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Structural damage detection using a hybrid particle swarm algorithm

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Abstract. Damage in the form of line crack in a thin plate is detected using an inverse time domain formulation. The damage is modeled using an orthotropic finite element model based on the strain energy equivalence principle. The identification is carried out using time domain acceleration responses. The principle is to minimize the difference between the measured and theoretically predicted accelerations. For this purpose a new hybrid of Particle Swarm Optimization (PSO) and Genetic Algorithm (GA) was used. A quarter substructure of the plate was identified which contained 72 damage variables, which is significant for any optimization algorithm. Using numerically simulated experiments, multiple cracks in a plate were reliably detected using this method. The effect of noise in the experimental values were also simulated. The hybrid algorithm proved more accurate in damage prediction than pure PSO and GA.

Keywords: damage detection, hybrid algorithm, optimization

1 Introduction

Inverse problems often occur in many branches of engineering where the values of some physical model parameters must be obtained from observed data. System identification (SI) for structural damage detection and health monitoring comes under the category of inverse problems. The application of SI technique presented here is the time domain vibration signal based damage detection of a uniform thin plate. Doebling et al. has presented a comprehensive survey of vibration based damage detection methods[2]. The use of modal analysis techniques for damage detection is more traditional. Young Shin Lee et al. used the first lower four natural frequencies of the cantilever beam to identify a crack[8]. Wang et al. presented a damage detection scheme using static deformation and natural frequencies of planar truss and beam using an interactive optimization algorithm to asses locate and extent of damages[9]. Damage in the form of a crack in an isotropic finite element represented as an equivalent orthotropic element using Strain Energy Equivalence principle was presented[7]. This model was used later to predict crack damage in a plate using frequency response data[6]. Surface crack detection in composite laminates by modal analysis and strain energy method was carried out[3]. Here the FE model of composite laminate was established using ANSYS and results are validated with experimental results. Substructuring formulation in the time domain is presented in[1] and based on this Koh et al. [4] proposed a substructure system identification scheme for engineering structures such as trusses and frames. Substructure approaches in the time domain, including variations such as progressive and direct methods, are presented in[5]. A good substructure method has two advantages (a) it is able to identify the parameters inside a structural zone without the need to measure the responses outside that zone i. e., a saving in sensors and (b) reduced computational effort resulting from dealing with fewer DOF's.

This paper presents the application of a new hybrid of Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) to a time-domain damage detection scheme. Although a thin plate is chosen in this paper for damage identification, the same approach can be equally well applied to beams and trusses. Because the size of the plate global model is large with many damage variables, a substructure scheme is used. The algorithm

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estimates the damage parameters through minimization of an error function defined by the mean squared error between the measured and theoretically estimated accelerations at all time steps and all sensor locations inside the substructure. The plate can be divided into substructures and each one checked for existence of crack damage variables inside its zone. In this paper the "measured" values are obtained from simulated experiments using a known numerical model. The estimated values are obtained from the same numerical (i. e finite element) model with the (unknown) damage indices treated as optimization variables. When the estimated model coincides with the experimental model, the error vanishes to zero.

The original contributions of the paper are summarized here. Time domain signals have been used to identify cracks in a plate instead of frequency domain in earlier literature. Additionally, a substructure approach has been used. A hybrid of Particle Swarm and Genetic Algorithm has been used instead of pure Genetic Algorithm or Least Square Method in the earlier literature. This has enabled us to apply the method for problems with larger number of damage variables.

2 Time domain substructuring

The equation of motion for a structural system is given by:

$$M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = P(t), \tag{1}$$

where M, K and C represent the mass, stiffness and Rayleigh damping parameters; and P the external forces in the time domain t. Fig. 1 shows the substructural representation of the simply supported plate considered

