ISSN 1 746-7233, England, UK
World Journal of Modelling and Simulation
Vol. 8 (2012) No. 2, pp. 147-153

Application of the FRF curvature energy damage detection method to plate like structures

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(*Received August 26 2010, Accepted July 12 2011*)

Abstract. The objective of current work is to show the effectiveness of using frequency response function (FRF) curvature energy damage index and to establish its capability to detect and localize damage. The dimension of aluminium plate under consideration is 0.25m×0.25m×0.003m with fixed-fixed support condition Damage is simulated by reducing the thickness of one element for different damage cases. The mode shape and FRF energy data of the square plate with damage of different sizes are obtained by free vibration analysis using MATLAB 7.0. Testing the frequency intervals, it is found that the FRF curvature energy damage index method defined in the range of frequencies include the eigen frequencies. The damage index is found to be a function of the frequency bandwidth and variation of FRF curvature energy damage index versus frequency range (band width) seems to provide further information in choice of optimum frequency range response analysis. This typical analysis presented wider frequency ranges, including several higher modes, the difference of curvature energies of the damaged and undamaged model becomes less significant. The influence of noise in the FRF data seems to be quite small. It also found that influence of excitation location is not important for the damage detection. It is observed that by using FRF curvature energy damage index data, damage location can be identified provided the reduction in elemental thickness is more than 10%.

Keywords: structural health monitoring, modal analysis, damage detection, FRF curvature energy damage index method

1 Introduction

Significant work has been done in the area of detecting damage in structures using changes in dynamic response of the structure. Because the natural frequencies and mode shapes of a structure are dependent on the mass and stiffness distribution, any subsequent changes in them should, theoretically, be reflected in changes in the frequencies and mode shapes and their sensitivities to damage level. Consequently, structural safety and functionally will be significantly improved and a condition-based maintenance procedure can be developed.

More recently resonant methods based on modal data have been used both to identify that damage exists, and to locate it. Some techniques, such as those treat frame work as discrete systems, and compare the modal behavior of the damaged structure with that of the undamaged structure[1]. Experimental modal analysis proposed in [5] referred to here as modal testing. Approach for not only detecting but also locating a structural fault[7] and also presented a method to determine mass, stiffness and damping properties from measured frequencies response functions and showed how changes in those parameters could be used to locate the fault. Presented a method based on the decreases in modal strain energy between two structural degrees of freedom as defined by the curvature of the measured mode shapes. This method as been successful applied to data from a damaged bridge[12]. The absolute changes in mode shape curvature can be a good indicator of damage for the FEM beam structure they considered[9]. The displacement functions converted into curvature function, which are further processed to yield a damage index.

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A comprehensive literature review of damage detection methods using vibration signals for structural and mechanical systems is provided[4]. Later in place of using the displacement mode shapes. Strain or curvature shapes (surface strain in a beam is proportional to curvature) are more effective at identifying the location of damage[8]. A method which requires that the mode shapes before and after damage be known, but the modes do not need to be mass normalized and only a limited to structures that are characterized by one dimensional curvature[3]. A method for locating structural damage using experimental vibration data uses measured frequency response functions to obtain displacement as function of frequency[10].

Conducted some experimental aspects of dynamic response - based damage detection technique on Carbon/epoxy composites are addressed. Smart piezoelectric materials are used as sensors (or) actuators to acquire the curvature mode shapes of the structures. These materials are surface bonded to the beams. An impulse hammer is used as an actuating source as well four types of damage algorithms are evaluated for several possible damage configurations with two different excitations sources. The quality of damage identification with the four different detection algorithms is discussed[6]. A neural network approach that uses vibration and thermal signatures to determine the condition of a composite sandwich structure is proposed. Method can work jointly to complement each other in detecting the state of a sandwich composite structure[2]. Structural damage identification methods based on changes in the dynamic characteristics of the structure are examined and new methodology is also developed[11]. Acceleration responses energy based damage detection strategy, numerical analysis on long-span cable stayed bridge is performed by using the proposed method and the traditional mode shape curvature strategy, and at the same time damage quantification analysis and robustness analysis for noise pollution are carried out[14]. An overview of some of the methods of health monitoring for damage detection, applications of different techniques to bridges and critical issues for further research and development[13].

