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The optimum order strategy from multiple suppliers with alternative quantity discounts*

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Abstract. In this study, the coordination of one buying company and multiple suppliers in a fuzzy random environment is studied. In order to reduce the purchase risk, many buying companies purchase raw materials from multiple suppliers. The quantity distribution become particularly important in this problem. We present a multi-objective programming model (MOPM) to optimize the purchase orders. After setting up the model with fuzzy random varieties (FRVs), we use the dependent chance operator to deal with the objective functions. In order to find the optimal solutions of this problem, the equivalent model which can be solved by a improved Particle Swarm Optimization algorithm based on fuzzy random simulation (IPSO-based FRS). A numerical example is presented to illustrate the effectiveness of our proposed approach.

Keywords: supply chain management, purchasing order optimization decision, multi-objective programming model, fuzzy random, PSO

1 Introduction

The supply chain (SC) can be defined as an integrated structure involving the procurement, production, storage, distribution and control of goods. A typical supply chain comprises suppliers, manufacturing sites, distribution centers, and customers. SCs are generally complex and are characterized by numerous activities spread over multiple functions and organizations, which cause challenges for effective SC coordination. To meet these challenges, SC members must work towards a unified system and coordinate with each other.

In the field of optimal order decision, there are many studies like: Guder et al. [3] to solve a buyer's multiple item material cost minimization problem with incremental discounts offered by a single supplier. The buyer has a single resource constraint; Chauhaun and Proth [1] start by analyzing a single product purchasing problem for a manufacturer who attempts to source a fixed quantity from multiple suppliers. Each supplier quotes a unique pricing scheme which integrates a fixed cost plus a concave increasing cost in the quantity sourced. Cai et al. [4] evaluates the impact of price discount contracts and pricing schemes on the dual-channel supply chain competition. In this paper, we consider a purchasing system that contains one manufacturer and multiple suppliers. Facing the market demand, the manufacturer purchases raw materials from the suppliers, adds some value to the product, and sells it to the customers. We are to consider a complete supply chain system including procurement, production and sales. Specifically, we need to simultaneously determine the selling price and the production quantity, as well as acquire enough supplying capacity from multiple capacitated suppliers. The goal of the decisions is to maximize the total profit. The motivation of this paper comes from coordination of multiple suppliers.

Obviously, multiple suppliers helps to reduce the supplying uncertainty and maintain a reliable supply channel. Multiple suppliers can also stimulate competition among suppliers, which will reduce the supplying

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co and improve the service quality, the existence of others suppliers bring about the market competition and impels the technique improvement[2]. And from another hand, multiple suppliers is a necessity in some situations. This happens when a single supplier may have a limited supplying capacity. So, in this paper, we give a multiple objective programming model (MOPM) to solve this kind of problem.

It's common to know that uncertain information exists everywhere all the time, how can we do if we face such statements, "It is about 60 yuan" or "It is 20 tons with 80% probability", the data we get in the actual construction projects is almost like these, so fuzzy theory or stochastic theory is helpful. But the data like "It is about 20 tons with 80% probability", we should use fuzzy random theory to handle such uncertain information in fuzzy random environment. The uncertainty of SCM has been studied in some literatures, Liu [8] considered the random fuzzy variables in supplier selection; Paksoy et al. [9] presented an application of fuzzy mathematical programming model to solve network design problems for supply chains via considering aggregate production planning; Kabak and Ulengin [10] presented a possibilistic linear programming model is used to make strategic resource-planning decisions using fuzzy demand forecasts and fuzzy yield rates as well as other inputs such as costs and capacities; Hu et al. [11] shows that manufacturer's repurchase strategy can achieve the increase in the whole supply chain profit. The influence of the fuzzy randomness of the demand and the defective rate on the optimal order quantity, the whole supply chain profit and the repurchasing price is analyzed via numerical examples etc.

