ISSN 1 746-7233, England, UK
World Journal of Modelling and Simulation
Vol. 8 (2012) No. 2, pp. 135-141

Improving local and regional earthquake locations using an advance inversion Technique: Particle swarm optimization*

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(*Received December 25 2010, Accepted July 17 2011*)

Abstract. The estimation of the hypocentral parameters in seismology has remained as one of the best-studied and challenging problem. In this paper a simple procedure has been presented to obtain the improved locations of local and regional earthquakes with advance inversion technique with minimum seismograph recording geometry. The problem is formulated as a nonlinear optimization problem in which the decision variables are the hypocentral parameters and the objective function to be minimized is the sum of squares of the differences between the observed and calculated times at specified locations. The objective of this paper is demonstrating the use of the latest heuristic technique for optimization namely "Particle Swarm Optimization" for solving the stated inversion problem. The earthquakes have triggered and recorded in NW Himalayan region are taken for experiments. The results obtained are discussed in this paper.

Keywords: PSO, seismic location, earthquake, QPSO

1 Introduction

Finding seismic location of an earthquake with better accuracy is always an important question for the seismologists. There are several techniques and algorithms which are being used to find the hypocentral parameters of an earthquake. The location of an earthquake is generally refers as a location of its focus (longitude, latitude and depth). There are a variety of positioning methods are available such as Geiger law and double difference location method which are being used for finding earthquake location[6]. Lee and Lahr [13] have presented a computer program called HYPO71[13], for determining hypocentral parameters of local earthquakes. This program models the earth as being made up of horizontal layers. Few papers are also came which use the non-linear least square methods for solving this problem of finding earthquake location by inversion technique[4, 5, 7–10, 14, 15]. The results of earthquake locations have been widely used to earthquake prediction, earthquake engineering, the earth's crust stress field analysis and many other aspects. This needs rapid and accurate earthquake location methods which helps us to moderate the disaster and overcome the high level effects of the forthcoming earthquake by post-earthquake relief. Earthquake early warning technology also needs fast and accurate methods for the earthquake locations. To deal with this type of accuracy and quickness, in this paper we have developed a new procedure for improved locations of the earthquake.

In the present paper the objective is to use Particle Swarm Optimization (PSO)[11] search technique for determination and improvement of the spatial seismic locations from the observed arrival times of various waves at a number of stations for a given seismic speed model for the NW Himalayan region and we have compared the results of the two recent versions of the PSO. This constitutes an inverse problem; in order to solve this inverse problem the solution of the forward problem must first be established.

* Thanks to Council of Scientific and Industrial Research (CSIR) for financial support with grant number 09/143(0734)/2009-EMR-I.
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2 Forward problem model

Given a seismic wave structure in a medium it is possible to use analytic formulae to calculate the theoretical arrival times of body waves in terms of the coordinates of the origin of the earthquake, speed with which energy travels in the form of waves and the location of observation stations. This is called the forward problem. If the parameters of the hypocentre be (x_i, y_i, z_i), Where they represent the coordinate values of longitude, latitude and depth of the preliminary hypocentre. (x_j, y_j) be the longitude and latitude of the observation stations on the earth surface. v_1 is the average crustal velocity of the seismic wave (p-wave) in single layer. The theoretical travel time t_{ij}[1] and epicentral distances Δ_{ij}[1] are given by the Eqs. (1) and (2) respectively

$$t_{ij} = \frac{\sqrt{\Delta_{ij}^2 + Z_i^2}}{v_1}, \tag{1}$$

where

$$\Delta_{ij} = 111.199 \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 \frac{\cos^2(x_i - x_j)}{2}}. \tag{2}$$

