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Planning and operation of active radial distribution networks for improved voltage stability and loss reduction

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Abstract. The impending deregulated environment facing the electric utilities in the twenty first century is both a challenge and an opportunity for a variety of technologies and operating scenarios. Changing regulatory and economic scenarios, energy savings and environmental impacts are providing impetus to the development of an Active Distribution Networks (ADN), which is predicated to play an increasing role in the electric power system in the near future. Connecting Distributed Generator (DG) to a passive distribution network becomes an active distribution network. Distributed Generators effectively reduce the real power losses and improve the voltage profile in Radial Distribution Networks (RDN). This article proposes a novel methodology for finding the optimal size and location for installation of DG so as to minimize total power loss in radial distribution system. DG unit placement and sizing were calculated using fuzzy logic and new analytical method respectively. A detailed performance analysis has been carried out on IEEE 33-bus and IEEE 69-bus radial distribution networks to demonstrate the effectiveness of the proposed methodology. Effect of optimal DG placement on system voltage profile and branch power losses are also computed and reported.

Keywords: distributed generation, radial distribution network, fuzzy, voltage stability

1 Introduction

Utilities are continuously planning the expansion of their existing electrical networks in order to face the load growth and to properly supply their consumers. Distribution system provides a final link between the high voltage transmission system and the consumer. The direction of power flow in the distribution network was almost always from higher to the low voltage levels. This system architecture was a technical and economic choice, and with a new technology and changed economic and commercial environment, the power system is now beginning to be modified by the reintroduction of generation connected the distribution networks[18]. The power loss is significantly high in distribution systems because of lower voltages and higher currents, when compared to that in high voltage transmission systems. Studies have indicated that as much as 13% of total power generated is consumed as I^2R losses in distribution level. Reactive currents account for a portion of these losses. Reduction of total loss in distribution systems is very essential to improve the overall efficiency of power delivery. The pressure of improving the overall efficiency of power delivery has forced the power utilities to reduce the loss, especially at distribution level[21]. Electricity networks are in the era of major transition from stable passive distribution networks with unidirectional electricity transportation to active distribution networks with bidirectional electricity transportation. Distribution networks without any DG units are passive since the electrical power is supplied by the national grid system to the customers embedded in the distribution networks. It becomes active when DG units are added to the distribution system leading to bidirectional power flows in the networks[6]. In an active distribution network the amount of energy lost in transmitting electricity is less as compared to the passive distribution network, because the electricity is generated very

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near the load centre, perhaps even in the same building. DG plays an important role in the active distribution networks. The term DG also implies the use of any modular technology that is sited throughout a utility's service area to lower the cost of service[7]. The DG benefits are numerous and the reasons for implementing DGs are energy efficiency or rational use of energy, deregulation or competition policy, diversification of energy sources, availability of modular generating plant, the ease of finding sites for smaller plants and proximity of the generation plant to heavy loads, which reduces transmission costs[5]. Also it is accepted by many countries that the reduction in gaseous emissions offered by DGs is major legal driver for DG implementation[1]. Hence, utilities and distribution companies need tools for proper planning and operation of Active Distribution Networks. The most important benefits are reduction of line losses and voltage stability improvement. They are crucially important to determine the size and location of DG unit to be placed. There have been number of studies to define the optimal sizing and placement of DG units. In [28], an analytical approach has been presented to identify appropriate location to place single DG in radial as well as loop systems to minimize losses. But, in this approach, optimal sizing is not considered. In [9] optimal placement of DG units is determined exclusively for the various distributed load profiles to minimize the total losses. They have iteratively increased the size of DG unit at all buses and then calculated the losses; based on loss calculation they ranked the nodes. Top ranked nodes are selected for DG unit placement. The Genetic Algorithm (G. A) based method to determine size and location of DG unit is used in [15]. They have addressed the problem in terms of cost, considering cost function may lead to deviation of exact size of the DG unit at suitable location, but they are computationally demanding and slow in convergence. Particle Swam Optimization (PSO) has been applied to DG placement for either, sizing or location of DG's[13]. In [2] the optimal size and location of DG is calculated based on exact loss formula and compared with successive load flows and loss sensitivity methods. The method is computationally less demanding for radial and networked systems, however, it requires the calculation of the bus impedance matrix, Z bus, the inverse of the bus admittance matrix, Y bus. It should be noted that due to size, complexity and specific characteristics of distribution systems, the method could not be directly applied to distribution systems. It fails to meet the requirements in robustness aspects in the distribution system environment[24]. This paper proposes a combined method of minimizing the loss associated with the active and reactive component of branch currents by placing optimal DG units at proper location. The method first finds the location of the DG unit using fuzzy logic. Once the DG unit locations are identified, the optimal DG unit size at each location is determined through optimizing the loss saving equation. The proposed method was tested on two different distribution systems. The methodology is computationally very demanding and simple to implement.