Despite of the extensive studies of vibration analysis on damaged structures, only few effective and practical techniques are found for very small damage identification. This paper, therefore, focuses on the study of the FRF curvature energy damage index method for damage detection purposes. The results indicate that the present method is quite sensitive to assessing damage in addition to identifying location of damages.

2 Formulations

2.1 Frequency response function (FRF)

The general mathematical representation of a single degree of freedom (SDOF) system is expressed by

$$m\ddot{x}(t) + c\dot{t} + kx(t) = F(t). \tag{1}$$

Assuming that the forcing is harmonic of the form $F(t) = F_O e^{i\Omega t}$.

In more general case the Receptance matrix for MDOF systems with viscous damping, can be expressed as

$$[H(\Omega)] = [[K] - \Omega^2[M] + i\Omega[c]]^{-1}. \tag{2}$$

Similarly without viscous damping the above equation can written as

$$[H(\Omega)] = [[K] - \Omega^2[M]]^{-1}. \tag{3}$$

The Receptance matrix is symmetric for linear systems and therefore

$$H_{rz}(\Omega) = \frac{\bar{X}_r}{F_z} = H_{zr} = \frac{\bar{X}_S}{F_z}, \tag{4}$$

where $\bar{X}_k$ and F_k are, respectively the Fourier transform of the displacement and applied force time histories at the k_{th} degree of freedom. The functions $H_{rz}(\Omega)$ can be arranged in matrix form. This leads to a Receptance matrix defined as

$$[H(\Omega)] = \begin{bmatrix} H_{11} & H_{12} & \cdots & H_{1n} \\ H_{21} & H_{22} & \cdots & H_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ H_{nl} & H_{n2} & \cdots & H_{nn} \end{bmatrix} \quad (5)$$

2.2 The frequency response function (FRF) curvature method

This method presented by Pandey et al. [9] have found that in place of using a displacement mode shape, strain or curvature shapes (surface strain in a beam is proportional to curvature) are more effective at identifying the location of damage. Sampaio et al. [8] showed in place of using displacement mode shapes. Strain or curvature shapes (surface strain in a beam is proportional to curvature) are more effective at identifying the location of damage]. This paper is extended to plate like structures by using FRF curvature data rather than mode shape data. The FRF-curvature for any frequency is defined by

$$H''_{i,j}(\Omega) = \frac{-H_{i-2,j}(\Omega) + 16H_{i-1,j}(\Omega) - 30H_{i,j}(\Omega) + 16H_{i+1,j}(\Omega) - H_{i+2,j}(\Omega)}{2h^2}, \quad (6)$$

where $H''_{i,j}(\Omega)$: FRF curvature measured at location i due to a force input at position j. h: The distance between two consecutive measurement points.

2.3 The FRF curvature energy damage index

In this section a new damage index based on the concept of FRF curvature energy is proposed. The damage indices are based on the variation of the FRF curvature energy at the element of the structure for a given excitation frequency. In a plate structure the FRF curvature energy can be defined as

$$\eta(\Omega) = \int_0^L [H''(x;\Omega)]^2 \mathrm{d}x, \quad (7)$$

where L is Span of the plate and $H''(\Omega)$ is FRF curvature for a frequency Ω.

x and y are the horizontal and vertical directions of the plate. In above equation showed only x direction.

The structure system is assumed to be divided into n elements ($j = 1, 2, 3, \cdots, n$). For the j^{th} element the FRF curvature energy can be written as

$$\eta(\Omega) = \int_{x_k}^{x_{k+1}} [H''(x^{\cdot};\Omega)]^2 \mathrm{d}x, \quad (8)$$

where x_k and x_{k+1}: coordinates of the nodes of the elements j.

For applied force at point j, the absolute difference between the FRF curvature energy of damaged and undamaged structure at a location i, in a predetermined frequency range is defined as

$$\Delta\eta = \sum_{\Omega} |\eta^*(\Omega) - \eta(\Omega)|, \quad (9)$$

where $\eta^*(\Omega)$: FRF curvature energy of damaged plate. $\eta(\Omega)$: FRR curvature of undamaged plate.

3 Simulation results and discussions

The dimension of plate under consideration is 0.25m×0.25m×0.003m with Fixed-Fixed support condition as shown in Fig. 1.

