

## A probabilistic model for the spread of HIV infection among injection drug users\*

Changiz Eslahchi<sup>1</sup>, Hamid Pezeshk<sup>2,3</sup>, Mehdi Sadeghi<sup>4,5</sup>, Philippe Giabbanelli<sup>6</sup>, Fateme Movahedi<sup>1</sup>, Vahid Dabbaghian<sup>6†</sup>

<sup>1</sup> Faculty of Mathematics, Shahid Beheshti University, Tehran 19839, Iran

<sup>2</sup> Department of Statistics and Center of Excellence in Biomathematics, University of Tehran, Tehran 14155, Iran

<sup>3</sup> School of Mathematics, Institute for Research in Fundamental Science, Tehran 19395-5746, Iran

<sup>4</sup> Department of Biophysics, National Institute of Genetic Engineering and Biotechnology, Tehran 14178, Iran

<sup>5</sup> School of Computer Science, Institute for Research in Fundamental Science, Tehran 19395, Iran

<sup>6</sup> MoCSSy Program, The IRMACS Centre, Simon Fraser University, Burnaby V5A 1S6, Canada

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**Abstract.** By sharing contaminated needles, injecting drug users contribute in a significant manner to the spread of the human immunodeficiency virus (HIV) in Asia and in some European countries. Furthermore, injecting drug users may also be sex workers, and risky sexual activities allow the virus to spread to other parts of the population. Mathematical models of needle sharing have been used to evaluate the success of needle exchange programs, and have led to advances such as new legislations. We designed a compartmental model to analyse how injecting drug users may start or cease sharing needles under social influences, and may become infected with HIV when sharing. While similar models have been proposed for various aspects of HIV, our approach differs by using discrete Markov chains in the analysis instead of the differential equations or next-generation matrix commonly employed. Our simulations showed that the prevalence of HIV depended very little on the probability of transmission of HIV when sharing a needle, but almost only on the encouragement and discouragement regarding needle sharing in the community. By measuring the cost of resources required to decrease factors encouraging needle sharing and to increase discouraging ones, our model can be refined to provide an estimate of the expected prevalence of HIV among injecting drug users.

**Keywords:** HIV, injection drug users, public intervention, Markov property, transition dynamics

### 1 Introduction

Sharing contaminated injecting equipment among drug users is a major mode of HIV transmission. While the use of contaminated needles has decreased in western Europe, it remains the main cause of HIV transmission in Poland as well as in the Baltic states<sup>[19]</sup>. Furthermore, significant increases in the prevalence of HIV in injecting drug users (IDUs) were found in Asia: from 11% in 2002 to 18% in 2004 in the Sichuan province of China, from less than 1% in 2004 to 26% in 2005 for the most populated city of Pakistan, and from 9% in 1996 to 34% in 2005 in Vietnam; further statistical data is available in [12]. Given that a number of IDUs are also being paid to have sex and may do it without protection, such as 64% in Sichuan for one month, it also allows the epidemic to spread to other parts of the population<sup>[18]</sup>. Thus, preventing IDUs from sharing contaminated equipment is a central element against the HIV epidemic: it initiated the creation of needle exchange programs

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† Corresponding author. Tel.: +1(778)782-7854, fax: +1(778)782-7065. E-mail address: vdabbagh@sfu.ca.

for numerous cities in the early 1990s<sup>[11]</sup> which, despite controversies, were shown to be efficient compared to cities that did not promote them<sup>[9]</sup>.

Sharing a needle is a complex process. It involves economic (e.g., the cost of a needle can instead be used to purchase drugs) as well as social factors: sharing a needle can bind a friendship in the subculture of IDUs and refusing to do so may be deemed offending to the group<sup>[13]</sup>. Thus, mathematical models offer an insight into the spread of HIV in this population by abstracting different factors. Models can be broadly classified in two kinds. Firstly, models can be discrete, and usually focus on the individual level: the individual is an explicit entity (e.g., a node in a graph, in cell in a cellular automaton, etc.) and is connected to another individual when a needle is shared<sup>[11, 15]</sup>. Secondly, models can be continuous: the population is then classified into classes and can flow through them with given probabilities<sup>[7]</sup>. Continuous models have been used to promote real-world changes: for example, Kaplan's model suggested that the prevalence of HIV among IDUs could be reduced by a third using needle exchange<sup>[10]</sup>, which led the Connecticut legislature to take actions such as decriminalizing syringe possession. They have also been employed to model other aspects of HIV such as the infection at the cell level, for which numerous references can be found in [5, 14].