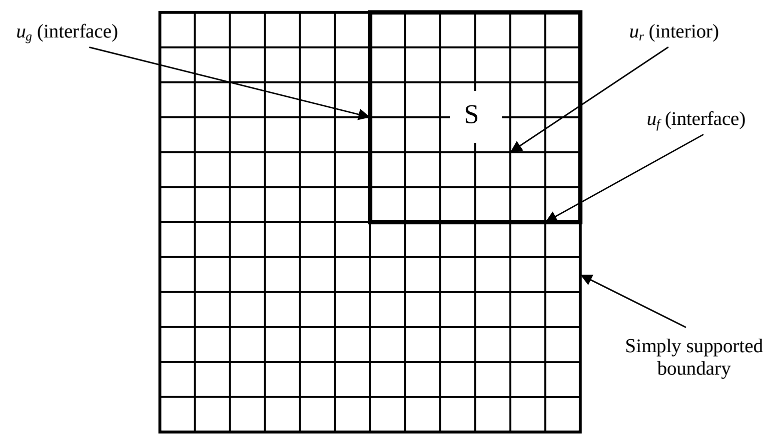

Fig. 1. Substructural representation of plate

here. The subscripts g and f represent the two interface DOFs and r the interior DOFs. The partitioned equations of motion may be represented by Eq. (2). The external force vector could act either in the interior or exterior of the substructure. The subscript 'j'' denotes all interface DOFs (i. e. f and g included).

$$\begin{bmatrix} M_{rf} & M_{rr} & M_{rg} \end{bmatrix} \begin{Bmatrix} \ddot{u}_f \\ \ddot{u}_r \\ \ddot{u}_g \end{Bmatrix} + \begin{bmatrix} C_{rf} & C_{rr} & C_{rg} \end{bmatrix} \begin{Bmatrix} \dot{u}_f \\ \dot{u}_r \\ \dot{u}_g \end{Bmatrix} + \begin{bmatrix} K_{rf} & K_{rr} & K_{rg} \end{bmatrix} \begin{Bmatrix} u_f \\ u_r \\ u_g \end{Bmatrix} = \{P_r\}, \tag{2}$$

$$M_{rr}\ddot{u}_r(t) + C_r\dot{u}_r(t) + K_r u_r(t) = P_r - M_{rj}\ddot{u}_j(t) - C_{rj}\dot{u}_j(t) - K_{rj}u_j(t). \tag{3}$$

Eq. (2) can be rearranged to bring the 'interior' partitions to the left and interface effects in the form of a force on the right as Eq. (3). P_r is the external force applied the interior node(s) of substructure. If the force is applied outside the substructure, P_r is set to 0. In the above form it is required to calculate the substructure interface displacements $\ddot{u}_j$, velocities and accelerations. These interface accelerations have to be obtained experimentally (simulated numerically here), and thereafter integrated to obtain the displacements and velocities. We also require the experimentally measured acceleration response $\ddot{u}_m$ at a few interior points M.

The estimated (or predicted) accelerations $\ddot{u}_e$ at those M points are obtained from the mathematical model from the left hand side of Eq. (3). For correct identification they have to match with the experimentally measured acceleration response $\ddot{u}_m$.

Using an optimization algorithm the following fitness function is minimized, which is the sum of the square of deviations between the measured and estimated interior accelerations at all locations and all time steps,

$$f = \frac{\sum_{i=1}^{M}\sum_{n=1}^{L} |\ddot{u}_m(i,n) - \ddot{u}_e(i,n)|^2}{ML}, \tag{4}$$

where subscript 'm' and 'e' denote measured and estimated quantities respectively, L is the number of time steps and M is the number of measurement sensors used. Ideally it must be minimized to zero, but usually it approaches a small value close to zero.

The number of interior measurements M in Eq. (4) has an effect on fitness function. More interior points will increase the identification accuracy. Four interior points were chosen from the interior of the substructure after getting less satisfactory results with 2 points. The effect of changing the location of the 4 points have not been investigated.

3 Crack damage representation

There are many finite element based crack damage models. In this paper we use the model proposed in [7] according to which a small material volume (SMV) with a line crack behaves effectively orthotropically in a small zone. A small material volume with a crack can be represented as an equivalent continuum model with orthotropic properties having the same strain at the boundaries of that volume (see Fig. 2). Consider an elastic thin plate with thickness h and the width Lx and Ly in the x and y directions, respectively with a crack of length $2l$ and oriented at an angle θ with the global x axis. The dimensions of the element are $2\bar{x}$ and $2\bar{y}$.

