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A Matlab/Simulink-based method for modelling and simulation of split Hopkinson bar test*

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Abstract. The split Hopkinson bar (SHB) technique is widely used to determine dynamic properties of materials at high strain rates. There are several basic assumptions, especially the assumption of uniform stress distributed in specimen, in the foundation of SHB. Thus, test design, analysis and data validation are significantly important to get enough accurate results. One-dimensional (1-D) simulation is such a basic approach for those purposes. But generally, the simulation needs complicated processes of deriving and programming by the theory of 1-D stress wave. This paper develops a brief method for 1-D simulation of SHB test using the software package Matlab/Simulink. The examples show its validity and the features of visualization, modularization, high concision and high efficiency. This study successfully employs Matlab/Simulink into the field of modelling and simulation of 1-D stress wave propagation for the first time. It would have a theoretical contribution for further studies.

Keywords: split Hopkinson bar (SHB), one-dimensional (1-D) stress wave theory, Matlab, Simulink, simulation, dynamic system

1 Introduction

Presently, split Hopkinson bar (SHB) tests have been widely used to determine dynamic properties of materials at high strain rates[3, 5, 8], and also been extended to other applications of dynamic tests[6, 7]. As we know, since there are several basic assumptions (such as the assumption of uniform stress distributed in the specimen) in the foundation of SHB[12], the results are not always reliable. So, most of the researchers paid much attention to test design, analysis and data validation in using SHB facilities[2, 4, 9–11, 13]. One-dimensional (1-D) simulation of stress wave propagation in SHB test is a basic and useful approach for those purposes. But unfortunately, the simulation often needs deriving and programming by the theory of 1-D stress wave, which is a rather complicated process[9, 13].

Simulink® is a program package running in Matlab® as a companion[1]. This software is very efficient and convenient in modelling and simulation of dynamic systems, especially of control systems, signal-processing systems, etc. Using it, the users need only focus on the most important things, such as developing algorithms, analyzing and visualizing simulations, customizing the simulation environment and defining parameters. But pitifully, we found nobody made use of this tool in modelling and simulation of stress propagation.

This paper, for the first time, develops a Matlab/Simulink-based method for modelling and simulation of 1-D stress wave propagation in SHB tests. After the presentation of the background knowledge and the basic method of modelling and simulation, several examples are given to illustrate the features of our method.

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2 Foundation of 1-D stress wave and SHB

2.1 Basic behaviours of 1-D stress wave

The basic behaviours of 1-D elastic stress wave include reflection, transmission and linear superposition.

Reflection and transmission occur when a stress wave propagates at an interface with varying generalized mechanical impedance. If a stress wave propagates from bar 1 to bar 2, their generalized mechanical impedances are $(\rho CA)_1$ and $(\rho CA)_2$, respectively, and the relative cross-sectional area and relative impedance are denoted as

$$\alpha = \frac{A_1}{A_2}, \ \ \beta = \frac{(\rho CA)_1}{(\rho CA)_2}, \tag{1}$$

the reflection and transmission coefficients can be expressed as

$$R_{12} = \frac{1-\beta}{1+\beta}, \ \ T_{12} = \frac{2\alpha}{1+\beta}, \tag{2}$$

respectively. Thus, if the incident stress pulse is σ_I , the reflected and transmitted waves can be denoted as $R_{12}\sigma_I$ and $T_{12}\sigma_I$, respectively.

Furthermore, the principle of superposition is available for 1-D elastic stress wave. That is to say, the total stress at a certain point equals to the linear superposition of all the values of stress waves propagating there.

2.2 Foundation of SHB

There are three kinds of conventional SHB: split Hopkinson pressure bar (SHPB), split Hopkinson tension bar and split Hopkinson torsion bar. Here as an example, we briefly introduce the foundation of SHPB.

As shown in Fig. 1 (a), a short specimen is sandwiched between the input bar and the output bar. An incident pulse (often obtained by impact) propagates from the input bar into the output bar through the specimen. The strain gauges glued on the two bars are used to measure the incident, reflected and transmitted strain waves, i.e. ε_I, ε_R, ε_T, respectively, for calculating the stress and strain histories in/of the specimen. At last the stress-strain relationship is obtained by time-elimination.

Actually, the stress wave in the specimen reflects and transmits back and forth a number of times until equilibration, as shown in Fig. 1 (b). Here the variables σ_I, R_{BS}, R_{SB}, T_{BS} and T_{SB} are defined same to the above sub-section 2.1, and $\bar{t} = t/\tau$ denotes non-dimensional time (t is the time and τ is the duration for a stress wave to travel through the specimen). Hence, the reflected strain wave ε_R equals to the superposition of all the left-handed strain waves in the input bar and the transmitted strain wave ε_T equals to the superposition of all the right-handed strain waves in the output bar.