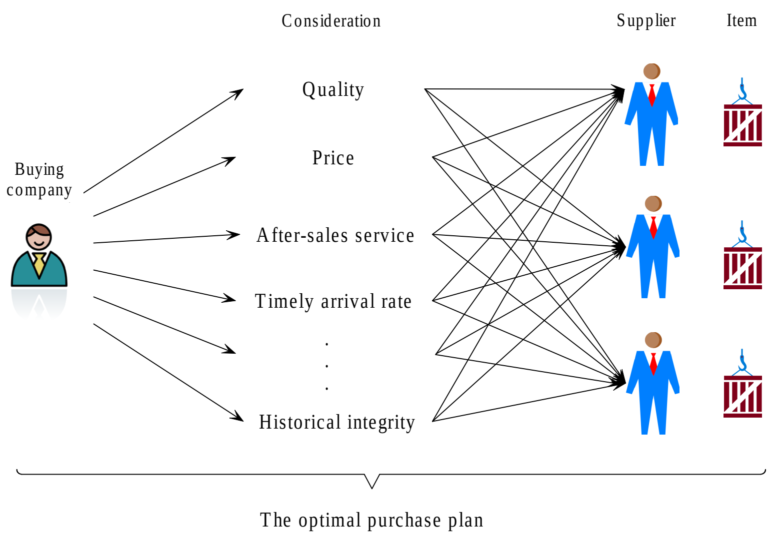

Fig. 1. The example of multiple supplier purchase problem

About fuzzy random variables, Kwakernaak [5, 6] introduced a mathematical model which has been later formalized in a clear way by Kruse and Meyer [7]. In the approach of Kwakernaak/Kruse and Meyer, a fuzzy random variable is viewed as a fuzzy perception/observation/report of a classical real-valued random variable. For better understanding this paper, some basic knowledge about fuzzy random variables are stated. These results are crucial for the rest of this paper. The model is stated as following:

Definition 1. *Given a probability space* $(\Omega, \mathcal{A}, P)$*, a mapping*

$$\mathcal{X} : \Omega \to \mathcal{F}_c(\mathbb{R})$$

is said to be a fuzzy random variable (or FRV for short) if for all $\alpha \in (0, 1]$ *the two real-valued mappings*

$$\inf \mathcal{X}_\alpha : \Omega \to \mathbb{R}, sup\mathcal{X}_\alpha : \Omega \to \mathbb{R}$$

(defined so that for all $\omega \in \Omega$ *we have that* $\mathcal{X}_\alpha(\omega) = [infX_\omega)_\alpha, supX_\omega)_\alpha]$ *are real-valued random variables.*

Example 1. Let X be a random variable defined on probability space $(\Omega, \mathcal{A}, Pr)$. If we define that for every $\omega \in \Omega, \xi(\omega) = (a, X(\omega), b)$, $X(\omega) \sim \mathcal{N}(\varepsilon, \sigma^2)$ which is a triangular fuzzy variable defined on some possibility space $(\mathcal{T}, \mathcal{P}(\mathcal{T}), Pos)$. Then, ξ is a (triangular) fuzzy random variable.

In this paper, in order to study the uncertainty more clearly, we use this kind of approach to define the fuzzy random variables.

The propose of this paper is to get the coordination of one purchaser and multiple suppliers under an uncertain environment which combined fuzzy random variables (FRVs). This paper is organized as follows: in Section 2, the mathematical model is introduced; in Section 3, a IPSO-based FRS to solve this MOPM problem is given; in Section 4, a numerical example is presented to demonstrate the efficiency of the proposed method; finally, in Section 5, concluding remarks are outlined.

2 Problem statement and model formulation

Problem in this paper focuses on the coordination of one purchaser and multiple suppliers. The proposed fuzzy random MOPM has considered to optimize several aspects separately.

2.1 Problem description

In this paper, a purchaser in order to reduce risk, purchase items from multiple suppliers. Each suppliers can provide a quantity discount according to actual situation. And the purchaser will allocation its amount of purchased among the several suppliers to get his maximum profit. The goal of purchasing decision in SC is considering whether the total amount of purchasing can meet the required production amount, whether the material quality of purchasing is substandard the process standards, whether suppliers can timely supply the items. On the basis of these conditions, to get the total cost of procurement minimum. And this goal is not easy to achieve, because usually these requirements are contradictory, for instance, low price often quality is bad. So, based on purchasing order optimization decision's main goal is how to balance between these goals, and get a relatively more satisfied purchasing strategy or decisions. The Fig. 1 describes this kind of problem.

In practice, the fuzzy variables and stochastic variables have been applied in many areas for so long time. In this paper, we employ the FRVs because it is hard to describe these weighting factors of suppliers. In this situation, we consider the weighting factors as FRVs can be more effective for get the optimal result.

2.2 Model assumptions and notation

The model was built on the following assumptions:
(1) Suppliers can provide price discount, according to different procurement using different price discount.
(2) We only consider single stage, single product and can supplied by multiple suppliers.
(3) Suppliers all have cooperative relationship before, therefore, for each supplier has a weighting factor.
(4) We consider the supply delay rate and scrap rate.

The following notation is used.

Indications:

i : set of suppliers, $i = 1, 2, \ldots, I$;
j : set of section of different price discount correspondence different purchasing quantity,
: $j = 1, 2, \ldots, J$.

Parameters:

a_{ij} : procurement to a certain quotas from supplier i, get discount rate j in corresponding discount stage;
b_{ij} : cap on the number of procurement for discount rate a_{ij};
p_i : price of product from supplier i;
h : inventory holding cost for product;
$\tilde{\bar{W}}_i$: weighting factor of supplier i;
$m_i^{\max}$: maximum capacity for product at supplier i;

Q : the total amount of need to purchase;
t_i : timely arrival rate of supplier i;
T : expected timely arrival rate of purchaser;
C_i : the ordering cost of supplier i;
S_i : scrap rate of supplier i.