The material properties used in modeling the plate are $E = 68.9Gpa$, $\nu = 0.33$ and $\rho = 2710\text{Kg/m}^3$ which refer to Young Modulus, Poisson ratio and Mass Density respectively. The Length and width of plate is equally divided into 15 elements so the simulated model has total of 225 elements. Damage is simulated by

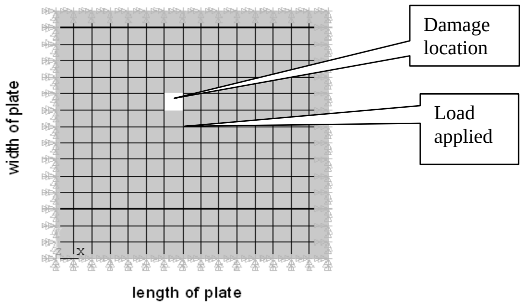

Fig. 1. Finite element mesh of a fixed-fixed plate with an area of reduced thickness

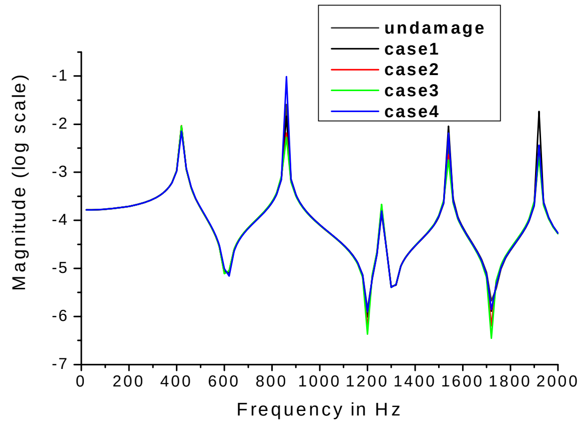

Fig. 2. FRF's of the undamaged and damaged model, measured at a point163 for an input force at the same point

Table 1. Represents different damage cases with element number

Damage cases	% of thickness reduction	Damaged element number
1	83.33	100
2	66.66	100
3	50	100
4	10	100

Table 2. Comparison of first five natural frequencies (Hz) for undamaged and damaged cases

Damage cases	Frequencies (Hz) for the mode number				
	1	2	3	4	5
Undamaged	422.56	860.56	860.56	1263.6	1541.1
Case 1	422.45	859.04	860.06	1263.1	1539.6
Case 2	422.37	857.81	859.67	1262.7	1538.4
Case 3	422.41	857.32	859.41	1262.5	1537.9
Case 4	422.1	856.92	858.21	1262.1	1536.7

reducing the thickness of one element (100th element) by 83.33, 63.33, 50, and 10% at location shown in Fig. 1 and details highlighted in Tab. 1.

The mode shape and Frequency response function FRF energy data of the square plate with damage of different sizes are obtained by using MATLAB 7.0. The first five natural frequencies resulting from modal analysis for a undamaged and all damaged cases are tabulated in Tab. 2.

Simultaneously harmonic analysis is done with frequency range 0-2000 Hz with load applied on structure 100N and the Frequency Response Function (FRF's) data is collected from same location where load applied is shown in Fig. 2 for undamaged and all damaged cases.

It is observed that there is no considerable shift in natural frequencies due to damage as observed from Tab. 2 and thus damage identification becomes difficult. Here the 2 and 3 modes are almost same frequencies because the structure is symmetric and it shows the coupling modes.

3.1 Influence of frequency range

While testing frequency intervals, it is found that the FRF curvature energy damage index method worked better for a range before the first anti-resonance or resonance. In fact for wider frequency ranges, including several modes the difference of curvature energies of the damaged and undamaged model becomes less significant when compared with the amplitude difference arising from the resonant frequencies shift.

From Fig. 3 shows the damage index variation for different damage cases in frequency range 0 to 500Hz which is good reliable in detecting and estimating the damage index.