This paper is structured as follows: Various load flow methods for distribution systems are briefly explained in Section 2. Section 3 presents theoretical and mathematical back ground of the article, methodology to assess the optimal location and identification of optimal size of DG. Section 4 is explained steps followed in algorithm. Finally, results of the study and conclusions are presented in Section 5 and 6, respectively.

2 Load flow study

Conventional Newton_Raphson and Gauss_Seidel methods may become inefficient in the analysis of distribution systems, due to the special features of distribution networks, that is, radial structure, high R/X ratio and unbalanced loads, etc. These characteristic features make the distribution systems power flow computation different and somewhat difficult to analyze as compared to the transmission systems when the conventional power flow algorithms are employed[5]. Various methods are available in the literature to carry out the analysis of balanced and unbalanced radial distribution systems[1, 2, 9, 13, 15, 20, 22, 24–29]. Methods developed for the solution of ill-conditioned radial distribution systems may be divided into two categories. The first type of methods [1, 2, 9, 13, 15, 20, 22, 24–29] is utilized by proper modification of existing methods such as, Newton_Raphson and Gauss_Seidel. On the other hand, the second group of methods [3, 4, 8, 10–12, 14, 16, 17, 19, 23] is based on forward and/or backward sweep processes using Kirchhoff's Laws or making use of the well-known bi-quadratic equation. Due to its low memory requirements, computational efficiency and robust convergence characteristic, forward/backward sweep based algorithms have gained the most popularity for distribution systems load flow analysis. In the present study, network topology based forward/backward sweep algorithms have been used for load flow analysis.

Nomenclature
P_{TL} : total power loss;
P_{La} : total power loss due to active component of current;
P_{Lr} : total power loss due to reactive component of current;
P_{La}^{com} : total power loss due to active component of current in the compensated system;
I_a : active branch current Ir reactive branch current;
R : resistance of the branch;
X : reactance of the branch;
A : ampere;
MW : mega watt;
$Mvar$: mega volt ampere reactive.
Abbreviations
ADN : active distribution network;
DG : distributed generator;
RDN : radial distribution network;
T&D : transmission and distribution;
G.A : genetic algorithm;
WODG : without distributed generator;
WDG : with distributed generator;
FIS : fuzzy inference system ;
VSI : voltage stability index;
PLI : power loss index ;
DGSI : distributed generator suitability index;
Δ : small change in variable;
L : low ;
LM : low medium ;
M : medium ;
HM : high medium ;
H : high.
Subscript
i : node.

3 Back ground

The total I^2R loss (P_{TL}) in a distribution system having n number of branches is given by

$$P_{TL} = \sum_{i=1}^{n} I_i^2 R_i, \tag{1}$$

Here I_i and R_i are the current magnitude and resistance, respectively, of the ith branch. The branch current can be obtained from the load flow study. The load flow based on forward/backward sweep algorithm has been used for this purpose. The branch current has two components; active Ia and reactive component Ir. The total loss associated with the active and reactive components of branch currents can be written as

$$P_{TL} = P_{La} + P_{Lr}, \tag{2}$$

$$P_{TL} = \sum_{i=1}^{n} I_{ai}^2 R_i + \sum_{i=1}^{n} I_{ri}^2 R_i. \tag{3}$$

Note that for a given configuration of single source radial networks, the loss P_{La} associated with the active component of branch currents cannot be minimized because all active power must be supplied by the source at the root bus. However, by placing DGs, the active component of branch current is reduced. This paper presents a method that minimizes the loss due to the active component of the branch current by optimally placing the DGs and thereby reduces the total loss in the distribution system. A two stage methodology is applied here. In the first stage optimum location of the DGs are determined by using fuzzy approach and in the second stage an analytical method is to determine sizes of the DGs for maximum real power loss reduction.

3.1 Identification of optimal dg location—fuzzy approach

There are many uncertainties in various power system problems. Because of this it becomes very difficult to stick to mathematical formulae alone. To overcome this, fuzzy set theory has been applied to many power system related problems. Two objectives are considered while designing a fuzzy logic for identifying the optimal DG locations. They are (i) to minimize the real power loss and (ii) to maintain voltage within permissible limits. Voltage stability and power loss indices of distribution system are modeled by fuzzy membership functions. A fuzzy inference system (FIS) containing a set of rules is then used to determine the DG placement suitability of each node in the distribution system. The DG unit can be placed on the nodes with highest suitability index. A set of fuzzy rules has been used to determine suitable locations in a distribution system. Two input and one-output variables are selected. Input variable-1 is Voltage Stability Index (VSI) and Input variable-2 is the Power Loss Index (PLI). Output variable is DG suitability index (DGSI). Five membership functions are selected for VSI. They are L, LN, N, HN and H. These membership functions are trapezoidal and triangular as shown in Fig. 1. Five membership functions are selected for PLI. They are L, LM, M, HM and H. All the five membership functions are triangular as shown in Fig. 2. Five membership functions are selected for DGSI. They are L, LM, M, HM and H. These functions are also triangular as shown in Fig. 3. For the DG allocation problem, rules are defined to determine the suitability of a node for DG installation. Such rules are expressed in the following manner: IF premise (antecedent), THEN conclusion (consequent). For determining the suitability of DG placement at a particular node, a set of multiple antecedent fuzzy rules have been established. The inputs to the rules are the voltage and Loss indices and the output is the suitability of DG placement. FIS editor receives inputs from the load flow program. Several rules may fire with some degree of memberships. FIS is based on Mamdani max-min and max-prod implication methods of inference. These methods determine the aggregated output from the set of triggered rules. The max-min method involves truncating the consequent membership function of each fired rule at the minimum membership value of all the antecedents. A final aggregated membership function is achieved by taking the union of all truncated consequent membership functions of the fired rules. After calculating the suitability membership function, it is to be de-fuzzified using the centroid method to determine the optimal DG location[17].