Decision Variables:

Y_{ij} : binary variable indicating whether procure or not from discount stage j of supplier i;
X_{ij} : amount of procurement from discount stage j of supplier i.

2.3 Model formulation

SCM is a complex problem that involves taking decisions all time, to make a optimization strategy of purchasing order planning should consider many factors, such as cost, quality, timely arrival rate, etc. And the purchase quantity allocate in multiple suppliers should consider these criteria include quality, delivery, performance history, warranties, price, technical capability, and financial position.

2.3.1 Objective function

After analysis, we conclude that there are three objective functions in this problem.

The first one is the total cost is the minimum:

$$\min\ Z_1 = \sum_{i=1}^{I}\sum_{j=1}^{J} C_i\, Y_{ij} + \sum_{i=1}^{I}\sum_{j=1}^{J} a_{ij}\, p_i\, Y_{ij}\, X_{ij}.$$

The second one is the minimal scrap amount of procurement:

$$\min\ Z_2 = \sum_{i=1}^{I}\sum_{j=1}^{J} S_i\, X_{ij}.$$

The third one is to maximize the weights rate of the suppliers, the weighting factors are FRVs, we should translate them into determinate ones. There are many methods like expected value operator, chance-constrained operator, etc. According to actual meaning of this problem, we use dependent-chance model to deal the fuzzy random variables. Fuzzy random dependent-chance model (Fu-Ra DCM) is based on selecting the decision with maximize the chance to meet the event[16]. Here we first have a simple introduction of Fu-Ra DCM. The general model for Fu-Ra DCM is like this:

The decision maker determine the ideal objective values $\bar{f}_i$ for each objective $f_i(x,\xi)$, and maximize the chance measure of the fuzzy random events $f_i(x,\xi) \geq \bar{f}_i$ under a confidence level γ_i.

$$\begin{array}{l} \max \left\{ \begin{array}{l} Ch\{f_1(x,\xi) \geq \bar{f}_1(\gamma_1)\} \\ Ch\{f_2(x,\xi) \geq \bar{f}_2(\gamma_2)\} \\ \cdots \\ Ch\{f_n(x,\xi) \geq \bar{f}_n(\gamma_n)\} \end{array} \right\} \\ \text{s.t.}\ \ g_r(x,\xi) \leq 0 \quad r = 1,2,\ldots,p. \end{array} \tag{1}$$

wherein $\xi = (\xi_1, \xi_2, \ldots, \xi_n)$ is fuzzy random vector, $Ch\{\cdot\}$ denotes the chance measure of the fuzzy random event in $\{\cdot\}$, $\gamma_i, \eta_r, \theta_r \in [0,1]$ are the predetermined confidence level, $i = 1,2,\ldots,m, r = 1,2,\ldots,p$.

If we introduce the new variables $\delta_i, i = 1,2,\ldots,m$, then the Eq. (1) can be written as the following equivalent form,

$$\begin{array}{l} \max\{\delta_1, \delta_2, \ldots, \delta_m\} \\ \text{s.t.} \left\{ \begin{array}{ll} Ch\{f_i(x,\xi) \geq \bar{f}_i\}(\gamma_i) \geq \delta_i, & i = 1,2,\ldots,m, \\ Ch\{g_r(x,\xi) \leq 0\}(\eta_r) \geq \theta_r, & r = 1,2,\ldots,p. \end{array} \right. \end{array} \tag{2}$$

Definition 2. *If x^* is an efficient solution of problem (2), we call it as a fuzzy random dependent chance efficient solution[16].*

According to the definition of the chance measure, Eq. (2) is also can be written as:

$$\begin{array}{l} \max\{\delta_1, \delta_2, \ldots, \delta_m\} \\ \text{s.t.} \left\{ \begin{array}{ll} Pr\{\omega | Pos\{f_i(x,\xi) \geq \bar{f}_i\} \geq \delta_i\} \geq \gamma_i & i = 1, 2, \ldots, m, \\ Pr\{\omega | Pos\{g_r(x,\xi) \leq 0\} \geq \theta_r\} \geq \eta_r & r = 1, 2, \ldots, p. \end{array} \right. \end{array} \tag{3}$$

Let $x \in R^n$, for determined confidence level θ_r, η_r, if $x \geq 0$ and $Pr\{\omega | Pos\{g_r(x,\xi) \leq 0\} \geq \theta_r\} \geq \eta_r$ are tenable, then x is a feasible solution of Eq. (2) or (3).

