}{\theta_i}(\alpha_i(t)\\
&+L_i)\dot{\alpha}_i(t)+\sum_{i=1}^{N}\frac{1}{\varphi_i}(\beta_i(t)-\sum_{j=1}^{N}d_{ij})\dot{\beta}_i(t)\\
=&\sum_{i=1}^{N}(E^i(t))^T\Phi[f(x^i(t),t)-f(s(t),t)+\sum_{j=1}^{N}c_{ij}e^j(t)+\sum_{j=1}^{N}d_{ij}\dot{e}^j(t)-\alpha_i(t)e^i(t)-\\
&\beta_i(t)\dot{e}^i(t)]+\sum_{i=1}^{N}(\alpha_i(t)+L_i)(E^i(t))^TE^i(t)+\sum_{i=1}^{N}(\beta_i(t)-\sum_{j=1}^{N}d_{ij})(E^i(t))^T\dot{E}^i(t)\\
=&\sum_{i=1}^{N}(E^i(t))^T\Phi[f(x^i(t),t)-f(s(t),t)+\sum_{j=1}^{N}c_{ij}e^j(t)+\sum_{j=1}^{N}d_{ij}\dot{e}^j(t)]+\sum_{i=1}^{N}L_i(E^i(t))^TE^i(t)\\
&-\sum_{i=1}^{N}\sum_{j=1}^{N}d_{ij}(E^i(t))^T\dot{E}^i(t)\\
=&\sum_{i=1}^{N}(E^i(t))^T\Phi[f(x^i(t),t)-f(s(t),t)+\sum_{j=1}^{N}c_{ij}e^j(t)]+\sum_{i=1}^{N}(E^i(t))^T\Phi\sum_{j=1}^{N}d_{ij}\dot{e}^j(t)+\\
&\sum_{i=1}^{N}L_i(E^i(t))^TE^i(t)-\sum_{i=1}^{N}\sum_{j=1}^{N}d_{ij}(E^i(t))^T\dot{E}^i(t)\\
=&\sum_{i=1}^{N}(E^i(t))^T\Phi[f(x^i(t),t)-f(s(t),t)+\sum_{j=1}^{N}c_{ij}e^j(t)]+\sum_{i=1}^{N}(E^i(t))^T\sum_{j=1}^{N}d_{ij}\dot{E}^j(t)+\\
&\sum_{i=1}^{N}L_i(E^i(t))^TE^i(t)-\sum_{i=1}^{N}\sum_{j=1}^{N}d_{ij}(E^i(t))^T\dot{E}^i(t)\\
=&\sum_{i=1}^{N}(E^i(t))^T\Phi[f(x^i(t),t)-f(s(t),t)+\sum_{j=1}^{N}c_{ij}e^j(t)]+\sum_{i=1}^{N}\sum_{j=1}^{N}d_{ij}(E^i(t))^T\dot{E}^j(t)+\\
&\sum_{i=1}^{N}L_i(E^i(t))^TE^i(t)-\sum_{i=1}^{N}\sum_{j=1}^{N}d_{ij}(E^i(t))^T\dot{E}^j(t)\\
=&\sum_{i=1}^{N}(E^i(t))^T\Phi[f(x^i(t),t)-f(s(t),t)+\sum_{j=1}^{N}c_{ij}e^j(t)]+\sum_{i=1}^{N}L_i(E^i(t))^TE^i(t)
\end{aligned}$$

$$
\begin{aligned}
&\leq \sum_{i=1}^{N}(E^i(t))^T \Phi l_i e^i(t) + \sum_{i=1}^{N}(E^i(t))^T \Phi \sum_{j=1}^{N} c_{ij} e^j(t) + \sum_{i=1}^{N} L_i (E^i(t))^T E^i(t)\\
&= \sum_{i=1}^{N} l_i (E^i(t))^T E^i(t) + \sum_{i=1}^{N}(E^i(t))^T \sum_{j=1}^{N} c_{ij} \Phi e^j(t) + \sum_{i=1}^{N} L_i (E^i(t))^T E^i(t)\\
&= \sum_{i=1}^{N}(l_i + L_i)(E^i(t))^T E^i(t) + \sum_{i=1}^{N}\sum_{j=1}^{N}(E^i(t))^T c_{ij} E^j(t)\\
&= (E(t))^T Q E(t),
\end{aligned}
$$