In Section 2, we introduce a continuous model in which the dynamics of the population are driven by positive and negative socio-environmental influences. Compared to the previous approaches, we do not aim at evaluating a particular needle-exchange program but rather at analyzing the importance of the parameters underlying the model. For example, our work can be used in the design phase of a policy, to estimate the impact of promoting safer behaviours regarding drug injection. Furthermore, we use the technique of Markov chains, which is an interesting alternative to the mainstream approaches in epidemic modelling, namely studying differential equations or the next-generation matrix<sup>[6]</sup>. Markov chains have been used recently to model tuberculosis<sup>[20]</sup>, however the authors focussed on simulations. In order to provide a broader understanding of the dynamics in our model, we first study it formally, and then we perform simulations in Section 3. In particular, our simulations show that the prevalence of HIV in the population depends mainly on the socio-environmental influences. In order to provide a clear illustration, we perform both a simulation of the overall dynamic and a breakdown per classes. Finally, we summarize our main findings in Section 4 and discuss applications of our model as well as potential extensions. In particular, the technique of Markov chains illustrated in this article can be extended with a positional (non homogeneous) Markov chain which allows to consider that influences are exerted differently depending on the state of an individual.

## 2 Model and theoretical behaviour

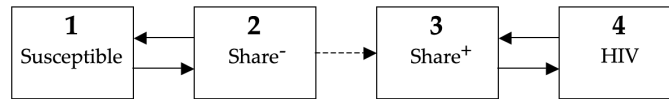
### 2.1 Compartmental model

Our model has four components, called labelled compartments or epidemiological classes<sup>[8]</sup>, which stand for different categories of the population:

- Susceptible. IDUs who do not share injection equipment and are HIV<sup>-</sup>.
- Share<sup>-</sup>. IDUs who share injection equipment and are HIV<sup>-</sup>. Individuals in this class can become infected when they share with individuals who are HIV<sup>+</sup>, and in this case they join such individuals in the class Share<sup>+</sup>. The other option is to stop sharing needle, in which case the individual moves back to the Susceptible class.
- Share<sup>+</sup>. IDUs who share injection equipment and are HIV<sup>+</sup>. Since it is not possible for such individuals to revert to HIV<sup>-</sup>, the only possibility is to stop sharing needles, in which case they move to the HIV class.
- HIV. Individuals who became HIV<sup>+</sup> by sharing contaminated needles and subsequently ceased sharing. Since individuals may be influenced to share again, they can return to the state Share<sup>+</sup>.

This model focuses on injection drug users, thus it entails that the only way for such individuals to become HIV<sup>+</sup> is by sharing needles. Exogenous factors such as sexual relationships are clearly at work in the real world, and could be encompassed in this model provided additional classes such as the ones discussed in [3]. Our model is illustrated in Fig. 1, in which classes are represented as boxes and arrows denote transitions between classes. We distinguish two types of transitions: solid arrows represent transitions due to socio-environmental influences, such as the incentive to start sharing injection equipment, and the dashed arrow

represents the probability  $p$  of becoming infected with HIV by sharing injection equipment. The assumption of models based on epidemiological classes is that all transitions that are not explicitly specified are either indirect or impossible. For example, we consider that one cannot directly transit from sharing and being HIV<sup>-</sup> to not sharing and being HIV<sup>+</sup>: such a transition is done in an indirect manner by first becoming HIV<sup>+</sup> while sharing and then ceasing to share. Similarly, one cannot transit directly from being HIV<sup>+</sup> to being HIV<sup>-</sup>: in this case, since no indirect transitions allow it then such a change is considered impossible.



**Fig. 1.** Our model is based on 4 epidemiological classes specifying whether individuals share needles and are infected by HIV

### 2.2 Transitions using a discrete Markov chain

We abstract the socio-environmental factors exerted on individuals in two types:  $\alpha$ , accounting for how much individuals are discouraged from sharing needles, and  $\beta$  accounting for how much individuals are encouraged to share needles. The primary goal of our model is to describe the impact of influences  $\alpha$  and  $\beta$  on the prevalence of HIV due to sharing infected needles in the community. In order to do so, we study the theoretical behaviour of our model in this section, using a discrete Markov chain, and we analyze particular values using simulations in the next section.