So we can get the determined objective function which dealt with Fu-Ra DCM:

$$\begin{array}{l} \max\{\delta_1\} \\ \text{s.t. } Ch\left\{ \sum_{i=1}^{I} \sum_{j=1}^{J} \tilde{\bar{W}}_i X_{ij} \geq \bar{f}_1 \right\}(\gamma_1) \geq \delta_1 \end{array} \tag{4}$$

2.3.2 Constraint

The amount of product purchased from all suppliers should equal to the demand likes Eq. (5):

$$\sum_{i=1}^{I} \sum_{j=1}^{J} X_{ij} = Q. \tag{5}$$

The amount supplied of each supplier should less or equal to its maximum capacity amount. So the constraint likes Eq. (6):

$$\sum_{j=1}^{J} X_{ij} \leq m_i^{\max}. \tag{6}$$

The amount of purchasing product more than 0 likes Eq. (7):

$$X_{ij} \geq 0. \tag{7}$$

The timely arrive rate of each supplier should achieve the requirements of the enterprize, Eq. (8):

$$\sum_{i=1}^{I} \sum_{j=1}^{J} t_i X_{ij} \geq T Q. \tag{8}$$

The discount rate of purchasing should corresponding to the quantity of suppliers' discount like Eq. (9):

$$Y_{ij} B_{ij-1} \leq X_{ij} \leq Y_{ij} B_{ij}. \tag{9}$$

For the same supplier, only chose one discount stage to purchase:

$$\sum_{j=1}^{J} Y_{ij} \leq 1. \tag{10}$$

2.3.3 Multi-objective programming model

The whole multi-objective programming model under fuzzy random environment (A1) is established based on the above discussion.

$$
(A1)\begin{cases}
\min\ Z_1 = \sum_{i=1}^{I}\sum_{j=1}^{J} C_i\, Y_{ij} + \sum_{i=1}^{I}\sum_{j=1}^{J} a_{ij}\, p_i\, Y_{ij}\, X_{ij} \\
\min\ Z_2 = \sum_{i=1}^{I}\sum_{j=1}^{J} S_i\, X_{ij} \\
\max\ \{\delta_1\} \\
\text{s.t.} \begin{cases}
Ch\left\{\sum_{i=1}^{I}\sum_{j=1}^{J} \tilde{\bar{W}}_i\, X_{ij} \ge \bar{f}_1\right\}(\gamma_1) \ge \delta_1 \\
\sum_{i=1}^{I}\sum_{j=1}^{J} X_{ij} = Q \\
\sum_{j=1}^{J} X_{ij} \le m_i^{\max} \\
X_{ij} \ge 0 \\
\sum_{i=1}^{I}\sum_{j=1}^{J} t_i\, X_{ij} \ge T\, Q \\
Y_{ij}\, B_{ij-1} \le X_{ij} \le Y_{ij}\, B_{ij} \\
\sum_{j=1}^{J} Y_{ij} \le 1 \\
Y_{ij} \in \{0,1\},\ \forall i \in I,\ j \in J
\end{cases}
\end{cases}
$$

3 Modified PSO program based on fuzzy random simulation

For the multi-objective optimization problems, more and more scholars have done some significant work and made some progress. Because it is difficult to find a solution such that every objective gets the optimization, Pareto introduced the non-dominated solutions or Pareto optimal solutions to obtain optimal objectives without sacrificing other objective functions. However, for a complex multi-objective optimization problem, it is also difficult to obtain its Pareto optimal solution. Recently, some scholars have considered Particle Swarm Optimization (PSO) algorithm as an efficient method to find it's Pareto optimal solution.

PSO algorithm developed by Eberhart and Kennedy [15] in 1995. It is a form of swarm intelligence in which the behavior of a biological social system like a flock of birds or a school of fish is simulated. This technique uses collaboration among a population of simple search agents (called particles) to find optima in some search space, and has been shown to be effective in optimizing difficult multidimensional problems in a variety of fields, the detail process shows in Fig. 2[12, 15].

3.1 Simulation for Fu-Ra DCM

In order to solve the Fu-Ra DCM, we use the Fu-Ra simulation to compute the objective function of Fu-Ra DCM $Ch\{f_i(x,\xi) \ge \bar{f}_i\}(\gamma_i), i = 1, 2, \ldots, m$, the confidence level γ_i is predetermined.

In this paper, the objective function in the lower level dealt with DCM, according to the definition of the $Pr - Pos$ chance of fuzzy random variable which is,

$$Ch\{f_i(x,\xi) \ge \bar{f}_i\}(\gamma_i) = sup\{\delta_i | Pr\{\omega \in \Omega | Pos\{f_i(x,\xi(\omega)) \ge \bar{f}_i\} \ge \delta_i\} \ge \gamma_i\}$$

wherein $\omega = (\omega_1, \omega_2, \ldots, \omega_n), \xi(\omega) = (\xi_1(\omega_1), \xi_2(\omega_2), \ldots, \xi_n(\omega_n))$.