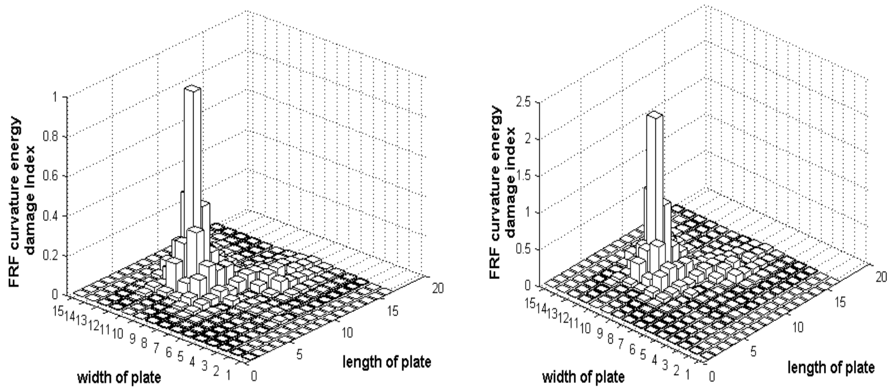

Fig. 3. FRF curvature energy damage index for a frequency range of 0-500 Hz. (a) Case 1 (b) Case 2

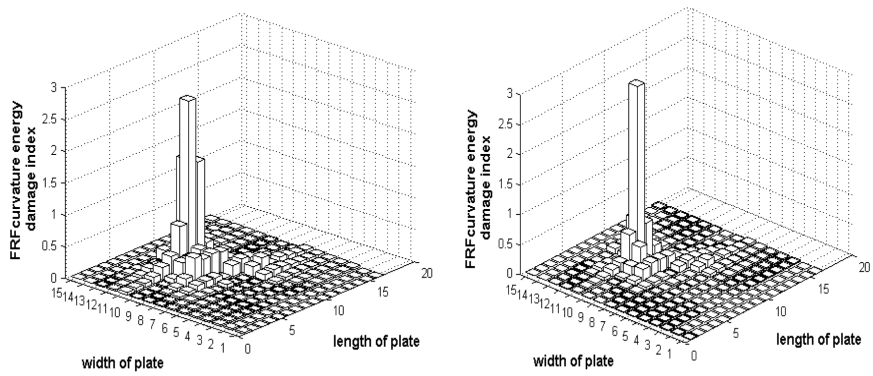

Fig. 4. FRF curvature energy damage index for a frequency range of 0-900 Hz. (a) Case 1 (b) Case 2

Further Fig. 4 shows damage index amplitude increases in frequency range 0 to 900Hz for different damage cases but damaged beside element also having amplitude which affects the quantification of damage.

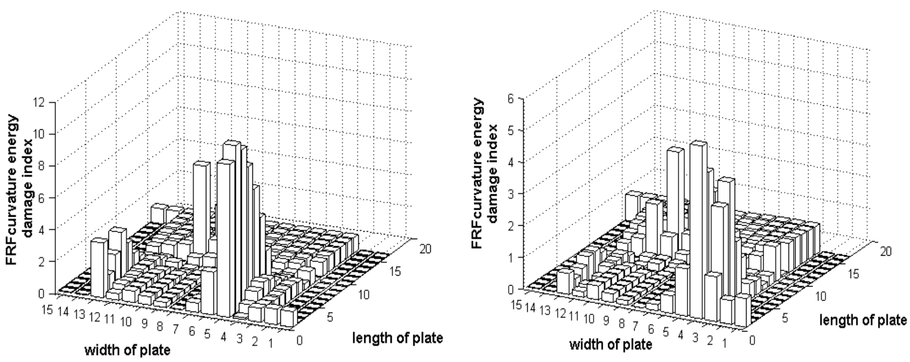

Fig. 5. FRF curvature energy damage index for a frequency range of 0-1280 Hz. (a) Case 1 (b) Case 2

Further increases the frequency range from 0 to 1280 Hz there is no localization for damage as shown in Fig. 5 for different damage cases. The sensitivity of damage identification mainly depends on damage location. In this analysis case 1 is sensitive because damage location n mode displacement location.

3.2 Influence of noise

To find out the influence of adding the noise to the numerical data (noise is always present on experimental data) it is decided to pollute our modal data with a multiplicative error of 15% of rms.

Fig. 6 shows that this method is quite insensitive to noise for damage identification.