We represent the transitions between classes using the transition matrix below, which corresponds to the discrete Markov chain underlying our model:

$$P = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 1 - \beta & \beta & 0 & 0 \\ \alpha(1 - p) & (1 - \alpha)(1 - p) & p & 0 \\ 0 & 0 & 1 - \alpha & \alpha \\ 0 & 0 & \beta & 1 - \beta \end{bmatrix} \end{matrix}$$

This matrix reads as follows: susceptible individuals are influenced by  $\beta$  to share injection equipment (transition from state 1 to state 2 with probability  $\beta$ ), and so are HIV<sup>+</sup> individuals who temporarily ceased sharing (transition from state 4 to 3 with probability  $\beta$ ). Individuals sharing needles but not yet infected may become infected with probability  $p$ , and those who did not (i.e., the other  $1 - p$  individuals) may cease sharing with probability  $(1 - p) \times \alpha$  (transition from state 2 to state 1).

The state of an individual at time  $t$  is denoted by the random variable  $X_t = 1, 2, 3, 4$ . For example, the probability to go from state 2 (Share<sup>-</sup>) to state 3 (Share<sup>+</sup>) is denoted by  $P(2, 3) = P\{X_{t+1} = 3 | X_t = 2\} = p$ , which reads as the probability that the individual is in state 3 at time  $t + 1$  knowing that he was in state 2 at time  $t$ . We assume that the probability of a transition  $P(i, j)$  is independent of the time  $t$ : in other words, the system is specified by a time-homogeneous Markov chain (also called stationary Markov chain). Thus, the probability transition function is denoted by:

$$P(i, j) = P\{X_{t+1} = j | X_t = i\} = P\{X_t = j | X_{t-1} = i\} \text{ for } i, j = 1, 2, 3, 4.$$

Given two states  $i$  and  $j$ ,  $j$  is said to be accessible from  $i$  if there is a positive probability to go from  $i$  to  $j$ . Using this definition, the following accessibility matrix of the Markov chain expresses states that are accessible by a +:

$$P = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} + & + & + & + \\ + & + & + & + \\ 0 & 0 & + & + \\ 0 & 0 & + & + \end{bmatrix} \end{matrix}$$

A state is said to be transient when there is a positive probability that, starting from this state, one never returns to it. Thus,  $T = \{1, 2\}$  is the set of transient states, since one may become infected by HIV and can never return to a state requiring HIV<sup>-</sup>. A state that is not transient is said to be recurrent, thus  $R = \{3, 4\}$  is the set of recurrent states. By definition ([2], page 90), the probability to start in a transient state and to end in a recurrent state is 1 (i.e., the probability of being in a recurrent state tends to 1 as the time tends to infinity). In other words, all individuals in states 1 or 2 will ultimately move to states 3 or 4. Intuitively, if there is a positive probability that individuals in a fixed population share needles and also a positive probability that individuals become infected with HIV when sharing, then, for a long enough time, all individuals end up infected with HIV.

To specify the behaviour for a given time  $k$ , we define  $f^k(\alpha, \beta)$  as the probability that an individual moves from state 2 to 3 after exactly  $k$  time units. It follows that if there are  $n$  individuals in states  $T = \{1, 2\}$  at time  $t = 0$ , then we expect to have  $f^k(\alpha, \beta) \times n$  individuals moved to states  $R = \{3, 4\}$  at time  $t = k$ . Therefore, we study the value of  $f^k(\alpha, \beta)$  in the next section in order to gain an insight into the likeliness that individuals become infected with HIV due to needle sharing.