On the basis of Eq. (4), it can be written as

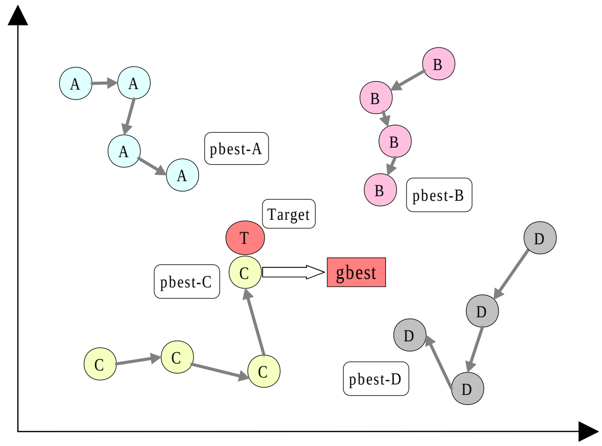

Fig. 2. The concrete process of PSO

$$Ch\left\{\sum_{i=1}^{I}\sum_{j=1}^{J}\tilde{\bar{W}}_i X_{ij} \geq \bar{f}_1\right\}(\gamma_1) \geq \delta_1 = sup\left\{\delta_i | Pr\{\omega \in \Omega | Pos\left\{\sum_{i=1}^{I}\sum_{j=1}^{J}\tilde{\bar{W}}_i X_{ij} \geq \bar{f}_1\right\} \geq \delta_i\right\} \geq \gamma_i\}.$$

The detail procedure as follows[16], the procedure of the simulation for Fu-Ra DCM:

Step 1. Generate vector $\omega_k = (\omega_{k1}, \omega_{k2}, \ldots, \omega_{kn})^T$ randomly from Ω according to the probability measure $Pr, k = 1, 2, \ldots, N$.

Step 2. Compute the possibilities of fuzzy events $\delta_{ik} = Pos\{f_i(x, \xi(\omega_k)) \geq f_i\}$ for $k = 1, 2, \ldots, N$ by fuzzy simulation.

Step 3. Set N' be the integer part of αN.

Step 4. Return the $N'th$ largest element in the sequence $\{\delta_{i1}, \delta_{i2}, \ldots, \delta_{iN}\}$. The value δ_i can be taken as the $N'th$ largest element in the sequence $\{\delta_{i1}, \delta_{i2}, \ldots, \delta_{iN}\}$.

3.2 Initializing method

A good initial solution set can help the algorithm converge efficiently and effectively. First of all, we randomly generate the initial binary phenotype particle $y \triangleq (Y_{ij,1}, Y_{ij,2}, \cdots, Y_{ij,n})$ as follows:

Constraint check

Step 1. Set $i = 1$;

Step 2. Random generate a number from (0, 1);

Step 3. If $rand(\ ) \geq 0.5$, $Y_{ij,n} = 1$, else, $Y_{ij,n} = 0$;

Step 4. If $Y_{ij,n} = 1$, then $X_{ij}^n = rand(0, Q)$, else $x_{ij}^n = 0$;

Step 5. Set $i = i + 1$, if stop criterion is met, go to Step 6, else, go back Step 2;

Step 6. Out put the $Y_{ij,n}$, x_{ij}, where $rand(\ )$ is a random number coming from the uniform distribution over the interval $[0, 1]$. Where $rand(a, b)$ is a uniformly distributed random number over the interval $[a, b]$.

3.3 Constraints check

In order to check that whether the initial value can meet the constraints or not, we use following programm to judge it.

Constraint check

Step 1. Set $flag = 1$;

Step 2. Get the initial values;

Step 3. Substitute them into every constraint;

Step 4. If they can meet the constraint, $flag = 1$, else, $flag = 0$.

Step 5. If $flag = 1$, go to next procure; If $flag = 0$, go back and reinitialization.

3.4 Update and the improvement against fall into local optimal

When a particle found a current group optimal location, other particles will quickly draw close to it. If the particle is the local optimal solution, particle swarm cannot research in the solution space. So the algorithms fall into local optimization, the problems is so-called premature convergence. So, we add a random disturbance to the current optimal solution of the whole particle swarm, which will be helpful to jump out the local optimal, and avoid premature convergence[13].

Hypothesis κ is a random variable which obey the standard normal distribution, $\kappa \sim \mathcal{N}(0,1)$, so