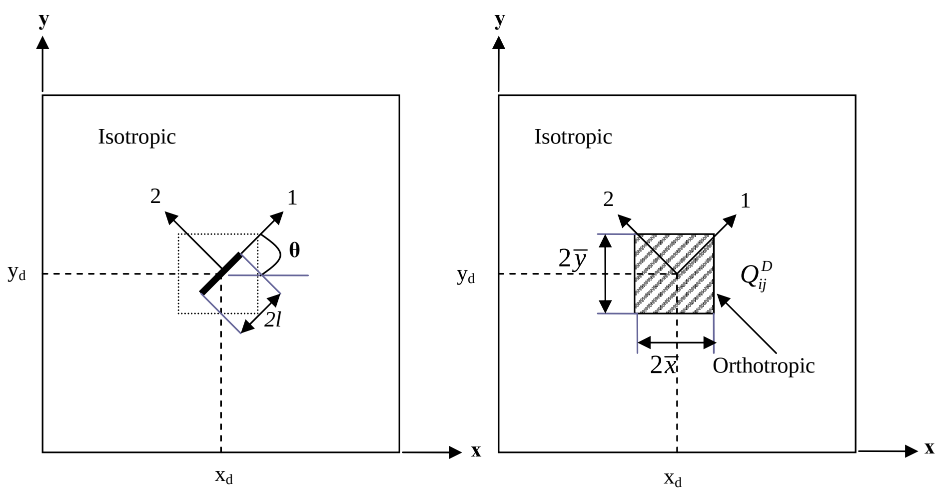

Fig. 2. Crack damage representation

The effective elastic stiffness Q_{ij}^D for the SMV containing a line crack damage is

$$Q_{ij}^D = Q_{ij}(1 - e_{ij}D), \quad i,j = 1,2,6, \tag{5}$$

where Q_{ij} are the reduced stiffnesses for the intact isotropic material in the plane stress state and e_{ij} are effective material directivity parameters given by:

$$e_{11} = \frac{2v^2}{1-v^2}, e_{22} = e_{12} = e_{21} = \frac{2}{1-v^2}, e_{16} = e_{26} = e_{61} = e_{62} = 0, e_{66} = \frac{2}{1+v}. \tag{6}$$

D represents the averaged severity of damage within an SMV, which is called herein the effective damage magnitude. It is the ratio of volume of the effective cracked material to the small material volume (SMV). Effective volume of a crack of depth h' and length $2l$ is $\pi h' l^2$ (there is a circular area of radius l containing the crack) and SMV is volume of the finite element $4xy$ containing the crack. D may have two extreme values,

$$D = \frac{\pi h' l^2}{4\bar{x}\bar{y}h} \begin{cases} 0, & \text{for intact state} \\ 1, & \text{for complete material failure} \end{cases} \tag{7}$$

where $h' = h$ for a through crack. Thus a crack in an element is defined by two indices, D varying from 0 to 1 and θ varying from 0 to 90°. $D = 0$ represents the case when there is no crack, and $D = 1$ represents the case when the volume of cracked material equals the finite element volume. For the purpose of damage detection it is assumed that the crack does not propagate and the damping is not significantly changed by the crack. The effective elastic stiffness with respect to the global coordinates x and y can be obtained by using the coordinate transformation as follows:

$$\bar{Q} = T(\theta)^T Q^D T(\theta), \tag{8}$$

where $T(\theta)$ is the coordinate transfer matrix, in which θ denotes the crack orientation (degrees) with respect to the global coordinate x.

The assumptions of this model are given below,

(1) The length of the crack is less than length of the element;
(2) Upon the occurrence of damage, the structure continues to behave linearly;
(3) Crack is of the static type i. e., there is no measurable growth of crack;
(4) The mass of the plate is unaffected due to crack damage;
(5) Damping is unaffected due to crack of damage.

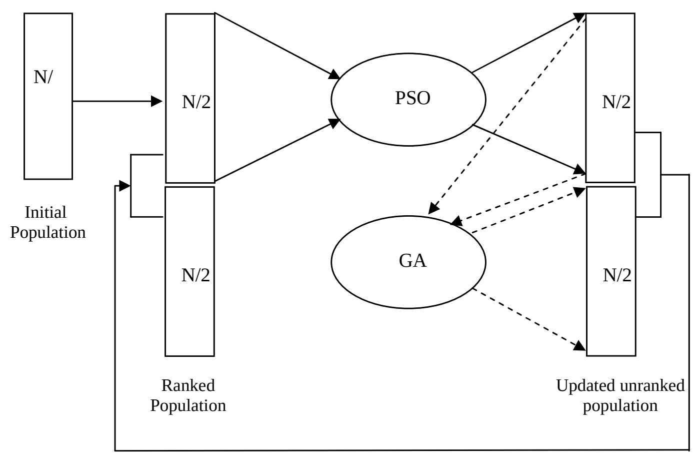

Fig. 3. Representation of Hybrid-PSO algorithm

4 Heuristic algorithms

Genetic Algorithms (GA) are exploration algorithms based on the mechanism of natural selection and survival of the fittest (evolutionary principle). GA combines the explorative ability of large search spaces as well as reasonable guided search. GA creates an initial random sample within the specified domain of variables, called 'population'. It then ranks them in the order of fitness and conducts crossover operations from