### 3 Simulations

#### 3.1 Probability of becoming seropositive

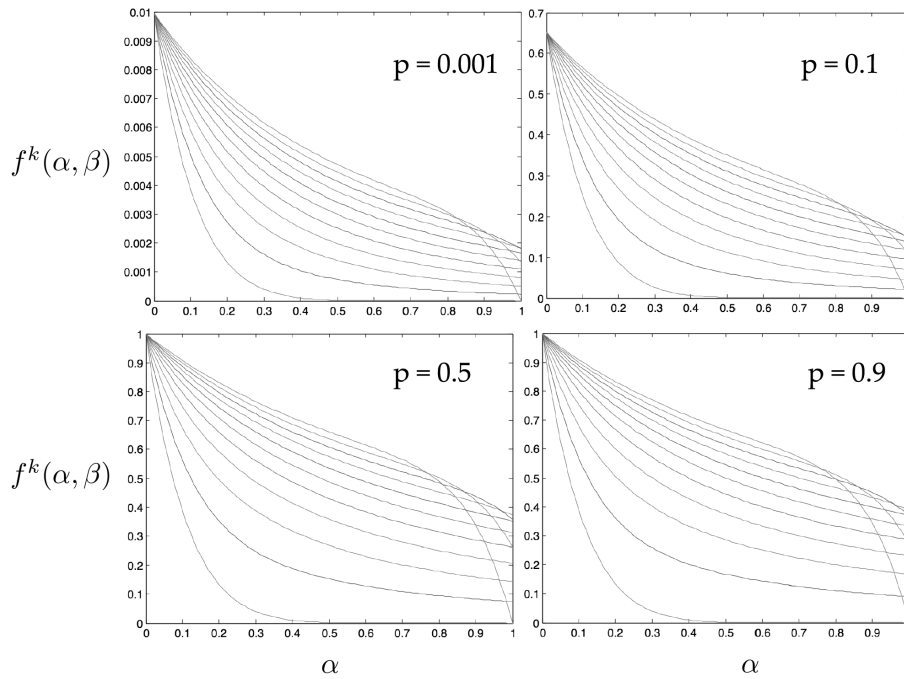
As shown in the previous section, the behaviour of the system mainly depends on the probability  $f^k(\alpha, \beta)$  that individuals move from non-seropositive to seropositive. In this section, we investigate the influence of the parameters  $\alpha$ ,  $\beta$  and  $p$  on  $f^k(\alpha, \beta)$ . Figs. 2 and 3 show the value of  $f^k(\alpha, \beta)$  for  $k = 10$  and  $k = 1000$  respectively, as a function of  $\alpha$ . The curves correspond to values of  $\beta$  from  $\beta = 0$  (lowest initial curve) to  $\beta = 1$  (highest initial curve), by steps of 0.1.

Two features of the expected behaviour are confirmed by the curves. Firstly, if individuals are not encouraged to share needles ( $\beta = 0$ ) then a small value of the factor  $\beta$  encouraging safe behaviours suffices to reduce the proportion of seropositive individuals to almost zero. Secondly, the fraction of seropositive individuals decreases with  $\alpha$ : the more individuals are encouraged toward safe behaviours, and the less they are infected. An interesting observation from the curves is that the probability  $p$  of becoming infected with HIV by sharing injection equipment has very little impact on the values for a short forecasting  $k = 10$ , and almost no impact on the long forecasting  $k = 1000$ .

#### 3.2 Quantity of individuals in each class over time

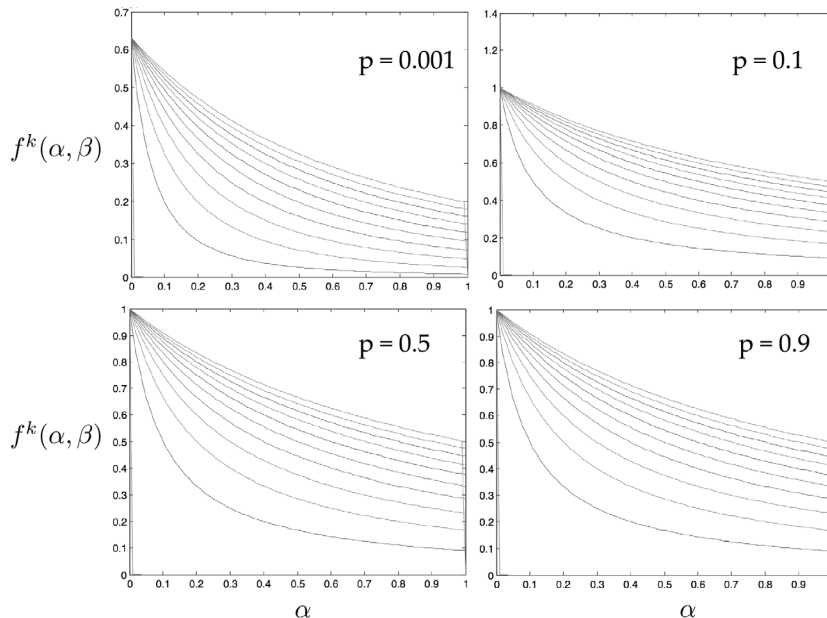
In this section, we investigate the influence of  $\alpha$ ,  $\beta$  and  $p$  on the number of individuals in each class after a given time. This detailed breakdown per class provides a concrete example to the dynamics investigated in Section 3.1, which were interested in the probability of going from non-infected classes 1 and 2 to infected classes 3 and 4. In our simulation, we follow the evolution of 10000 susceptible individuals, who thus start in class 1. Our results for different combinations of parameters values are summarized in Tabs. 1 to 3. Each cell contains the distribution of the population using the format  $[x_1, x_2, x_3, x_4]$ , where  $x_i$  is the number of individuals in class  $i$ .