$$\psi_{g(h+1)} = \psi_{gh}(1+\kappa). \tag{11}$$

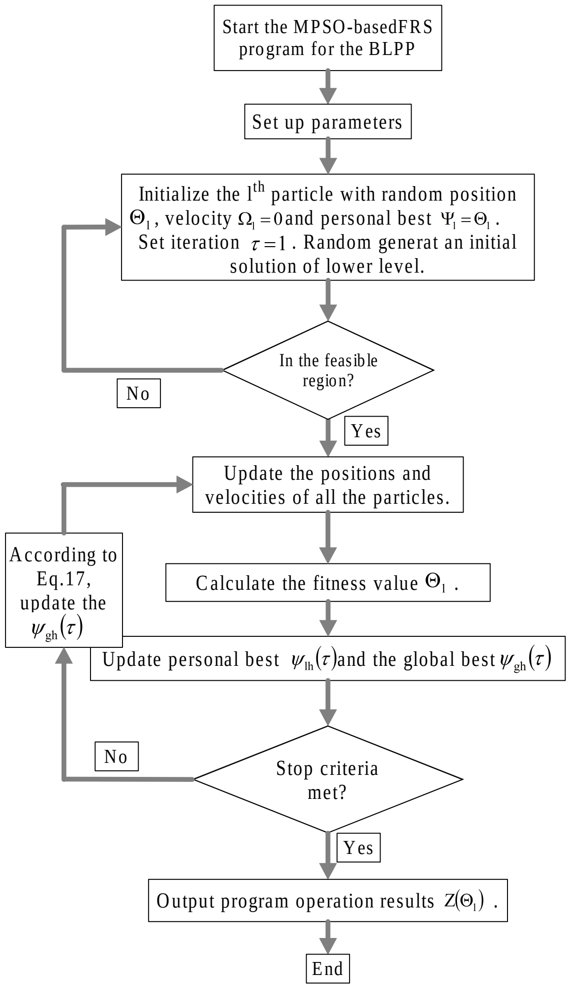

Fig. 3. The overall IPSO-based FRS program flow diagram

So, update the ψ_{gh} to $\psi_{g(h+1)}$ with Eq. (11), than we update the velocity and the position of each particle as below. The particle positions in each dimension are held to a maximal position $\theta^{\max}$ and a minimal position $\theta^{\min}$, the position in that dimension is limited to $\theta^{\max}$ and $\theta^{\min}$[14, 17].

Update the velocity of particle.

Step 1. calculate the inertia weight in the τth iteration use: $w(\tau) = w(T) + \frac{\tau - T}{1-T}[w(1) - w(T)]$

Step 2. update the velocity of particle: $\omega_{lh}(\tau+1) = w(\tau)\omega_{lh}(\tau) + c_p u(\psi_{lh} - \theta_{lh}(\tau)) + c_g u(\psi_{g(h+1)} - \theta_{lh}(\tau))$.

Update the position of particle

Step 1. update the position of particle use: $\theta_{lh}(\tau+1) = \theta_{lh}(\tau) + \omega_{lh}(\tau+1)$

Step 2. judge the position whether it in the feasible region or not: if $\theta_{lh}(\tau+1) > \theta^{\max}$, then: $\theta_{lh}(\tau+1) = \theta^{\max}$, $\omega_{lh}(\tau+1) = 0$, if $\theta_{lh}(\tau+1) < \theta^{\min}$, then: $\theta_{lh}(\tau+1) = \theta^{\min}$. $\omega_{lh}(\tau+1) = 0$.

3.5 Evaluation

In this paper, we adopted the weight-sum evaluation procedure to deal with the multi-objective model. For a given individual x , the weighted-sum objective function is given by the following equation:

$$Eval(Z) = r_1 Z_1 + r_2 Z_2 + r_3 Z_3, \tag{12}$$

wherein, the weight r for each objective is given by the decision maker, they reflect the importance of each objective in the decision makers' mind, and the weights should satisfy the equation $r_1 + r_2 + r_3 = 1$.

3.6 The overall IPSO-based FRS algorithm

For this MOPM, we use a IPSO-based FRS for searching optimal results. And we also add a random disturbance to avoid the premature convergence. To describe the algorithm model, the procedure of the proposed algorithm for solving the problem is presented as follows. And the overall algorithm's program flow diagram is presented as Fig. 3.

Table 1. Essential parameters of suppliers

Supplier i	Scrap rate S_i	Unit price p_i	Ordering cost C_i	deliverability $m_i^{\max}$	Supply timely arrival rate t_i
1	0.04	3	1000	500	96%
2	0.03	5	2000	600	98%
3	0.05	2	800	550	97%
4	0.06	3	700	750	98%

Table 2. Fuzzy random weighting factor

Note	Weighting factor	Parameter $d(\omega)$
$\tilde{\bar{W}}_1$	$(0.43, d(\omega), 0.56)$	$d(\omega) \sim \mathcal{N}(10, 16)$
$\tilde{\bar{W}}_2$	$(0.38, d(\omega), 0.47)$	$d(\omega) \sim \mathcal{N}(14, 7)$
$\tilde{\bar{W}}_3$	$(0.55, d(\omega), 0.67)$	$d(\omega) \sim \mathcal{N}(10, 25)$
$\tilde{\bar{W}}_{34}$	$(0.46, d(\omega), 0.53)$	$d(\omega) \sim \mathcal{N}(8, 15)$

Table 3. Discount stage of supplier 1

Quantity stage j	Quantity	Discount rate a_{ij}
1	$1 \leq X_{ij} < 300$	1.0
2	$300 \leq X_{ij} \leq 500$	0.9