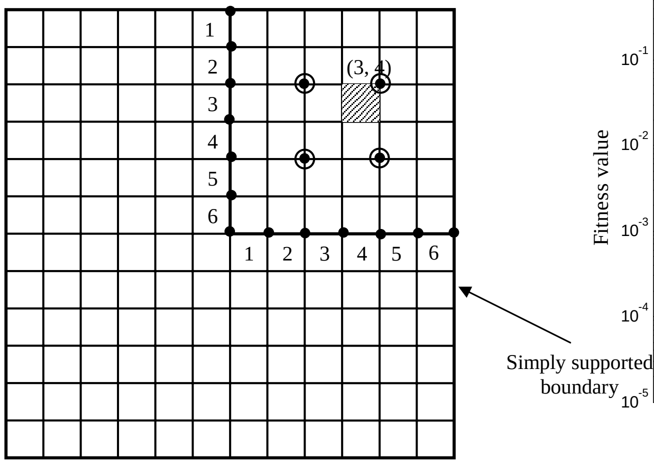

Fig. 4. Numerical model of plate, case A

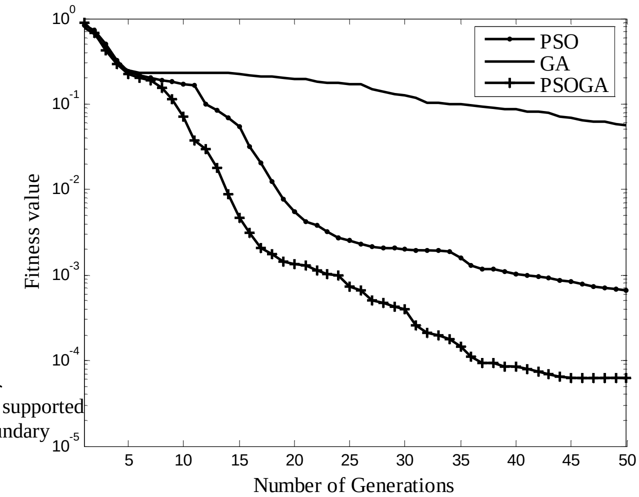

Fig. 5. Comparison of convergence of PSO, GA and Hybrid

among a pool of 'parents' through the Roulette wheel selection. Parents having higher fitness have a greater probability of being selected and their offsprings contribute to the next generation. GA can be programmed in the Binary or Continuous versions. Here GA in the continuous (decimal numbers) version is used.

Particle swarm optimization (PSO) is a population based continuous optimization technique, inspired by the social behavior of bird flocking or fish schooling (i. e., behaviorally inspired). The system is initialized with a population of random solutions and searches for optima by updating generations. However, unlike GA, PSO has no evolution operators such as crossover and mutation. PSO also explicitly keeps track of the globall best (gbest) and locally best moves of the particles (pbest) which accounts for its reported superiority over GA.