Eq. (3) is the most widely used formula group for stress-strain relationship calculation in SHPB tests, where only the reflected wave ε_R and the transmitted wave ε_T are required.

$$\begin{cases} \sigma_S(t) = \frac{E_B A_B}{A_S}\varepsilon_T(t), \\ \varepsilon_S(t) = -\frac{2C_B}{L_S}\int_0^t \varepsilon_R(t)\mathrm{d}t, \end{cases} \tag{3}$$

where, σ_S is the average stress in the specimen, ε_S is the average strain of the specimen, E_B is the Young's modulus of the bars material, C_B is the elastic wave speed in the bars, A_S is the cross-sectional area of the specimen, A_B is the cross-sectional area of the bars, and L_s is the length of the specimen.

In Eq. (3), the average stress in the specimen is the arithmetic average of the stresses at the two ends. And, an assumed approximate relationship, $\sigma_I = \sigma_r = \sigma_T$, is introduced into the derivation of Eq. (3). The basic assumption of uniform stress distributed in specimen just comes from here.

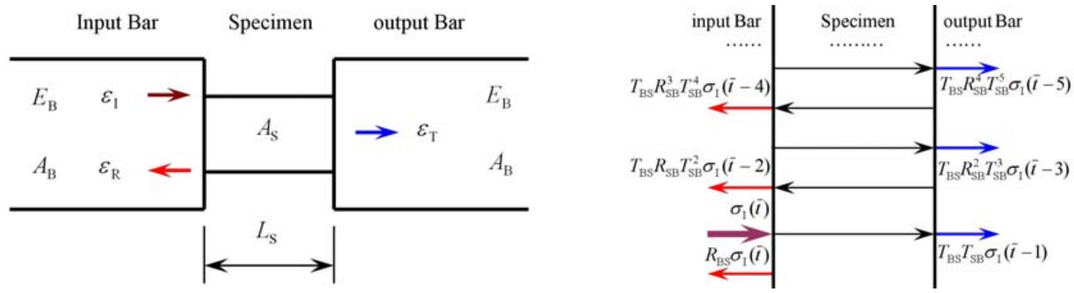


Fig. 1. Foundation of SHPB and its stress wave propagation. (a) Foundation of an SHPB test. (b) Propagation process of the stress wave in an SHPB test

3 Basic method for modelling and simulation

3.1 Modelling and simulation for basic behaviours of 1-D stress wave

Fig. 2 gives the basic method for modelling and simulating the behaviours of reflection, transmission and superposition using Matlab/Simulink. The implementation for simulating reflection and transmission is shown in Fig. 2 (a). There, the arrowed lines denote stress signals. When the signal of the incident pulse arrives at an interface, it is separated into two signals (both equal to the original). Then they pass two Gain blocks (which can be regarded as amplifiers), Gain 1 and Gain 2, with coefficients of reflection and transmission, respectively. The linear superposition of two stress waves can be implemented as Fig. 2 (b). There, the two stress waves (stress wave 1 and stress wave 2) pass a Sum block and therefore their linear superposition (stress wave 3) is outputted.

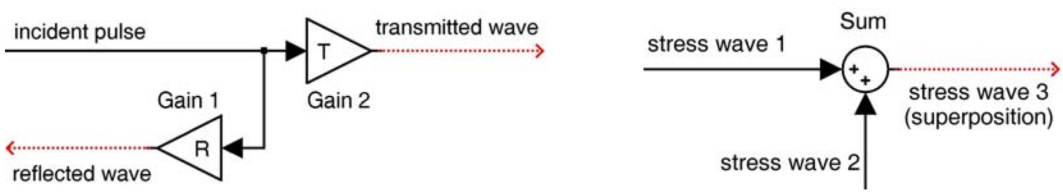


Fig. 2. Simulation of the basic behaviours of 1-D stress wave. (a) Reflection and transmission of a stress wave propagating at an interface. (b) Linear superposition of two stress waves