The overall procedure of the proposed algorithm:
Step 1. Set up parameters including population size L (the number of particles), maximal and minimum position value $\theta^{\max}$ and $\theta^{\min}$, inertial weight (ω). Two acceleration constant, c_p and c_g and a uniform random number u is in the interval $[0, 1]$.
Step 2. Initialize the l^{th} particle with random position Θ_l in the range $[\theta^{\min}, \theta^{\max}]$, velocity $\Omega_l = 0$ and personal best $\Psi_l = \Theta_l$ for $l = 1, \ldots, L$. Set iteration $\tau = 1$.
Step 3. Constraints check. If in the feasible region, go to Step 4, otherwise, back to Step 2.
Step 4. Perform the following actions to all the particles.
Step 4.1. Update the velocity and position for each l^{th} particle.
Step 4.2. Substitute the y and x into the objective functions. For $l = 1, \ldots, L$ compute the performance

measurement of R_l, and set this as the fitness value of Θ_l,represented by $Z(\Theta_l)$.
Step 4.3. Update the pbest: For $l = 1, \ldots, L$, update $\Psi_l = \Theta_l$, if $Z(\Theta_l) < Z(\Psi_l)$.
Step 4.4. Update the gbest: For $l = 1, \ldots, L$, update $\Psi_g = \Theta_l$, if $Z(\Psi_l) < Z(\Psi_g)$.
Step 4.5. Constraints check. If in the feasible region, go Step 5, otherwise, back Step 4.1.
Step 5. If the stopping criterion is met, i.e., $\tau = T$, go to Step 6. Otherwise, $\tau = \tau + 1$ and go to step 6.
Step 6. According to Eq.11 update the ψ_{gh} to $\psi_{g(h+1)}$. Go to Step 4.
Step 7. Output program operation results $Z(\Theta_l)$. Algorithm to the end.

4 Numerical example

In this section, a numerical example is given, which shows the application of the proposed models and algorithms. There are 1 buying company and 4 suppliers and each supplier has a capacity limit. They can give quantity discount due to different purchase amount. The detail data is showed in Tab. 3.6. The weighting factors are translated into fuzzy random variables, as shown in Tab. 1. The 4 suppliers quantity discount is shown in Tabs. 2 $\sim$ 4.

Table 4. Discount stage of supplier 2

Quantity stage j	Quantity	Discount rate a_{ij}
1	$1 \leq X_{ij} < 300$	1.0
2	$300 \leq X_{ij} < 500$	0.9
3	$500 \leq X_{ij} \leq 600$	0.8

Table 5. Discount stage of supplier 4

Quantity stage j	Quantity	Discount rate a_{ij}
1	$1 \leq X_{ij} < 400$	1.0
2	$400 \leq X_{ij} < 650$	0.9
3	$650 \leq X_{ij} \leq 750$	0.8

Table 6. Result from IPSO-based FRS

Variable	x_{11}	x_{12}	x_{21}	x_{22}	x_{23}	x_{31}	x_{41}	x_{42}	x_{43}
value	0	500	0	0	600	150	0	0	750
Variable	y_{11}	y_{12}	y_{21}	y_{22}	y_{23}	y_{31}	y_{41}	y_{42}	y_{43}
value	0	1	0	0	1	1	0	0	1
Function value	7284.76134								

It should be noted that the FRVs in the tables are obtained by the following steps:
(1) collect previous data and divide them into several groups for certain periods;
(2) the lower bound of the fuzzy random number (i.e., α) is the minimum value of all groups;
(3) the upper bound of the fuzzy random number (i.e., β) is the maximum value of all groups;
(4) suppose the median value of every group to be a random variable (i.e., $d(\omega)$) and follow a normal distribution, and use maximum likelihood method to estimate the parameters of its distribution;
(5) use goodness-of-fit testing to justify the appropriateness of the normal distribution in modeling the observed data;
(6) finally, the fuzzy random number, $(\alpha, d(\omega), \beta)$, is derived.

5 Results and analysis

Now, let us consider the Model (A1) with the above data and use the IPSO-based FRS to deal with it. In this paper, we use MATLAB 7.0 on a Pentium 4, 1.83GHz clock pulse with 2048 MB memory. And we set the parameters as follows: Population Size $L = 10$, Iteration number $T = 100$, $c_p = c_g = 2$, Inertia weight $w_{(1)} = 0.9$ and $w_{(T)} = 0.1$, respectively. After running the program 10 times, we get the best solution as Tab. 4.

Fig. 4. shows that the detailed distribution of objective value obtained by IPSO-based FRS in different generations. It shows that the objective function is gradually smaller from one generation to another, which keeps consistent with the evolutional idea of IPSO-based FRS.