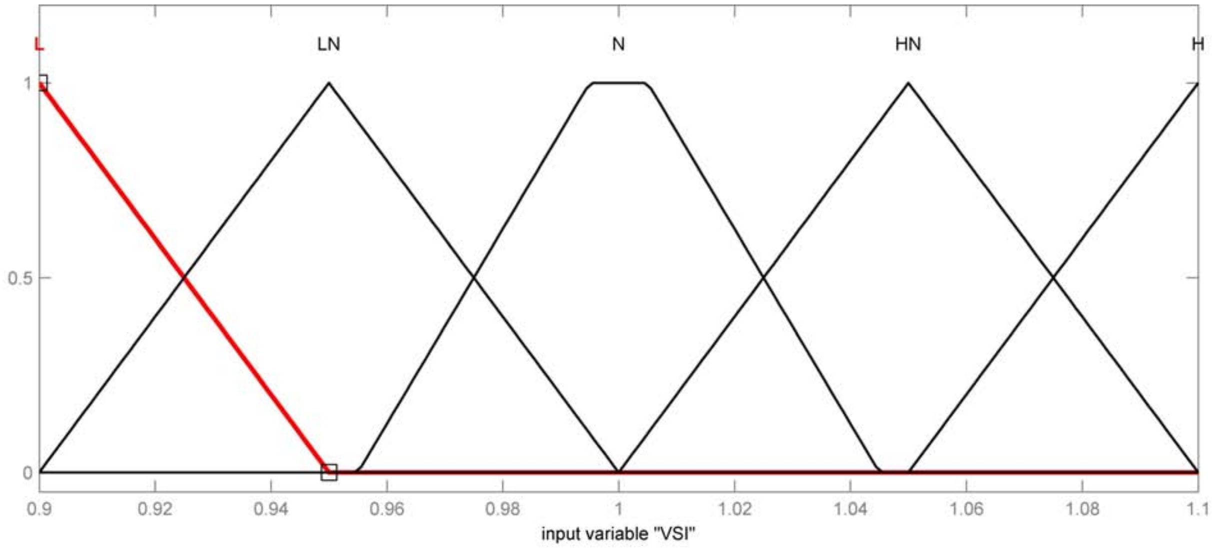

Fig. 1. Membership function plot for VSI

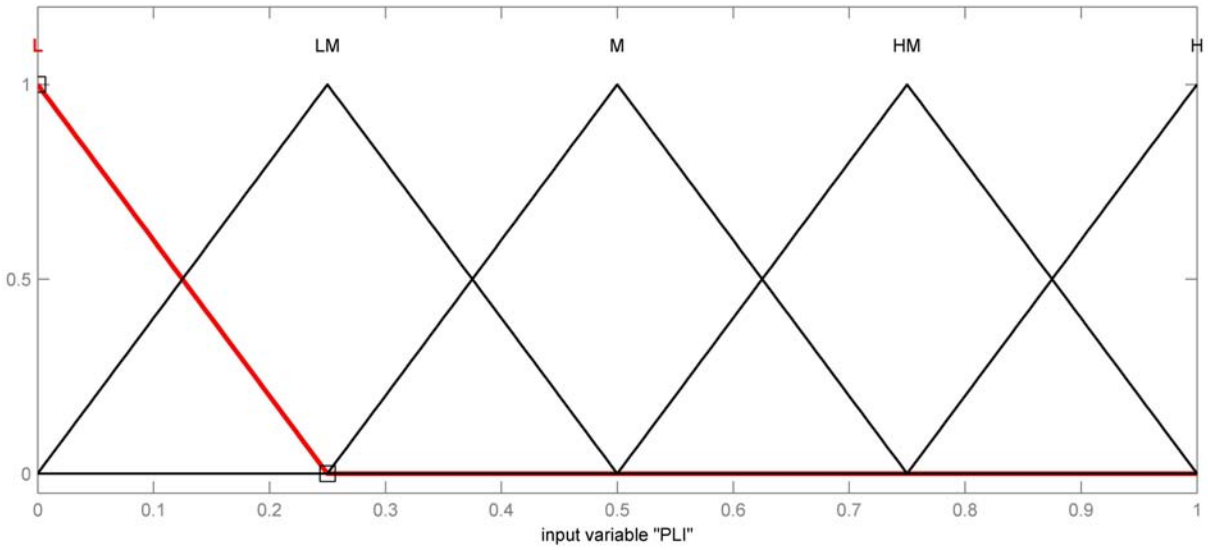

Fig. 2. Membership function plot for PLI

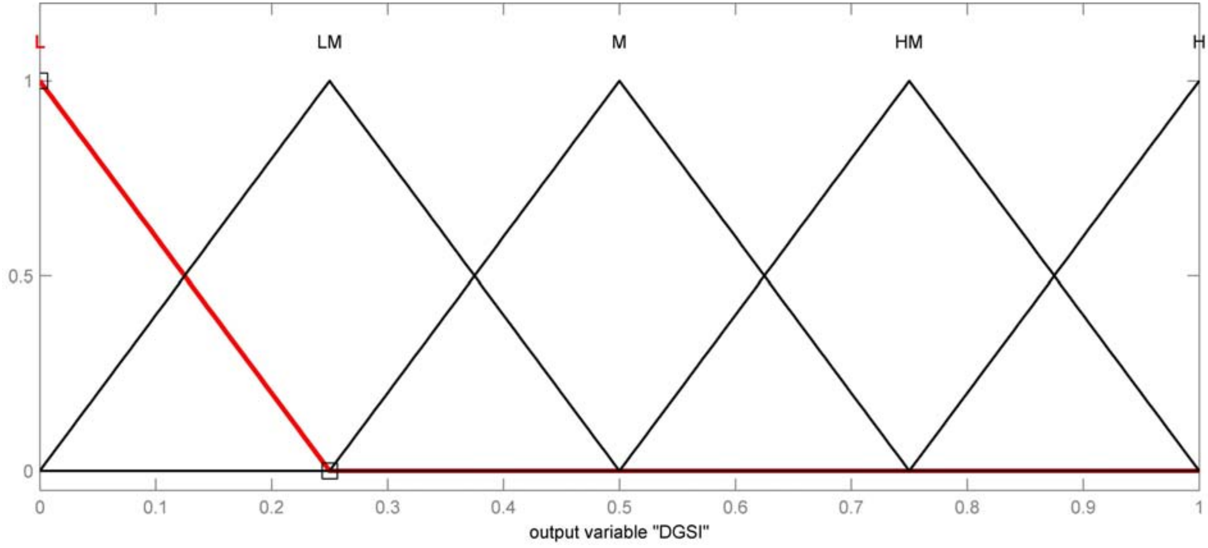

Fig. 3. Membership function plot for DGSI

3.2 Identification of optimal dg size

Consider a single source radial distribution system with n branches. Let a DG be placed at bus m and β be a set of branches connected between the source and DG (bus m). The DG unit supply active current IDG, and for a radial system it changes only the active component of current of branch set β. The current of other branches are unaffected by the DG. Thus the new active current new I_{ai} of the ith branch is given by

$$I_{ai}^{new} = I_{ai} + DG_i I_{DG}, \quad DG_i = 1 \text{ if } i \in \beta \text{ otherwise } 0. \tag{4}$$

I_{ai} is the active component of current of ith branch in the original system obtained from the load flow solution. The loss P_{La}^{com} associated with the active component of branch currents in the compensated system. For a DG unit (upf) placed at node k then the system loss can be written as