3.2 Basic method for modelling and simulation of SHB tests

The above sub-section 3.1 gives the method for modelling and simulating the basic behaviours of a stress wave. As shown in Fig. 1 (b), the stress wave reflects and transmits back and forth until homogenization in an SHB test. This phenomenon is necessary to be simulated for the purpose of SHB test simulation. Here a feedback loop technique is utilized to simulate the back-and-forth reflections and transmissions. Fig. 3 gives the module for implementing the method. In this figure, after the signal of the incident pulse arrives at the input bar-specimen interface, the first reflection and transmission occur. The block Gain 1 is used to obtain the first transmitted signal. The block Delay 1 is used to simulate the duration for stress wave to travel through the specimen and then arrive at the specimen-output bar interface. Here, the signal is transmitted and reflected again. The blocks Gain 2 and Gain 3 generate the two corresponding signals, which propagate into the output bar and back into the specimen, respectively. Then the back-propagating stress signal arrives at the specimen-input bar interface after passing the block Delay 2. Gain 4 outputs the reflected signal and Sum 1 superposes it with the originally transmitted signal outputted by Gain 1. Gain 5 outputs the transmitted signal and Sum 2 superposes it with the originally reflected signal outputted by Gain 6. Then, all the above forms a feedback loop. In the looping process, the signals of the total reflection and transmission can be obtained.

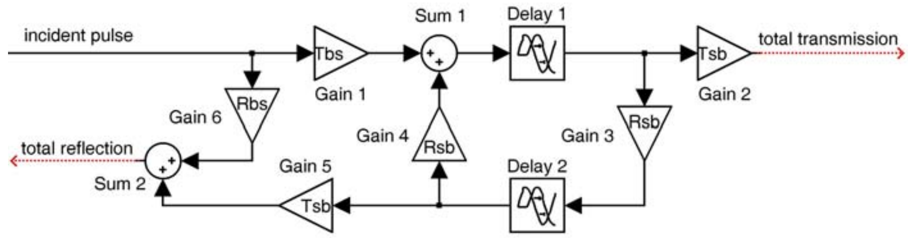


Fig. 3. Modelling and simulation of the back-and-forth reflections and transmissions of stress wave in an SHB test

The module shown in Fig. 3 can accomplish the main task of the simulation. But except that, the math-operating and displaying modules are also necessary for calculating and displaying the simulation results. Those will be introduced in the following Section 4.

4 Examples

4.1 Modelling

An SHPB test configuration is modelled as shown in Fig. 4, using the basic modelling and simulation method given in Section 3. This model consists of the basic module shown in Fig. 3 and other assistant blocks for calculation and visualization. As an example, the block Step is used to generate a rectangular signal, assumed as the incident stress pulse. Other kinds of signal shapes can be generated by other source blocks provided by the Simulink library. The blocks Scope 1, Scope 2 and Scope 3 are used to display the incident, transmitted and reflected stress waves. The block Mux is used to multiplex the three stress waves into a bus therefore to be displayed together by Scope 4. The block Gain 7 calculates the stress in the specimen from the transmitted stress in the output bar. The blocks Integrator and Gain 8 are united to calculate the strain of the specimen which is to be shown by Scope 5. At last, the stress-strain relationship, calculated by Eq. (3), can be displayed by the block XY Graph.

Before running this model, the variables, such as the reflection and transmission coefficients, need to be valued and stored in the workspace of Matlab by inputting or calculation. Then the Simulink model can use them and run automatically. The general parameters of the SHPB configuration were set as: $E_B = 200E9\,Pa$, $C_B = C_S = 5064\,m/s$, $L_S = 5.064E{-}3\,m$ and $\alpha = \beta = 1/10$. Furthermore, the simulation results also can

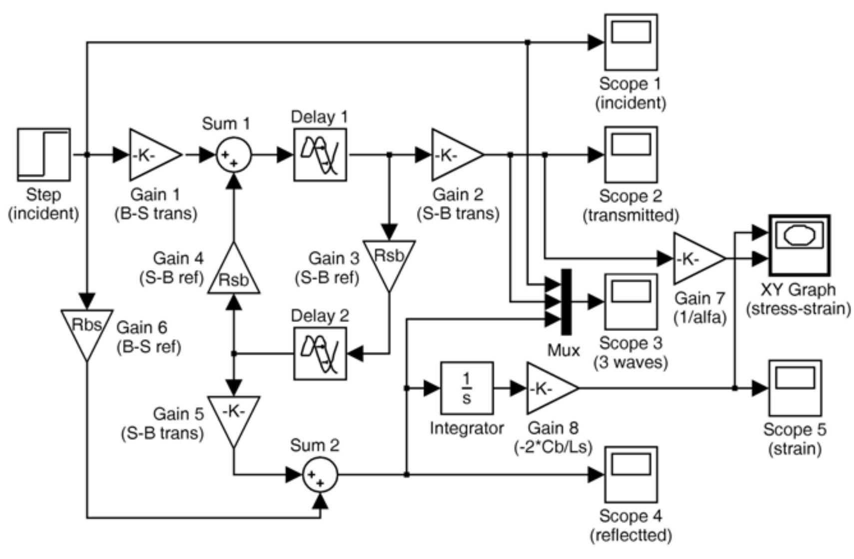


Fig. 4. Simulation model of the SHPB configuration, where the step signal source generates an incident stress pulse