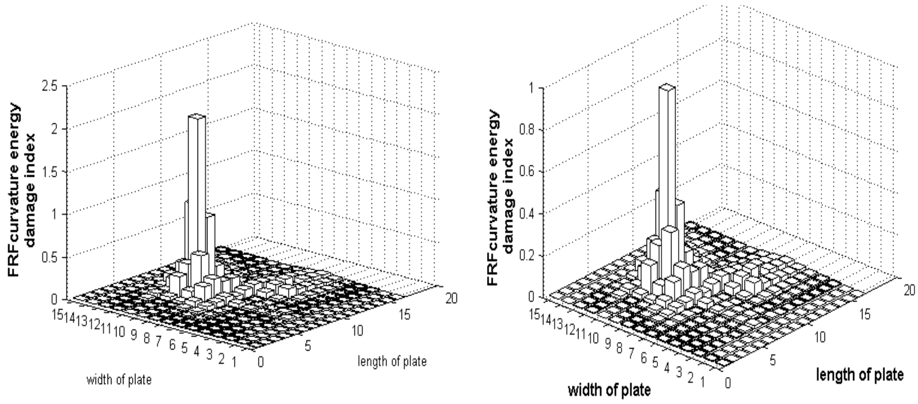

Fig. 6. FRF curvature energy damage index for a frequency range of 0-500 Hz. (a) Case 1 (b) Case 2

3.3 Influence of the input force location

It also found that it is necessary to assess the influence of the input force location on the damage identification. In this analysis exciting the system at different locations in the frequency range 0 to 500Hz and getting the responses of the FRF damage index.

From Fig. 7 shows for damage case 1 it seems that the influence of the position of the exciting force is not important.

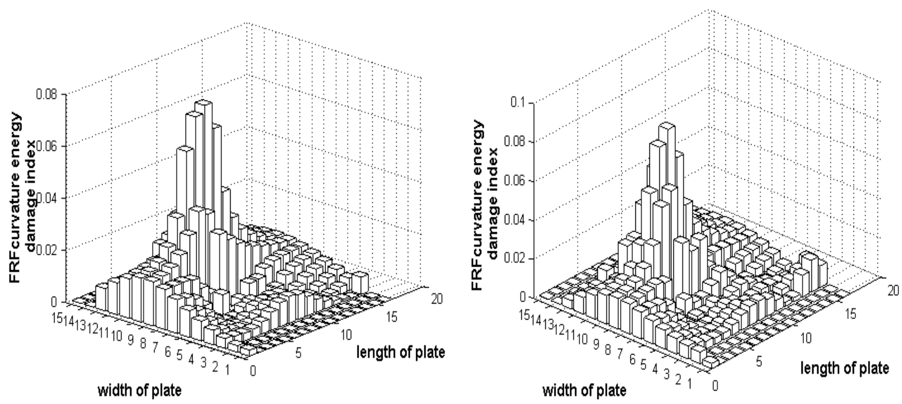

Fig. 7. FRF curvature energy damage index for a frequency range of 0-500 Hz. Case 1

4 Sensitive study of the damage

For the quantification of damage and also to ascertain the sensitivity of particular frequency range, the variation of maximum value of FRF curvature energy damage index versus percentage of damage for different frequency ranges is plotted as shown in Fig. 8. It is observed that the damage index value increase with increase in damage severity for all the three range of frequencies. By comparing the sensitivity of different range of frequencies it is found that the 0 to 500Hz range and 0 to 900 Hz is much more sensitive compare to the range 0 to 1280 Hz.

5 Conclusions

The results show that the FRF curvature energy damage index method performed well in detecting, locating and quantifying damage. Its main advantage is its simplicity and no need of performing a modal

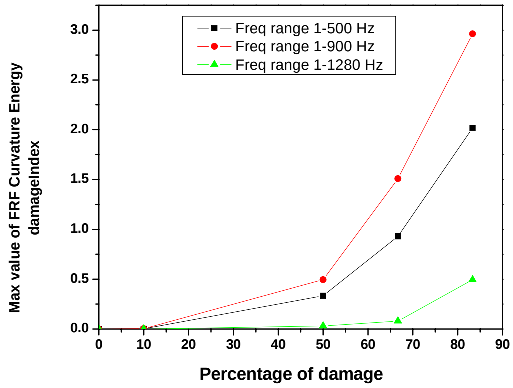

Fig. 8. Variation of percentage of damage versus maximum value of FRF curvature energy damage index for different frequency ranges

analysis for the identification of mode shapes or resonant. The damage index is found to be a function of the frequency bandwidth and variation of FRF curvature energy damage index versus frequency range (band width) seems to provide further information in choice of optimum frequency range response analysis. In case of influence of frequency range proved that the sensitivity of damage identification mainly depends on damage location and frequency range. In other case showed that even the system having noise will not affect the localization of damage. It proved that the influence of the position of the exciting force is not important for locate damage locations.

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