3 Inverse problem model

In the inverse problem of hypocentral location it is assumed that the observed arrival times of seismic waves are given at number of observations stations, and it is required to determine the hypocentral parameters of the earthquake. The position coordinates of earthquake are (x, y, z); longitude, latitude and depth (for a single earthquake). The way of solving the problem is to determine the values of x, y and z for which the sum of square of the differences between the calculated values and of arrival travel times of seismic waves at various observation stations and the actually observed times of these waves at the corresponding stations is minimized. Let C_k and O_k represents the calculated and observed travel time respectively. This may be represented as

$$\min f(x, y, z) = \left(\sum_i (C_k - O_k)^2\right)^{1/2}, \tag{3}$$

subject to

$$x_{\min} \leqslant x_{\max}, \quad y_{\min} \leqslant y_{\max}, \quad z_{\min} \leqslant z_{\max},$$

where $x_{\min}, x_{\max}$ etc. are, respectively, the probable lower and upper bounds within which the expected values of hypocentral parameters are expected to lie. Whereas the values of calculated travel times are to be computed analytically at each observation station, using Eq. (3) (Forward Problem), the values of observed travel times are to be taken from the actual observational data at each of observations stations.

The inverse problem for the location of hypocentral parameters is essentially an unconstrained non-linear optimization problem in which it is desired to find those values of the four three unknowns (x, y, z) for which the objective function $f(x, y, z)$ has the minimum possible value. Theoretically the global minimum of this optimization problem is obtained when the value of objective function is zero because of the errors in the observational data (recorded arrival time, inadequate knowledge of seismic wave speed distribution in the subsurface of and inadequacy of the model used to represent the real earth). Now any suitable non-linear optimization technique can be used to solve the inverse problem. We will use Particle Swarm Optimization to minimize the above objective function.

The whole calculation is done for the upper most layer of the earth crust (15 Km) and we have taken a velocity model[12] for the velocity of seismic waves in the upper most layer of the earth crust according to that the velocity is 5.80 km/sec. In the next section we will discuss about Nature Inspired Optimization Techniques.

4 Nature inspired optimization

Nature inspired optimization techniques are those techniques which are based on nature inspired phenomenon. Genetic Algorithm (GA), Ant colony optimization and PSO are few examples of nature inspired optimization. Genetic Algorithm is a population based search technique. In GA a population of strings (chromosomes) is used to encode an individual solution where as Ant colony optimization is based on social behaviour of ants.Particle swarm optimization is also a Nature inspired search technique which is inspired by Bird's flocking and Fish schooling. It was co-proposed James Kennedy and Russell Eberhart in [11].

4.1 Particle swarm optimization

Particle swarm optimization (PSO), which was first proposed by Kennedy and Eberhart in [11, 16], is a population based stochastic algorithm for continuous optimization. the algorithm is inspired by the social interaction behavior of bird's flocking and fish schooling. To search for the optimal solution, each individual, which is typically called a "particle" updates its flying velocity and current position iteratively according to its own flying experience and the other particles flying experience, By now, PSO has become one of the most popular optimization techniques for solving continuous optimization problems.

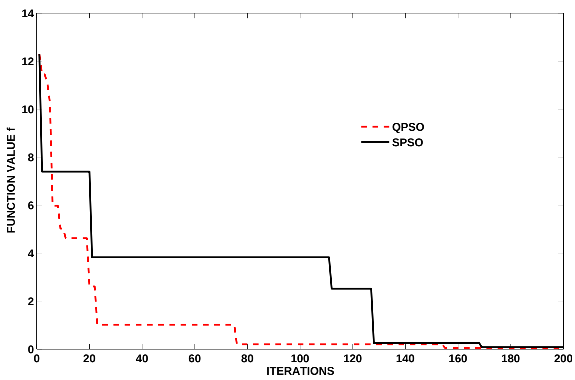

Fig. 1. Convergence of SPSO and QPSO and their comparison

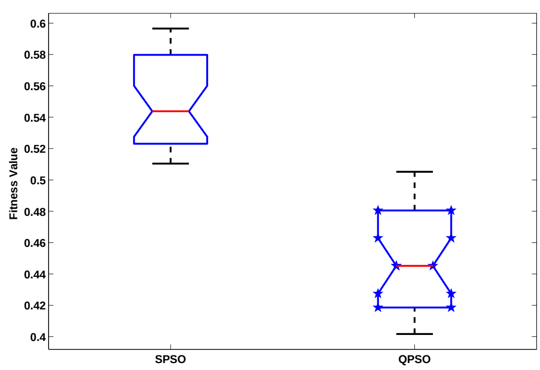

Fig. 2. Comparison of SPSO and QPSO for 30 runs

In original PSO S particles cooperate to search for the global optimum in the n-dimensional search space. The ith $(i = 1, 2, \cdots, M)$ particle maintains a position $X_i(x_i^1, x_i^2, ..., x_i^n)$ and velocity $V_i(v_i^1, v_i^2, ..., v_i^n)$. In each iteration,each particle uses its own search experience and the whole swarm's search experience to update the velocity and position. The updating rules are as follows:

$$v_i^j = v_i^j + c_1 r_1^i (pbest_i^j - x_i^j) + c_2 r_2^j (gbest^j - x_i^j), \quad (4)$$

$$x_i^j = x_i^j + v_i^j, \quad (5)$$

where $PBest_i(pbest_i^1, pbest_i^2, \cdots, pbest_i^n)$ is the best solution yielded by the ith particle and $GBest(gbest^1, gbest^2, \cdots, gbest^n)$ is the best-so-far solution obtained by the whole swarm. c_1 and c_2 are two parameters to weigh the importance of self-cognitive and social-influence, respectively. r_1^j and r_2^j are random numbers uniformly distributed in $[0, 1]$, and $j(j = 1, 2, 3, \cdots, n)$ represents the ith dimension.

4.2 Standard particle swarm optimization

In this section we present, in some detail, Standard Particle Swarm Optimization algorithm for global optimization which can be used to solve this inversion problem. The method seems to be eminently suited for such seismic inversion problems, as its uses only function evaluations and, unlike other methods does

Table 1. Travel Times calculated by forward problem model from artificial hypocenter to observational stations

Hypocentre (longi./latti./depth)	Location of observational Stations (Longitude, Latitude)	Travel Time (seconds)
	79.20 30.14	1.69040
	77.86 30.97	1.68687
	77.25 30.71	1.68416
$70^0, 30^0, 10$	79.61 31.10	1.68314
	78.10 30.54	1.68152
	78.43 29.71	1.68347
	77.58 30.49	1.69260
	77.73 30.53	1.68817
	78.74 30.33	1.68510

Table 2. Minimization of f using QPSO and SPSO by Inverse problem model and the revaluated hypocenter with function value f

Quadratic PSO				Standard PSO			
x^0	y^0	$z(Km)$	f	x^0	y^0	$z(Km)$	f
70.0028	30.0015	10.0001	0.0003	70.0034	30.0024	10.0012	0.0023
70.0026	30.0012	10.0010	0.0005	70.0054	30.0053	10.0024	0.0024
70.0029	30.0010	10.0009	0.0010	70.0067	30.0079	10.0056	0.0012
70.0010	30.0009	10.0007	0.0002	70.0045	30.0029	10.0069	0.0025
70.0030	30.0021	10.0015	0.0009	70.0089	30.0059	10.0083	0.0034
70.0022	30.0016	10.0004	0.0007	70.0123	30.0028	10.0023	0.0029
70.0019	30.0011	10.0003	0.0007	70.0023	30.0067	10.0069	0.0030
70.0015	30.0007	10.0011	0.0001	70.0054	30.0019	10.0024	0.0035
70.0027	30.0018	10.0009	0.0004	70.0045	30.0029	10.0060	0.0022
70.0015	30.0019	10.0007	0.0005	70.0012	30.0074	10.0081	0.0020

not make use of other mathematical properties of the function such as an assumption of its continuity and evaluations of its derivatives, which are often not easy to compute and - certain situations does not justified. The algorithm also tries to determine the global optimal solution which, in the absence of errors, must have zero as the objective function value. Also it does not require any initial guess values of the hypocentral co-ordinates on the part of the user to initiate the algorithm. The process of the algorithm is as follow: Let us consider that our search space is d-dimensional and the nth particle of the swarm can be represented by d-dimensional vector $X_i(x_i^1, x_i^2, \cdots, x_i^n)$, the velocity of the particle is denoted by $V_i(v_i^1, v_i^2, \cdots, v_i^n)$. Consider the best visited position for the particle is $PBest_i(pbest_i^1, pbest_i^2, \cdots, pbest_i^n)$ The best explored position is $GBest(gbest^1, gbest^2, \cdots, gbest^n)$ The position of the particle and its velocity is being updated using the following equations:

$$v_i^j = w * v_i^j + c_1 r_1^i (pbest_i^j - x_i^j) + c_2 r_2^j (gbest^j - x_i^j), \tag{6}$$

$$x_i^j = x_i^j + v_i^j, \tag{7}$$

where c_1 and c_2 are constants rand is random variable with uniform distribution between 0 and 1. $w =$ inertia weight, which shows that the effect of previous velocity vector on the new vector. $V_{\max}$ is an upper bound which is placed on the velocity in all dimensions. This limitation prevents the particle from moving too rapidly from one region in search space to another The algorithm of the Standard PSO is:

Initialize the swarm X_i^j,

the position of the particles is randomly initialized within the feasible space.

Evaluate the performance F of each particle using its current position $X_i^(j)$.

Compare the performance of each individual to its best performance so far: if $f(X_i^j) < f(Pbest_i), f(Pbest_i) = f(X_i^j), Pbest_i = X_i^j$.

Compare the performance of each particle to the global best particle: if $f(X) < f(Pbest)$.

$f(Gbest) = f(X), Gbest = X.$
Change the velocity of the particle according to.
Move each particle to a new position using.
Go to step to and repeat until convergence

4.3 Quadratic particle swarm optimization

This is a hybridized PSO proposed by Deep and Bansal in [3]. In this version of PSO we use a quadratic approximation operator which determines the point of minima of the quadratic hyper surface passing through three points in a D-dimensional space. It works as follows:
(1) Select the particle R_1, with the best objective function value. Choose two randomly selected particles R_2 and R_3 such that out of R_1, R_2andR_3, at least two are distinct.
(2) Find the point $R*$ of the quadratic surface passing through R_1, R_2 and R_3.The flow of QPSO is same as SPSO except the process of hybridization. In each iteration, the whole swarm is divided into two sub swarms