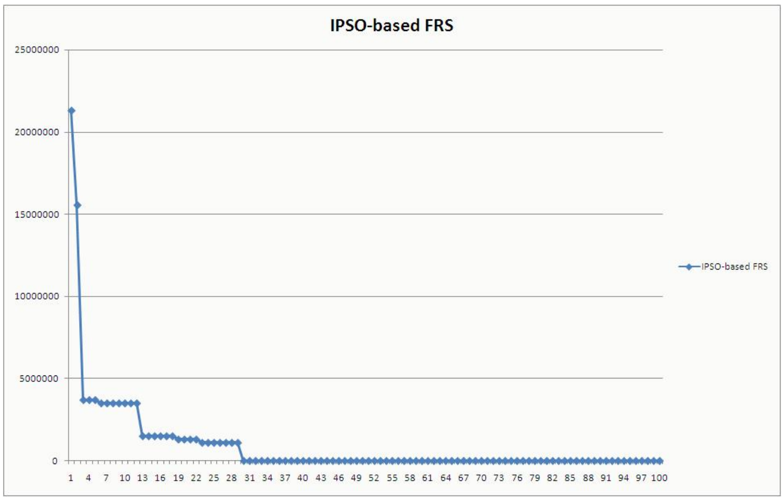

Fig. 4. The result from IPSO-based FRS

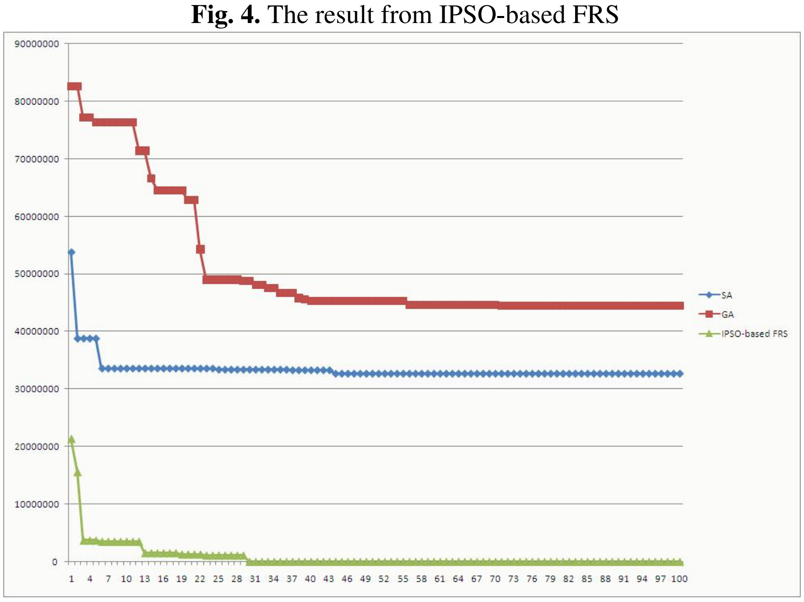

Fig. 5. Comparision

The genetic algorithm (GA), which was first introduced by Holland (1975), uses the Darwinian concept to solve the problems. GA belongs to the class of heuristic optimization techniques, and it is very useful when a large search space with knowledge of how to solve the problem is presented. And simulated annealing (SA) is a generic probabilistic metaheuristic for the global optimization problem of locating a good approximation to the global optimum of a given function in a large search space. We use GA and SA to compare with our IPSO-based FRS. After run each algorithm 10 times, we get the best results as shown in Fig. 5. The detail value of these intelligent optimization algorithms clearly show that IPSO-based FRS's best_fitness value is smaller than the GA's and SA's. After improve, we overcome the classical PSO will easily get into local optimal solution instead of globally optimal solution. The IPSO-based FRS can get the better best_fitness value and each operation results is very close. So the IPSO-based FRS is more steady and strong performer to solve this kind of MOPM.

6 Conclusion

The analysis of alternate supplier base pricing schemes in this paper provides guidance for a buying firm's optimal purchase strategy. Given a total order quantity Q that the buying firm must procure, a deterministic model is analyzed which highlights the importance of supplier capacity on the buying firm's sourcing decision. Specifically, if all of the suppliers in the base possess enough capacity to individually provide Q units, then it is optimal to single source from the least cost supplier evaluated at Q units. The case when the single supplier sourcing strategy is not optimal is when the minimum cost supplier does not have adequate capacity to fill an entire order. In practice, the uncertain information exists anytime and anywhere. The authors also expatiate the fuzzy random environment. And explain the necessity of using the fuzzy random theory to handle such kind of problem. Then we set up a MOPM to deal with this problem and propose the IPSO-based FRS to solve it. At last, we make a brief comparison between the IPSO-based FRS and classic intelligence algorithm to make out the merits of our algorithm.

Although this MOPM in this paper should be helpful for solving some real world problems, detailed analysis and further research is necessary to reveal more properties of a good method of solving other problems.

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