$$P_{La}^{com} = \sum_{i=1}^{k} (I_{ai} + DG_i I_{DG})^2 R_i + \sum_{k+1}^{n} I_{ai}^2 R_i + \sum_{i=1}^{n} I_{ri}^2 R_i, \quad I_{DG} = DG \text{ unit current.} \tag{5}$$

Subtracting Eq. (5) from Eq. (3), loss reduction due to the introduction of DG unit at node k is obtained.

$$\Delta P_k = -2I_{DG}\sum_{i=1}^{n} DG_i I_{ai} R_i - I_{DG}^2 \sum_{i=1}^{k} DG_i R_i. \tag{6}$$

Assuming no significant change in the node voltage after placing the DG unit power that can be generated at unity power factor is

$$P_{DG} = I_{DG} V_k. \tag{7}$$

The idea is to place a DG unit (upf) with a size and at a location such that the system loss reduction of maximized using Eq. (6). For system loss reduction to be maximized, the DG unit is to be placed at node k.

$$\frac{\partial P_k}{\partial I_{DG}} = 0, \tag{8}$$

$$I_{DG} = -\frac{\sum\limits_{i=1}^{k} DG_i I_{ai} R_i}{\sum\limits_{i=1}^{k} DG_i R_i}, \quad I_{DG} = \frac{\sum\limits_{i\in\beta} I_{ai} R_i}{\sum\limits_{i\in\beta} R_i}. \tag{9}$$

By substuting Eq. (9) in Eq. (7).