where $E(t) = (E^1(t), E^2(t), \ldots, E^N(t))^T \in R^{nN}$, $Q = (P + P^T)/2$ and $P = (l_i + L_i)I_{nN} + C$, I_{nN} is an identity matrix with dimension nN. It is obvious that there exists a sufficiently large positive constant L_i such that the symmetry matrix Q is negative definite. So, the set

$$
\Omega = \{E^i(t) = 0, \alpha_i(t) = -l_i, \beta_i(t) = \sum_{j=1}^{N} d_{ij}, i = 1, 2, \ldots, N\}
$$

is the largest invariant set of the set $\Omega' = \{\dot{V} = 0\}$ for the error system Eq. (3). Then according to the LaSalle's invariance principle of differential equation, starting with arbitrary initial values of the augment system, the orbits converge asymptotically to the set Ω. It implies that its error system achieves hybrid synchronization by using updating law Eq. (4). This completes the proof.

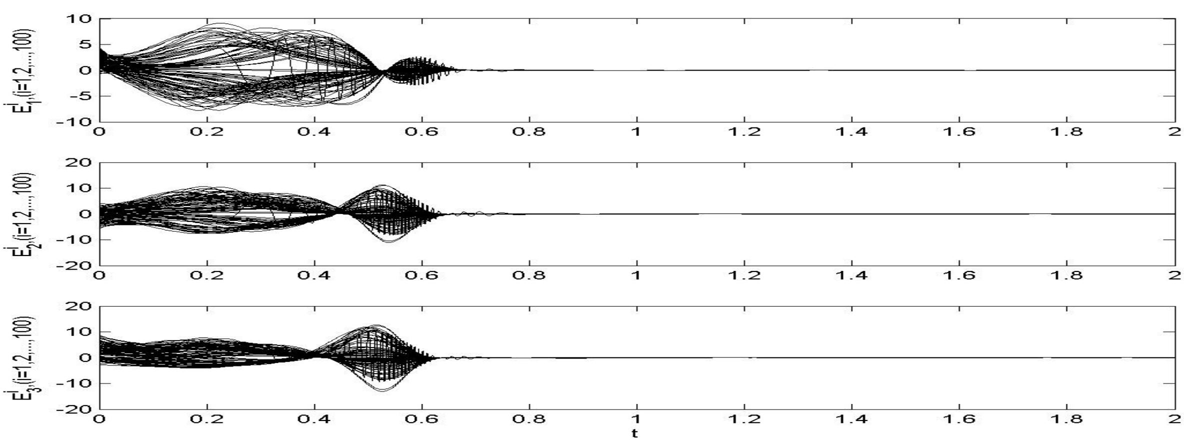

Fig. 1. When $\mu = 0$, the variance of the synchronization errors E^i

Remark 4. In our model, there exists non-derivative coupling and derivative, which is suitable to investigate and simulate more realistic complex networks.

Remark 5. It should be especially pointed out that the coupling configuration matrix doesn't need be symmetric, irreducible. To this end, this theorem is applicable to a great many complex dynamical networks.

Remark 6. The constant θ_i, φ_i can be chosen properly to adjust the synchronization speed. The larger the adaptive gains θ_i, φ_i, the faster is to achieve synchronization of networks.

If we let the coupling matrix $C = 0$ or $D = 0$, we will get two simple conditions:

$$\frac{\mathrm{d}x^i(t)}{\mathrm{d}t} = f(x^i(t), t) + \sum_{j=1}^{N} d_{ij}\dot{x}^j(t), \tag{6}$$

$$\frac{\mathrm{d}x^i(t)}{\mathrm{d}t} = f(x^i(t), t) + \sum_{j=1}^{N} c_{ij}x^j(t). \tag{7}$$

Then we have the following corollaries.