These simulations confirmed the very strong impact the socio-environmental influences  $\alpha$  and  $\beta$  compared to the probability of transmission  $p$ . For example, let's consider a probability  $p = 0.1$  (see Tab. 1), an influence  $\beta = 0.1$  towards needle sharing, and the distribution of the population after  $k = 10$  time steps. If there is no factor discouraging needle sharing (i.e.,  $\beta = 0$ ) then 6035 individuals end up seropositive and share needles. However, applying a small discouragement with  $\beta = 0.1$  yields a strikingly different result in which only 2535 individuals are infected. As we pointed out, in a closed population having no births or deaths, all individuals eventually become infected. In other words, the epidemic cannot be eradicated, and thus the efficiency of  $\alpha$  is evaluated as to how much the epidemic is slowed down. In this respect,  $\beta$  is particularly efficient: with  $\beta = 0$ , after a hundred time steps all individuals are seropositive and share needles, which indicates a very high risk of infecting newcomers; however, with  $\beta = 0.1$ , after a hundred time steps almost



**Fig. 2.**  $f^k(\alpha, \beta)$  as a function of  $\alpha$  (horizontal axis) and  $\beta$  (curves), for  $k = 10$  and different values of  $p$

all the population has been infected but only half of it still shares needles while the other half opted for a safer behaviour, which limitates the dangerousness of the epidemics. Furthermore, we notice that, for a large enough time, with  $\beta = 0.1$ , the quantity of infected individuals sharing and not sharing tends toward equilibrium (at  $k = 500$  we have 5287 and 4743 respectively, and at  $k = 1000$  we have 5188 and 4812 respectively) due to a slow increase of the latter category.



**Fig. 3.**  $f^k(\alpha, \beta)$  as a function of  $\alpha$  (horizontal axis) and  $\beta$  (curves), for  $k = 1000$  and different values of  $p$

Tabs. 1 and 2 show that increasing the probability  $p$  of becoming infected when sharing needles from  $p = 0.1$  to  $p = 0.5$  has only a minor impact. For example, for  $\alpha = \beta = 0.1$ , the final quantities differ by

around 2%. Similarly, for  $\alpha = 0.1$  and  $\beta = 0.7$ , the quantity of infected individuals changing needles changes from 9610 to 9608, whereas other infected individuals change from 390 to 392. These final quantities also lie in the same range for  $p = 0.9$ , as summarized in Tab. 3.

**Table 1.** Quantity of individuals per class after  $k$  time steps for  $p = 0.01$

$\alpha$	$\beta$	$k = 10$	$k = 100$	$k = 500$	$k = 1000$
0	0.1	[3505, 460, 6035, 0]	[1, 0, 9999, 0]	[0, 0, 10000, 0]	[0, 0, 10000, 0]
0	0.5	[6, 54, 9940, 0]	[0, 0, 10000, 0]	[0, 0, 10000, 0]	[0, 0, 10000, 0]
0	0.7	[0, 15, 9985, 0]	[0, 0, 10000, 0]	[0, 0, 10000, 0]	[0, 0, 10000, 0]
0.1	0.1	[5056, 2609, 1704, 831]	[187, 100, 5068, 4645]	[0, 0, 5287, 4743]	[0, 0, 5188, 4812]
0.1	0.7	[551, 3513, 5718, 218]	[0, 3, 9578, 419]	[0, 0, 9580, 420]	[0, 0, 9610, 390]
0.5	0.5	[6402, 2329, 2040, 1029]	[235, 90, 6473, 3202]	[0, 0, 6707, 3293]	[0, 0, 6594, 3406]