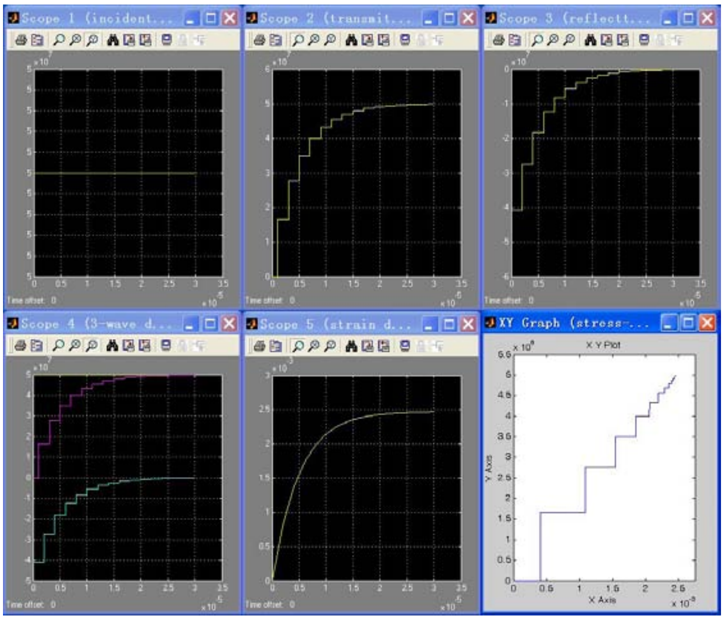
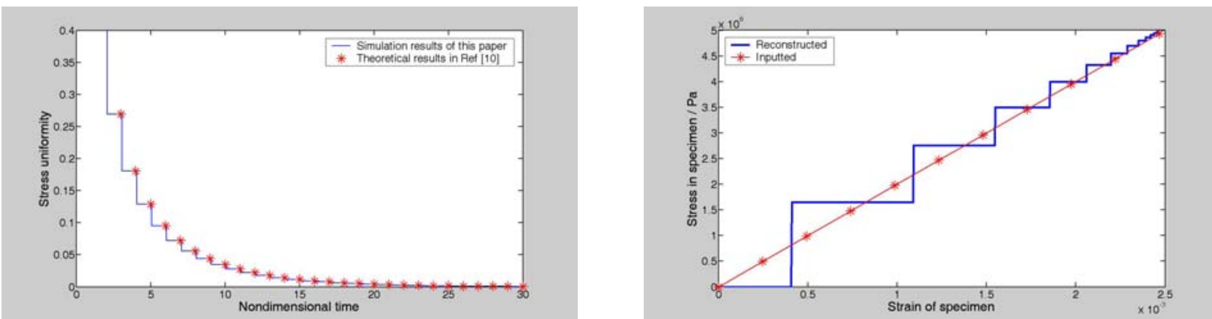


Fig. 5. Originally displayed simulation results (with a rectangular incident wave). The upper three windows display the incident, transmitted and reflected stress waves, respectively. The window Scope 4 displays all the three. Scope 5 displays the strain history of the specimen. The last window shows the reconstructed stress-train curve



(a) (b)

Fig. 6. Post-processed results of the simulation (with a rectangular incident wave). (a) Stress uniformity degree calculated by this paper and [13]. (b) Reconstructed stress-strain relationship and the inputted curve

be outputted to the Matlab workspace by using To Workspace blocks for post-processing and visualization. Here two important results are calculated by the post-processing program. The first one is the degree of stress uniformity[13] defined as

$$U(\bar{t}) = \frac{\Delta\sigma_S(\bar{t})}{\sigma_S(\bar{t})}. \tag{4}$$

For simple incident wave shapes, the results can be compared with the theoretical analyses in [13]. The second one is the real (inputted) stress-strain relation. In the model, the specimen is assumed linear and the Young's modulus can be determined by

$$E_S = \frac{\beta}{\alpha}\frac{C_S}{C_B}E_B. \tag{5}$$

Then, once E_S is obtained, the SHB test errors can be clearly observed by comparing it with the reconstructed curve.