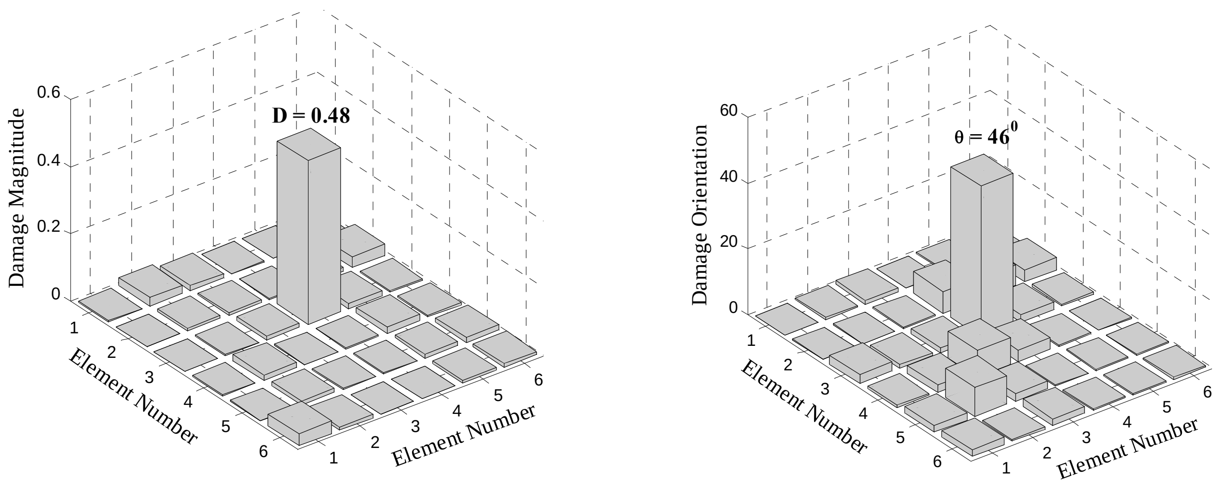

Fig. 6. Damage identification using PSO, single damage case

The novel hybrid method used here combines PSO and GA. The algorithm is capable of detecting 72 optimization (damage) variables in the numerical problem presented later. Very briefly, the hybrid algorithm combines the best features of GA and PSO i. e, the exploratory ability of GA (mutation, crossover) and the memory of historical best moves of PSO. The hybrid algorithm operates on a population size of N (see Fig. 3). It begins with a randomly created N population. It is ranked in the order of best fitness and the N/2 best particles are input to PSO. The normal PSO operations are applied to create a new output of N/2 particles.

These N/2 particles are saved in memory and a copy is input to GA in order to create another offspring set using the standard GA procedures of crossover and mutation.

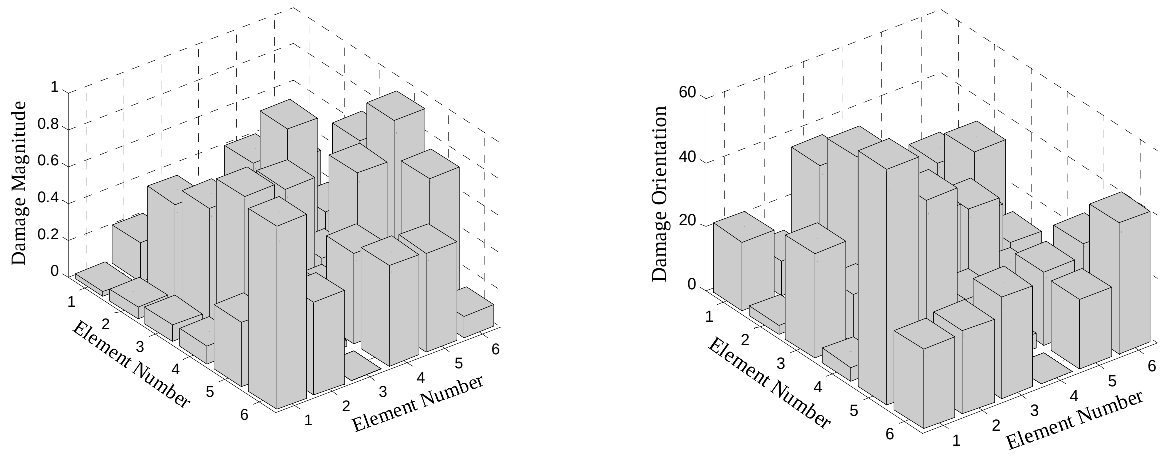

Fig. 7. Damage identification using GA, single damage case

These two sets are combined to give the new N population and ranked according to fitness. In doing so, if any particles of PSO which have low fitness than the new individuals created from GA, are replaced by new best individuals of GA. In the next generation, the particle replaced by GA will be treated as 'gbest'.

5 Procedure for identification

The procedure for identifying a crack in a plate is briefly presented here. First a substructure zone around the probable region of crack is marked out. If the probable region is not known, the plate can be divided into a few subructures and each zone checked for damage indices. Acceleration measurements at all nodes along the substructure interface are required to enforce the substructure compatibility conditions. The time domain excitation force (P_r) can be applied in the interior of the substructure. If the excitation is applied outside the substructure, is P_r set to zero. A few interior acceleration measurements are also required (4 in this case) to compare with the predicted values and the optimization algorithm tries to minimize the deviation between measured and predicted values. The optimization variables are the unknown damage indices which equal to double the number of elements in the substructure. If the resulting damage indices are all zero then there is no damage in the plate. Otherwise the values of D and θ indicate the crack length and orientation in the corresponding elements.