**Table 2.** Quantity of individuals per class after  $k$  time steps for  $p = 0.5$

$\alpha$	$\beta$	$k = 10$	$k = 100$	$k = 500$	$k = 1000$
0	0.1	[3505, 460, 6035, 0]	[1, 0, 9999, 0]	[0, 0, 10000, 0]	[0, 0, 10000, 0]
0	0.5	[6, 54, 9940, 0]	[0, 0, 10000, 0]	[0, 0, 10000, 0]	[0, 0, 10000, 0]
0	0.7	[0, 15, 9985, 0]	[0, 0, 10000, 0]	[0, 0, 10000, 0]	[0, 0, 10000, 0]
0.1	0.1	[3874, 421, 3911, 1794]	[4171, 0, 5308, 4691]	[0, 0, 5220, 4780]	[0, 0, 5302, 4698]
0.1	0.7	[10, 25, 9599, 366]	[0, 0, 9588, 412]	[0, 0, 9560, 440]	[0, 0, 9608, 392]
0.5	0.5	[454, 217, 6279, 3050]	[0, 0, 6653, 3347]	[0, 0, 6613, 3387]	[0, 0, 6752, 3248]

**Table 3.** Quantity of individuals per class after  $k$  time steps for  $p = 0.9$

$\alpha$	$\beta$	$k = 10$	$k = 100$	$k = 500$	$k = 1000$
0	0.1	[3455, 42, 6503, 0]	[1, 0, 9999, 0]	[0, 0, 10000, 0]	[0, 0, 10000, 0]
0	0.5	[6, 0, 9994, 0]	[0, 0, 10000, 0]	[0, 0, 10000, 0]	[0, 0, 10000, 0]
0	0.7	[0, 0, 10000, 0]	[0, 0, 10000, 0]	[0, 0, 10000, 0]	[0, 0, 10000, 0]
0.1	0.7	[0, 0, 9609, 391]	[0, 0, 9596, 411]	[0, 0, 9589, 411]	[0, 0, 9623, 377]
0.1	0.1	[3514, 40, 4377, 2069]	[0, 0, 5259, 4741]	[0, 0, 5303, 4697]	[0, 0, 5326, 4674]
0.5	0.5	[19, 3, 6628, 3350]	[0, 0, 6670, 3330]	[0, 0, 6654, 3346]	[0, 0, 6524, 3476]

#### 4 Summarizing remarks

We showed how to apply discrete Markov chains to analyze the dynamics of a population of injecting drug users specified by a compartmental model. We considered that the community could be discouraged or encouraged regarding needle sharing, by factors  $\alpha$  and  $\beta$  respectively. From the simulations, we observed that the fraction of individuals becoming seropositive depends almost only on  $\alpha$  and  $\beta$ , and not on the probability  $p$  of transmission when sharing needles. In other words, if we know the percentage of individuals who start sharing needles and the percentage who ceases sharing, then the simulations provide an estimate of the percentage of the population living with HIV. Thus, beside the alternative approach that we offer to compartment modelling, our model is also a first step toward helping policy makers at estimating the impact of measures in communities encouraged toward needle sharing for social or economic reasons. Applying the model on data recorded in different communities would be an important step forward. Indeed, it could quantify the accuracy of our predictions and offer a better understanding of the different phenomena at place that compose  $\alpha$  and  $\beta$ . Then, the model could be refined to better represent specific communities. However, this is a challenging task: there is a myriad of social reasons that may encourage an individual to start sharing needles<sup>[16, 17]</sup>, and choosing an appropriate scale to quantify some of these reasons is not straightforward. Furthermore, discouraging

needle sharing can be done in various ways, which should be combined in the model to better study the cost tradeoffs<sup>[21]</sup>. As it was already suggested<sup>[4]</sup>, it would thus be natural to foster the interdisciplinary approach in the development of this model by integrating knowledge from fields such as sociology and economy.

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