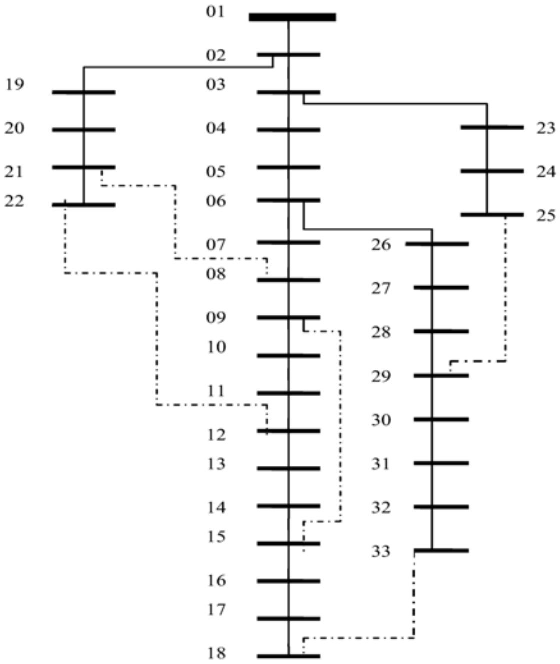

Fig. 4. 33-bus radial distribution test system

The expression for maximum loss reduction

$$\Delta P_{\max} = \frac{(\sum\limits_{i=1}^{k} DG_i I_{ai} R_i)^2}{\sum\limits_{i=1}^{k} DG_i R_i}. \tag{10}$$

It is noted that the loss reduction is always positive. Mathematical formulae for identification of DG unit size at any power factor are given in Appendix B. This process can be repeated for all the buses to get the highest possible loss saving for a singly located DG unit. When the candidate location is identified and DG is placed,