6 Numerical model

A simply supported thin square aluminum plate with thickness $h = 0.004\ m, 0.5\ m$ side, Young's modulus $E = 72\ GPa$, Poisson's ratio $\nu = 0.33$ and mass density 2800 kg/m3 is modeled with a finite element package. Two damage cases A and B are studied. In case A, a single line crack of length ($2l$) 0.0332m and $45°$ orientation occurs in element (3,4) as shown in Fig. 4 in the top-right quarter substructure. This crack is specified by damage index $D = 0.5$ and $\theta = 450$. In case B, there are 3 damaged elements (6,1), (2,2) and (4,5) containing cracks of damage indices $D = 0.3, 0.5$ and 0.7 at $\theta = 0°, 30°$ and $45°$ respectively. Gaussian noise of zero mean and 3% standard deviation is added to the synthetic experimental data to simulate errors. The full structure and quarter structure has 144 and 36 elements respectively. Since each element has two damage indices, the full structure has 288 and substructure have 72 damage variables to identify. The substructure damage variables are identified with various heuristic algorithms. PSO parameters are set as 1000 particles (swarm size), maximum of 50 generations (iterations), a linearly decreasing inertia function from 0.9 to 0.4,

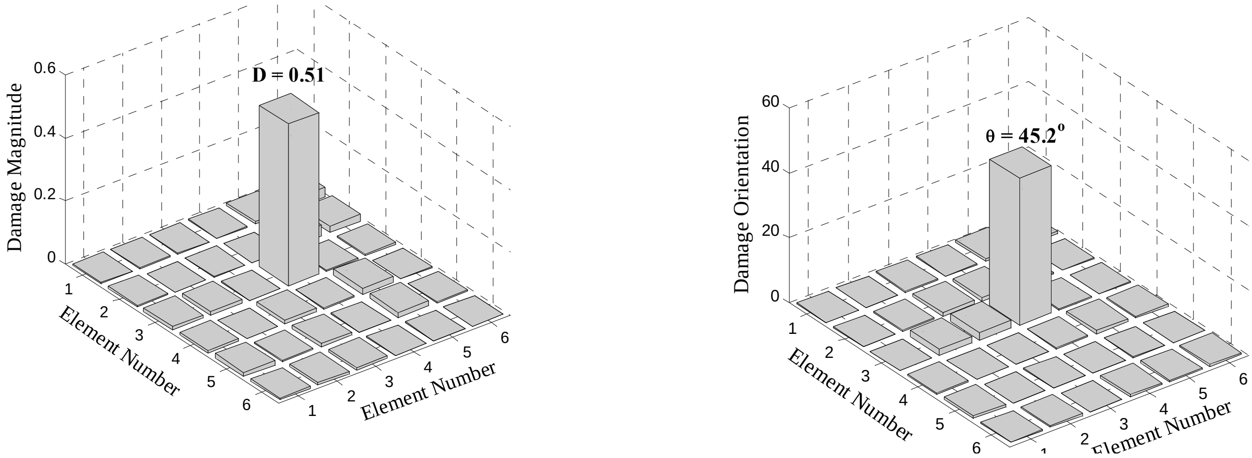

Fig. 8. Damage identification using PSO-GA hybrid, single damage case

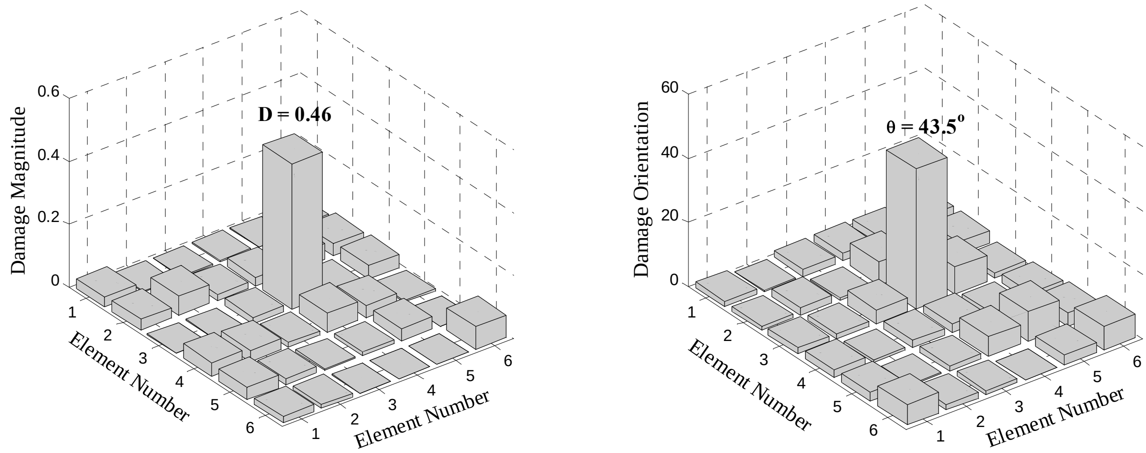

Fig. 9. Damage identification using PSO with 3% noise

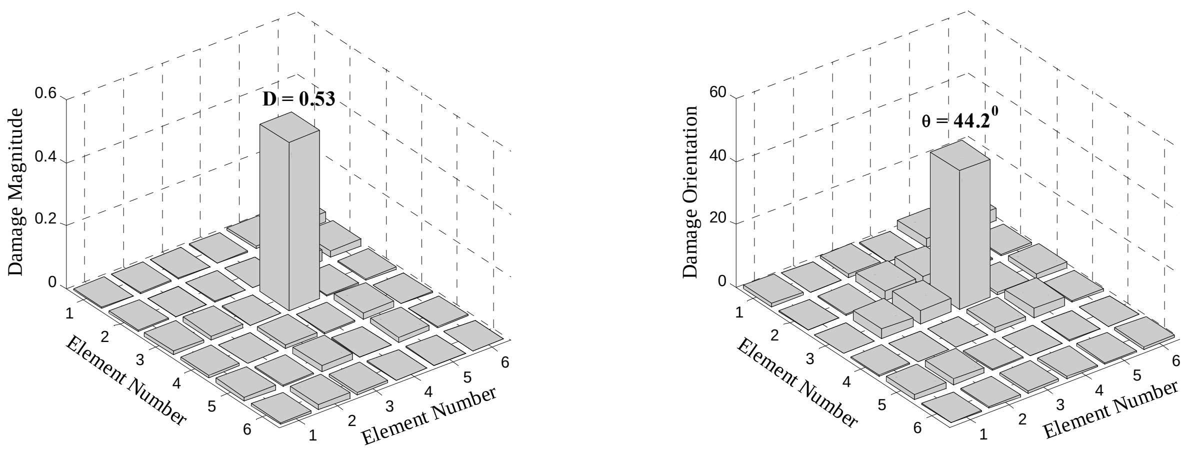

Fig. 10. Damage identification using Hybrid—with 3% noise