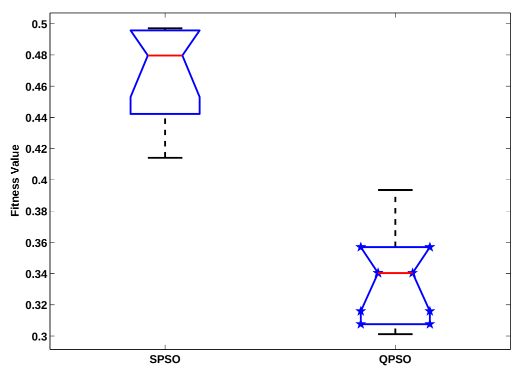

Fig. 3. Comparison of SPSO and QPSO for 50 runs

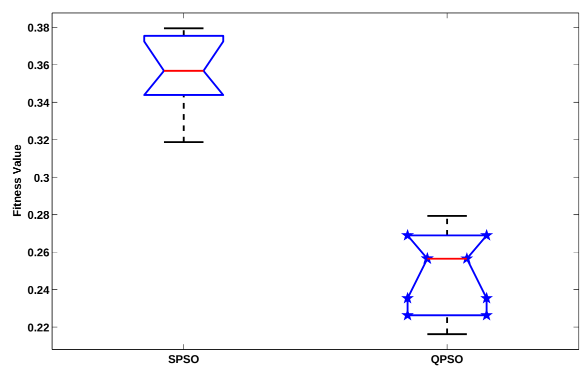

Fig. 4. Comparison of SPSO and QPSO for 70 runs

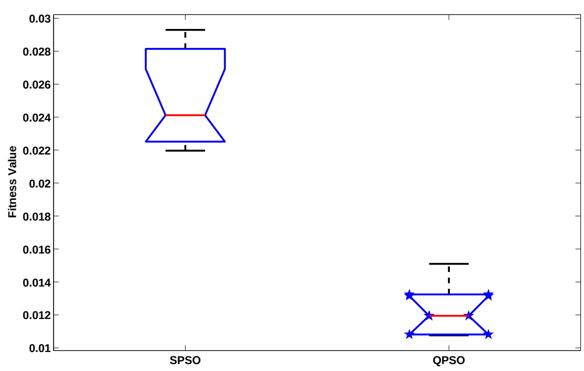

Fig. 5. Comparison of SPSO and QPSO for 90 runs

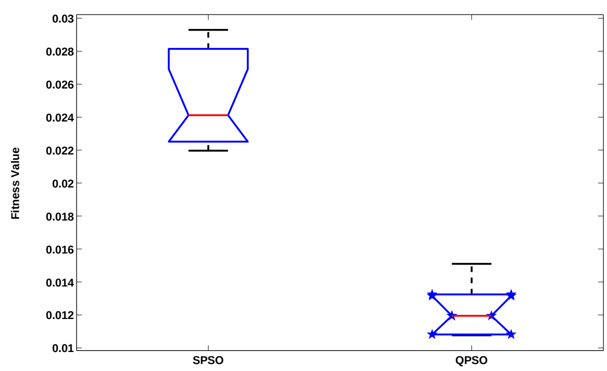

Fig. 6. Comparison of SPSO and QPSO for 100 runs

say (S_1 and S_2). From one generation to the next generation, S_1 is evolved using PSO, where as S_2 is evolved using Quadratic approximation operator and finally we compare the global best of S_1 and S_2. This we repeat unless until we get stopping criterion.

5 Parameter selection of QPSO and standard PSO

We will initialize the process randomly with zero velocity. For our problem number of variables are three therefore the dimension (D) of the search space is 3, i.e. $D = 3$. The size of the swarm is 13 $(10 + (int)(2 \times$

$sqrt(D)), c_1 = c_2 = 0.5 + log(2)$[2], w is defined as $w = 1/(2 \times \log(2))$[2]. Number of iterations are 100. All the parameters are same for both the version of PSO namely SPSO and QPSO except the coefficient of hybridization CH which is an important parameter for QPSO has been taken CH = 30[3]. As we are finding the earthquake location in NW Himalayan region so the following range of the particle X is to be set, more explicitly, X_{n1}, X_{n2} and x_{n3} are in the range of $[0, \pi]$, $[0, \frac{\pi}{2}]$ and $[0, 15]$ i.e

$$0 \leqslant x \leqslant \pi, \quad 0 \leqslant y \leqslant \pi/2, \quad 0 \leqslant z \leqslant 15.$$

6 Synthetic testing of algorithms on himalayan region data

To test the algorithms synthetically we will take a point inside the earth as an artificial Hypocenter and we will calculate the travel times of the seismic waves from this hypocenter to ten different stations Tab. 1. of the earth surface with the help of forward problem model then by inverse problem model we will inversely evaluate the value of initially taken artificial Hypocenter. This will test the availability of both the algorithms. TO compare the both versions of PSO we have statistically compared the performance of the algorithms for different-different runs from 30 to 100. The statistical plots shows the comparison.

7 Conclusions

The numerical results which are presented in Tab. 2. show that both the version of PSO algorithm (SPSO and QPSO) are able to determine the hypocentral coordinates of the earthquakes. As we have taken an artificial point as an hypocenter (700, 300, 10) and we calculated the travel time of seismic waves from this point two 10 different observation stations with the help forward problem model and then taking this travel time as a observed travel times we have obtained the hypocenter coordinates with the help of inverse problem model and the obtained values of hypocentral coordinates are very close to the initially taken hypocentral coordinates as shown in Tab. 1. These results shows the optimization ability of the both the algorithms. As Hang Dong-xue, Wang Gai-yun in [1] has given a paper for finding the seismic location using PSO. The version which we have used (QPSO) works better than the SPSO as it has a better and quick convergence than SPSO. This can be easily seen in the Fig. 5. Finally the results of the paper can be summarized as Forward problem is calculated using Travel time Model. Inverse problem is modelled in which objective function is the Root Mean Square of the observed and calculated travel times. Methodology used for Inverse problem is a new technique called Particle Swarm Optimization. Two versions of PSO-Standard PSO and another QPSO are used. It is observed that QPSO works better than SPSO as it takes less time to converge with better accuracy than SPSO.

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