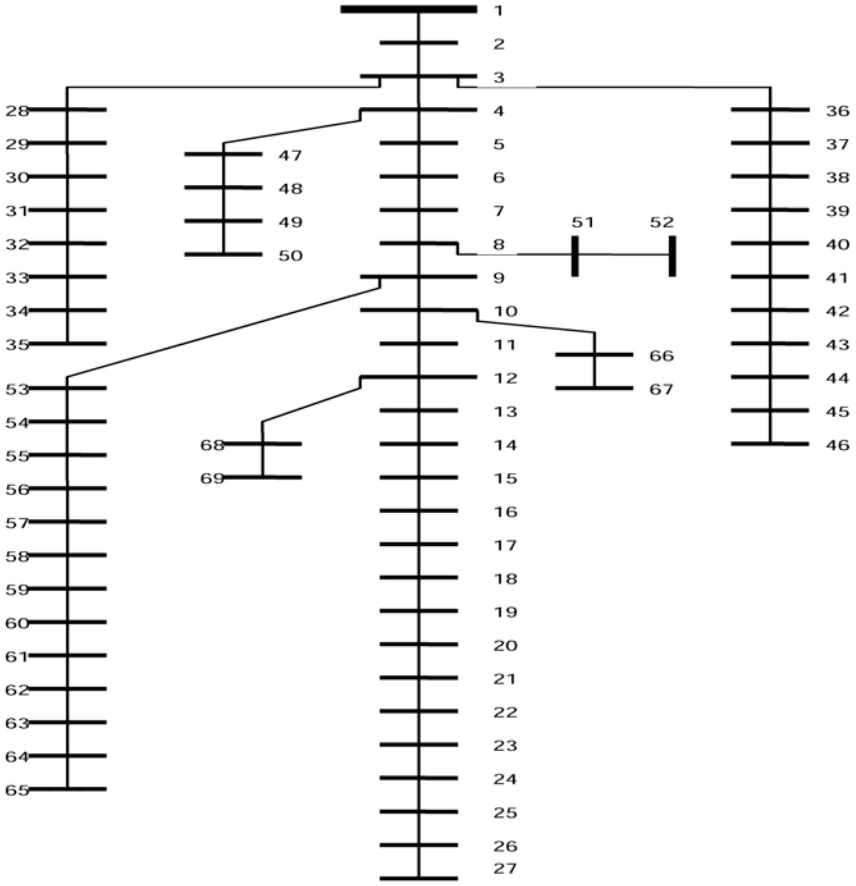

Fig. 5. 69-bus radial distribution test system

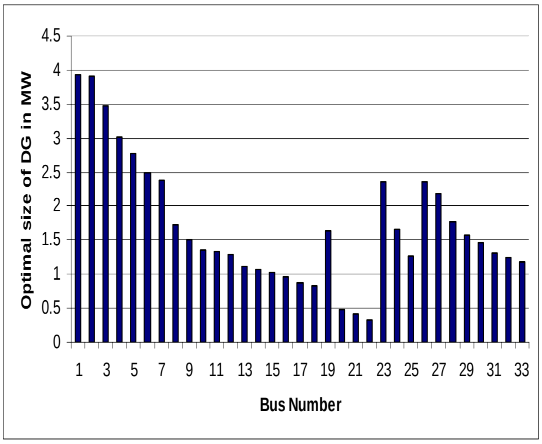

Fig. 6. Optimal DG unit sizes for 33-bus radial distribution system

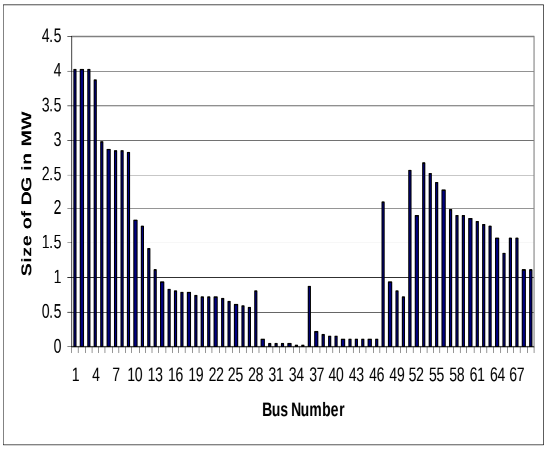

Fig. 7. Optimal DG unit sizes for 69-bus radial distribution system

the above technique can also be used to identify the next and consequent bus to be compensated for loss reduction.

With introduction of DG unit at any power factor

$$P^{com} = \sum_{i=1}^{n} (I_i + DG_i I_{DG})^2 R_i + \sum_{i=1}^{n} I_{ri}^2 R_i. \tag{11}$$

Substituting Eq. (11) from Eq. (3)

Table 1. PLI, VSI and DGSI ranges and their membership functions

Type of Indices	L	LM	M	HM	H
PLI	<0.25	0.0-0.5	0.25-0.75	0.5-1.0	0.75-1.0
VSI	<0.95	0.9-1.0	0.90-1.04	1.0-1.5	1.05-1.1
DGSI	<0.25	0.0-0.5	0.25-0.75	0.5-1.0	≥ 0.75

Table 2. Summary of critical bus analysis and comparison of active branch currents with and without DG for test systems

System Type	Bus number corresponding to lowest voltage	Voltage in p.u./VSI		Active branch current in branch1 in (A)	
		W.O.D.G	W.D.G	W.O.D.G	W.D.G
33-bus	18	0.9037/0.6672	0.9423/0.7886	310.11	157.25
69-bus	61	0.9091/0.6833	0.9817/0.9288	318.09	159.18

$$\varDelta P_k = -2I_{DG}\sum_{i=1}^{k} DG_i I_i R_i - I_{DG}^2 \sum_{i=1}^{k} DG_i R_i, \quad \frac{\partial P_k}{\partial I_{DG}} = 0,$$

$$S_{DG} = I_{DG}V_k, \quad I_{DG} = -\frac{\sum\limits_{i=1}^{k} DG_i I_i R_i}{\sum\limits_{i=1}^{k} DG_i R_i}. \tag{12}$$

The expression for maximum loss reduction is

$$\varDelta S_{\max} = \frac{(\sum\limits_{i=1}^{k} DG_i I_i R_i)^2}{\sum\limits_{i=1}^{k} DG_i R_i}.$$

4 Steps followed in algorithm

There are many computational steps involved in finding the optimal DG size and location to minimize losses in a radial distribution system. They are

Step 1. Conduct load flow program for the original system.

Step 2. Select the bus where the maximum loss and low voltage is obtained using fuzzy logic tool box.

Step 3. Corresponding DG size is calculated using Eq. (7) operated at unity power factor. And Eq. (12) for DG operated at any power factor respectively.

Step 4. Repeat this form $i = 2$ for all buses except source bus. Identify the bus using fuzzy logic that provides highest loss saving.

Step 5. Compensate the bus with the highest loss with the corresponding DG unit found form Eq. (7) and Eq. (12) respectively.

Step 6. Repeat the Step 1 to Step 5 to get the next DG size and hence sequence of buses to be compensated.

Step 7. Once the sequence of buses is known determine the optimum DG unit sizes and the corresponding loss saving.

Since the system load is time variant and load duration curve of the system can be approximated. It is assumed that load level is constant. The above algorithm provides the optimal DG sizes and locations for a given load level for DG operated at unity power factor and DG operated at any power factor.