and acceleration constant set to 2. The GA population is set to 1000, with mutation rate of 1% and crossover rate of 40%. The combined population of the hybrid algorithm is set to 1000, with GA and PSO parameters set as above. A point harmonic force of 10 $\sin(2\pi 100t)$ N applied at the centre of the plate. The excitation frequency (100Hz) is above the first natural frequency of 77Hz. The experiment is numerically simulated using a known mathematical model. The synthetic data of the numerical model are first calculated in terms of displacement, velocity and acceleration using Newmark's method with constant time step of 0.001 sec for 2 seconds. All the substructure interface DOF responses and four interior responses (Fig. 4) are required for substructure identification. The computer used has a 3Ghz Pentium-4 processor with 1Gb RAM

7 Results

The goal of the study is the correct identification of damage indices of each element in the substructure. A substructure may or may not contain the damaged elements. If the elements are undamaged, it will return zero damage indices for all elements. First the single damage case 'A' is discussed. Fig. 5 shows the convergence plot of the fitness function (Eq. (4)) when using different algorithms and it is clear that the hybrid has minimized it to the smallest value. The damage identification results are discussed next. Fig. 6 shows the damage index identification (D and θ respectively) using PSO. Convergence was achieved in 30 generations and the damage indices are fairly correctly identified as $D = 0.48$ and $\theta = 46°$. Fig. 7 shows the results for identification with pure GA. Here the results are clearly poor and GA fails to correctly identify the damaged elements. Next Fig. 8 shows the results of damage identification using Hybrid PSO-GA and the results are the best of the three algorithms (it has correctly identified zero damage for the undamaged elements). The time taken in minutes was 42 (PSO), 56 (hybrid) and 115 (GA). Thus the hybrid takes 30% more time than pure PSO, but compensates in accuracy.