Corollary 1. *Under Assumption 1. The controlled complex dynamical network Eq. (6) can achieve hybrid synchronization by using the following update laws of the coupling strengths:*

$$u^i(t) = -\beta(t)\dot{e}^i(t), \quad \dot{\beta}_i(t) = \varphi_i(E^i(t))^T\dot{E}^i(t).$$

Corollary 2. *Under Assumption 1. The controlled complex dynamical network Eq. (7) can achieve hybrid synchronization by using the following update laws of the coupling strengths:*

$$u^i(t) = -\alpha_i(t)e^i(t), \quad \dot{\alpha}_i(t) = \theta_i(E^i(t))^T E^i(t). \tag{8}$$

The proofs of Corollaries 1, 2 follow directly from Theorem 1, thus we leave out their proofs here.

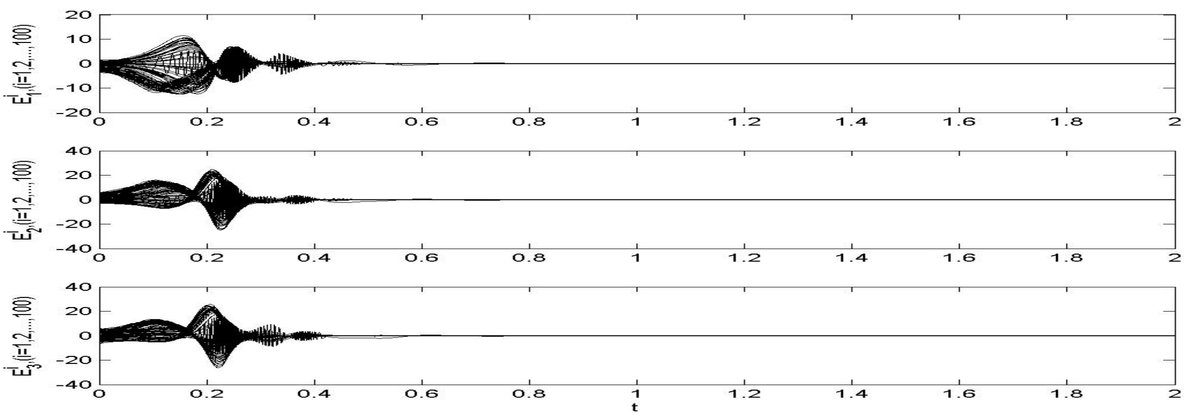

Fig. 2. When $\mu = 0$, the variance of the synchronization errors E^i

4 Illustrative example

In this section, we give an example to show the effectiveness of the method proposed in this paper. It is well known that the unified chaotic system is typical chaotic systems[7], which is described by

$$\dot{s} = \begin{pmatrix} -(25\mu+10) & 25\mu+10 & 0 \\ 28-35\mu & 29\mu-1 & 0 \\ 0 & 0 & -\frac{\mu+8}{3} \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \\ s_3 \end{pmatrix} + \begin{pmatrix} 0 \\ -s_1s_3 \\ s_1s_2 \end{pmatrix},$$

where $\mu \in [0, 1]$. When select $\mu = 0$, $\mu = 0.8$ and $\mu = 1$, respectively, the node dynamical system is Lorenz system, Lü system and Chen system[3, 10–12].

For any two state vectors y and z of the unified chaotic system, there exists a constant M satisfying $\|y_p\|, \|z_p\| \leq M$ for $1 \leq p \leq 3$ since the unified chaotic system is bounded by certain region[8], respectively. Therefore, one has

$$\|W(y) - W(z)\| \leq \sqrt{(-y_3(y_1 - z_1) - z_1(y_3 - z_3))^2 + (y_2(y_1 - z_1) + z_1(y_2 - z_2))^2} \leq 2M\|y - z\|.$$

Thus the unified chaotic system satisfy Assumption 1.