5 Results

The proposed method is tested on two different test systems of different sizes, to show that it can be implemented in distribution systems of various configurations and sizes. The first test system is 33-bus system,

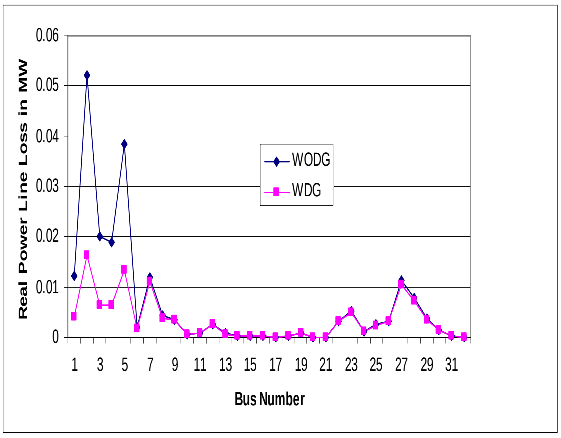

Fig. 8. Real power line losses of 33-bus radial distribution system with and without DG

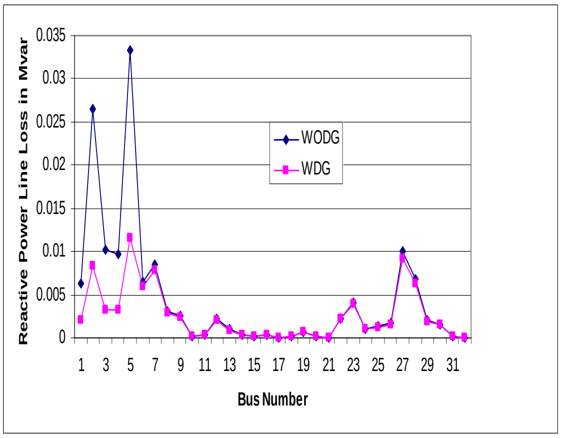

Fig. 9. Reactive power line losses of 33-bus radial distribution system with and without DG

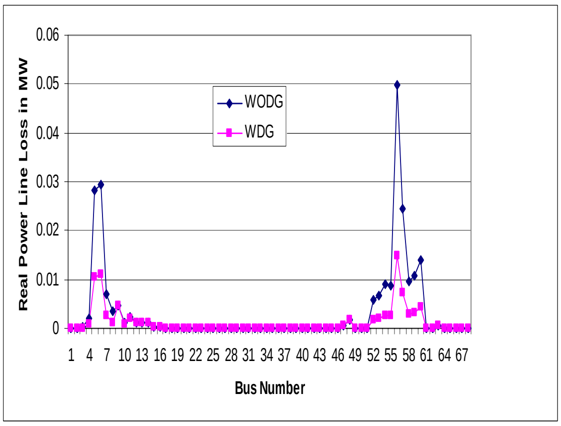

Fig. 10. Real power line losses of 69-bus radial distribution system with and without DG

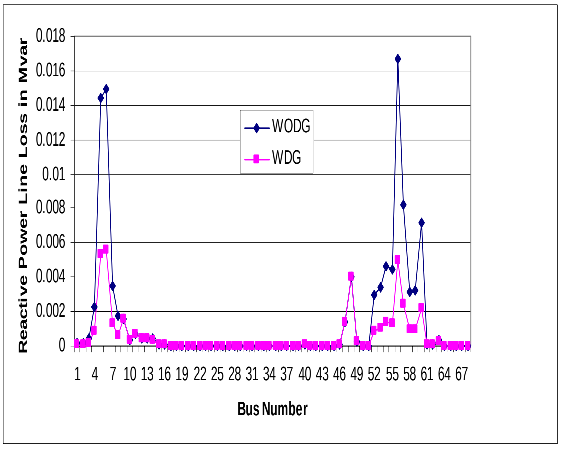

Fig. 11. Reactive power line losses of 69-bus radial distribution system with and without DG

Table 3. Summary of proposed method for test systems

System Type	33-bus radial distribution system	69-bus radial distribution system
Optimal location of DG	6-bus	61-bus
Optimal DG size in MW	2.59	1.87
Total real power loss in MW(WODG)	0.211	0.225
Total reactive power loss in MVar (WODG)	0.143	0.1021
Total real power loss in MW (WDG)	0.111	0.0832
Total reactive power loss in MVar (WDG)	0.0816	0.0405
Real power loss reduction in %	52.6	57.1
Reactive power loss reduction in %	36.9	39.4

as shown in Fig. 4. The total real power and reactive power load on this system is 3.72 MW and 2.3 Mvar. The initial real and reactive power loss in the system is 0.211 MW and 0.143 Mvar[23].

The second test system is a 69-bus system, as depicted in Fig. 5. The total real power and reactive power loads on this system is 3.80 MW and 2.69 Mvar. The initial real and reactive power loss in the system is 0.225 MW and 0.102 Mvar[12]. Optimal sizes of Dg units are calculated at each bus with DG unit at unity power factor and it may also calculate with DG unit at any power factor using Appendix. Optimal location is obtained using FIS editor. DG unit suitability index is greater than 0.8 are selected to place the DG unit. From FIS editor it is found that bus6 and bus 61 are suitable locations for 33 and 69 bus distribution test systems respectively. The ranges of power loss index and voltage stability index and DG unit suitability index