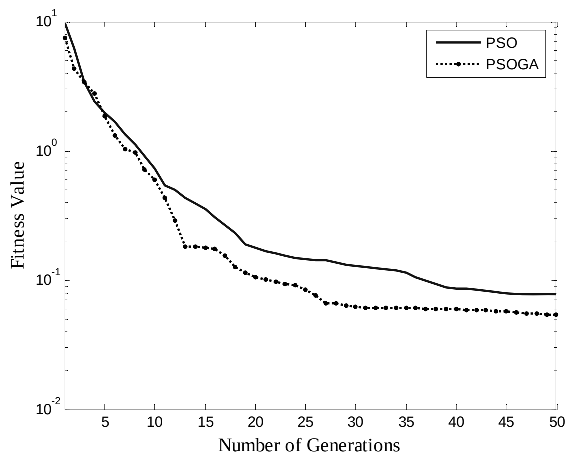

Fig. 11. Convergence of fitness function of PSO and Hybrid, with noise added

Next we discuss the case when 3% Gaussian noise is added. GA is omitted here because of its poor performance. Fig. 9 shows the result of identification using PSO and Fig. 10 shows the results with the Hybrid. Comparing Fig. 9 and 10 it is clear that PSO has incorrectly identified damages in many undamaged elements, whereas the Hybrid makes less errors in this regard. A comparison of the convergence of the fitness function of PSO and Hybrid when 3% noise is added, is shown in Fig. 11. It clearly shows the Hybrid has minimized the fitness function better than PSO.

After establishing the superiority of the hybrid model, a more complex example i. e., case B as discussed in section 5, with 3 crack damages is identified with 3% noise added. The results are shown in Fig. 12. The identified D values are 0.31, 0.48 and 0.68 (the correct values being 0.3, 0.5 and 0.7). The maximum error here is 4%. The θ values identified are $0.84°, 31.56°$ and $46.6°$ (the correct values being $0°, 30°$ and $45°$). The maximum error here is 5.2%. The computational time for the 3 damage detection was almost the same as for single damage. This is because all elements in the substructure are simultaneously checked for damage.

7.1 Conclusions

Using a time domain substructure method, single and multiple crack damages in a thin plate were correctly identified using an improved hybrid optimization algorithm. There were 72 damage (optimization) variables in the problem and the hybrid PSO-GA was most successful in predicting damaged elements, compared

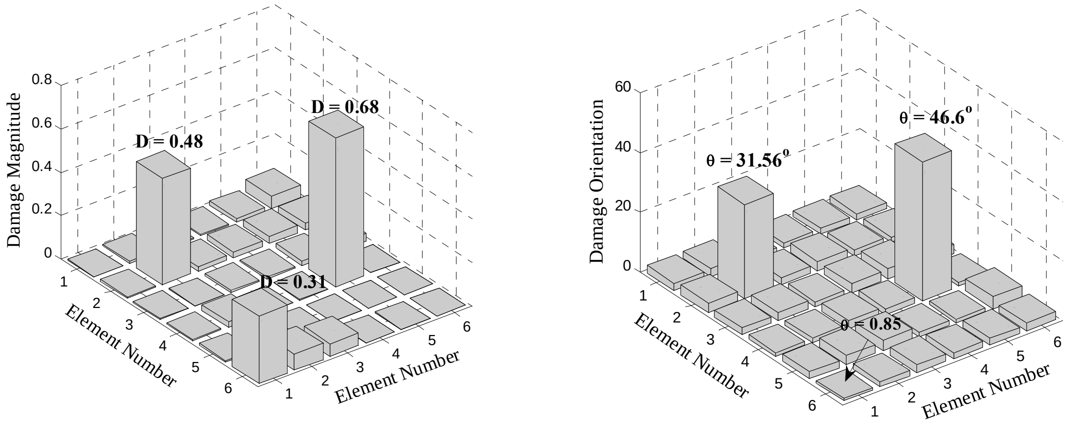

Fig. 12. Three damage identification using Hybrid—with 3% noise

to pure PSO and GA. With 3% noise added, the hybrid algorithm identified damages with a maximum of 4% error for D and 5.2% for θ. The study also shows the application of substructure approach to the damage identification of systems with a large number of damage variables.

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