We let $\Phi = \begin{pmatrix} 1 & -2 & 0 \\ 0 & 2 & 3 \\ 3 & 0 & 4 \end{pmatrix}$. According to Section 3, we show that a non-derivative and derivative coupling network with 100 nodes described by

$$\begin{cases} \frac{dx^i(t)}{dt} = f(x^i(t),t) + \sum_{j=1}^{100} c_{ij}x^j(t) + \sum_{j=1}^{100} d_{ij}\dot{x}^j(t) - \alpha_i(t)e^i(t) - \beta_i(t)\dot{e}^i(t) \\ \dot{\alpha}_i(t) = \theta_i(E^i(t))^T E^i(t) \\ \dot{\beta}_i(t) = \varphi_i(E^i(t))^T \dot{E}^i(t) \end{cases} \qquad (i = 1, 2, \ldots, 100)$$

where

$$C = \begin{pmatrix} -0.2 & 0.2 & 0 & \cdots & \\ 0.2 & -0.4 & 0.2 & 0 & \cdots \\ & & \cdots & & \\ \cdots & 0 & 0.2 & -0.4 & 0.2 \\ & \cdots & 0 & 0.2 & -0.2 \end{pmatrix}_{100\times100}, \quad D = \begin{pmatrix} -0.5 & 0.5 & 0 & \cdots & \\ 0.5 & -1 & 0.5 & 0 & \cdots \\ & & \cdots & & \\ \cdots & 0 & 0.5 & -1 & 0.5 \\ & \cdots & 0 & 0.5 & -0.5 \end{pmatrix}_{100\times100}.$$

In the numerical simulation, the parameters are given as follows: $\theta_i = 1$, $\varphi_i = 1$. All initial values are $Rand$, where $Rand \in (0,1)$ is a random number. Fig. 1 shows the variance of the synchronization errors for the parameter $\mu = 0$. Fig. 2 shows the variance of the synchronization errors for the parameter $\mu = 0.8$. Fig. 3 shows the variance of the synchronization errors for the parameter $\mu = 1$.

Numerical simulations of Corollary 1 and Corollary 2 can be illustrated in a similar way as shown in Theorem 1, thus we leave out numerical simulations here.

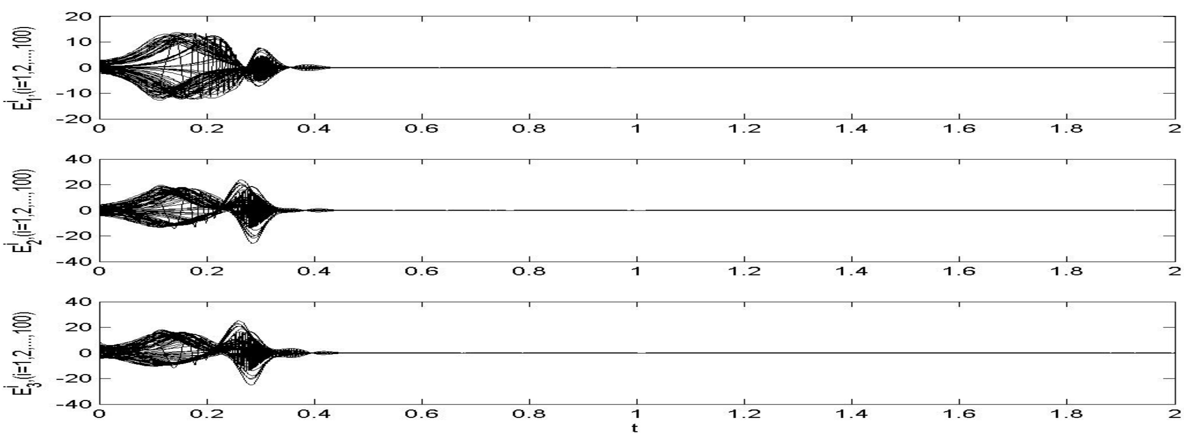

Fig. 3. When $\mu = 1$, the variance of the synchronization errors E^i

5 Conclusion

This paper handles the problem of adaptive hybrid synchronization of a general complex dynamical network with non-derivative and derivative coupling. Making use of Lasalle's invariance principle, adaptive synchronization criteria is obtained. Analytical result shows that under the designed adaptive controllers, a general complex dynamical network with non-derivative and derivative coupling can asymptotically hybrid synchronous to a given trajectory, and several useful criteria for synchronization are given. What is more, the coupling matrix is not assumed to be symmetric or irreducible. Finally, numerical simulations are then given to verify the effectiveness of the proposed adaptive schemes.

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