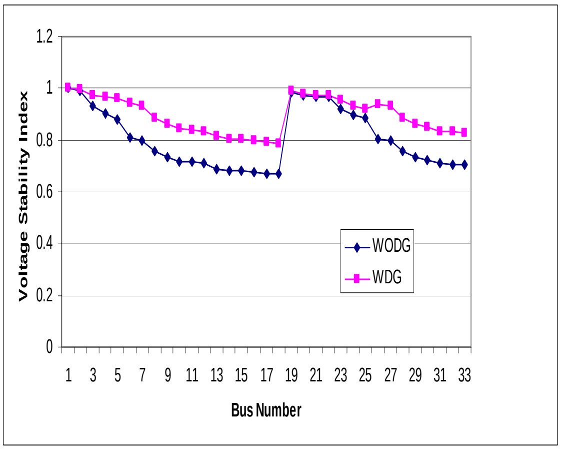

Fig. 12. Voltage stability index of 33-bus radial distribution system with and without DG

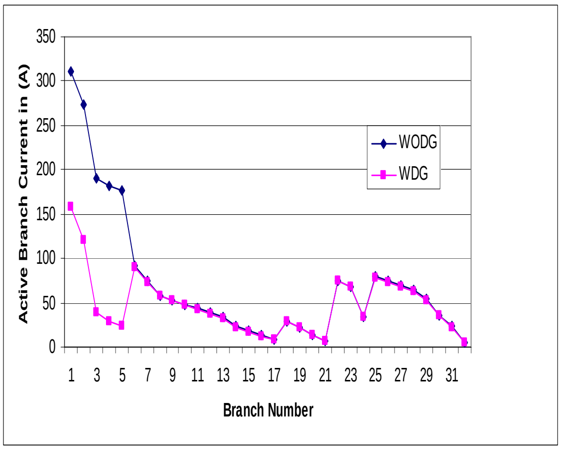

Fig. 13. Active branch currents with and without DG in 33-bus radial distribution system

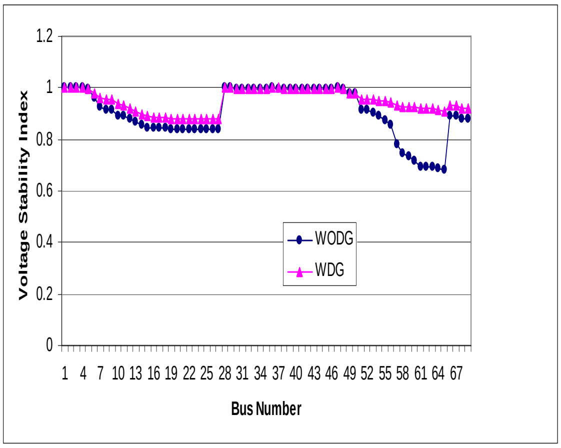

Fig. 14. Voltage stability index of 69-bus radial distribution system with and without DG

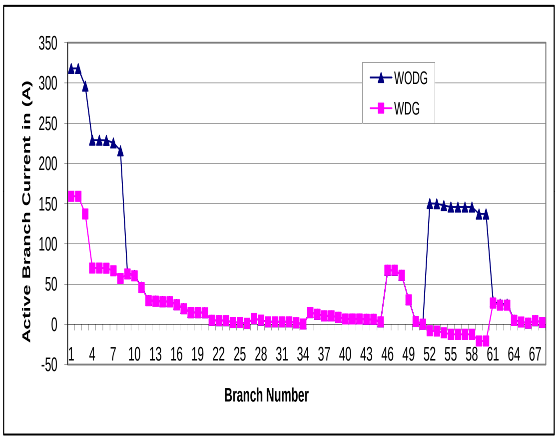

Fig. 15. Active branch currents with and without DG in 69-bus radial distribution system

and their membership function details are given in Tab. 1. With the help of FIS editor optimal location of DG unit is found where real power loss is more and voltage is low. Based on the method described previously, the optimal sizes of DG are calculated at all buses for the two test systems Fig. 6 and Fig. 7 respectively. Voltage profile improvement is observed where total power losses attain the minimum value.

The real and reactive power loss for the corresponding optimal DG unit sizes are shown in Fig. 8, Fig. 9, Fig. 10 and Fig. 11. Improvement in voltage stability and reduction in active branch currents are observed from Fig. 12, Fig. 13, Fig. 14 and Fig. 15 for 33 and 69 bus distribution test systems respectively. Results obtained are summarized in Tab. 2. In Tab. 3, for two different test system, the results of proposed method, total power losses with and without DG are presented. It is observed that the total power losses are significantly reduced due to reduction in branch currents for both test systems.

6 Conclusion

The size and location of DG unit are crucial factors in the planning and operation of active radial distribution networks. This study presents a new deterministic methodology which determines the optimal size and placement of DG. This method is easy to be implement and faster for the given accuracy. It is proved that the proposed method can saves huge amount of power and achieve significant improvement in voltage stability. Installation of DG unit at one location at a time is proved to be a valid assumption in the present study. However, this paper does not consider load and demand side management and the other benefits of DG as well as economics of it. Inclusion of the real time constraints such as time varying loads and different types of DGs and discrete DG sizes into the proposed method is the future scope of the work.

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