4.2 Results and discussions

This sub-section gives the simulation results for various incident waves. As shown in Fig. 4, the Step

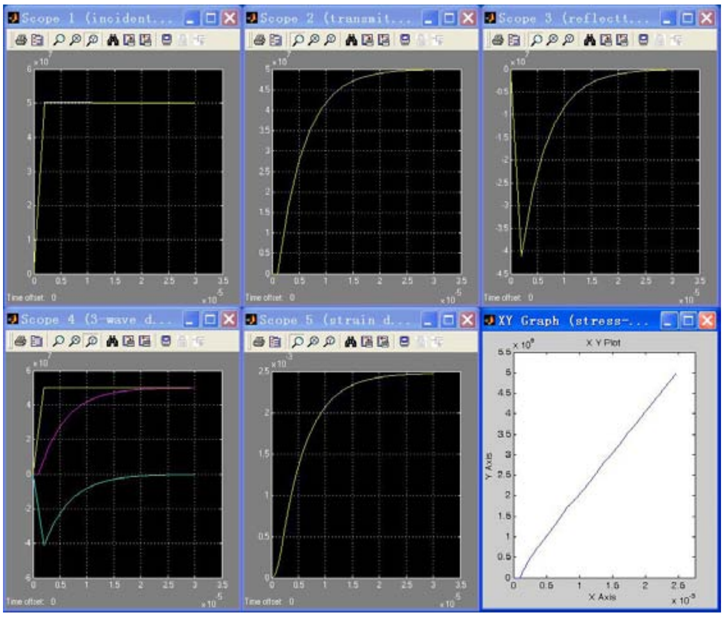
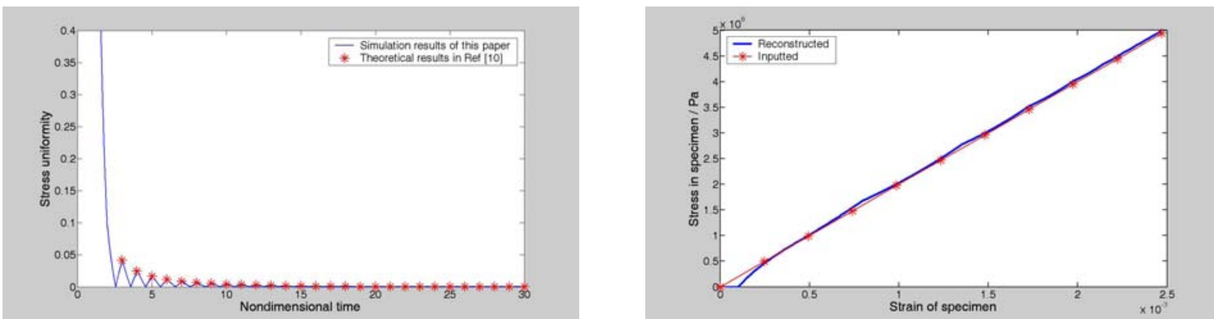


Fig. 7. Originally displayed simulation results (with a finite-rising incident wave). The upper three windows display the incident, transmitted and reflected stress waves, respectively. The window Scope 4 displays all the three. Scope 5 displays the strain history of the specimen. The last window shows the reconstructed stress-train curve



(a) (b)

Fig. 8. Post-processed results of the simulation (with a finite-rising incident wave). (a) Stress uniformity degree calculated by this paper and [13]. (b) Reconstructed stress-strain relationship and the inputted curve

block is used to generate a perfectly rectangular incident wave, where the stress amplitude is $50E6\ Pa$ and the signal length is $30E-6\ s$. The originally displayed simulation results are shown in Fig. 5 and the post-processed results are given in Fig. 6. Fig. 6 (a) shows the stress uniformity degree curve compared with the theoretical analysis in [13] and Fig. 6 (b) shows the comparison between the reconstructed stress-strain relation and the input.

Instead of the Step block, a Signal Builder block is used to generate an incident wave with rising time $2\tau = 2E-6\ s$, amplitude $50E6\ Pa$ and signal length $30E-6\ s$. The corresponding results are given in Figs. 7 and 8.

Also, a Ramp block is used to generate a sloping incident wave with final amplitude $50E6\ Pa$ and signal length $30E-6\ s$. The corresponding results are given in Figs. 9 and 10.

The above three simulations show that our method has the features as following:

• Convenience and high efficiency: the modelling processes are very convenient and a running only costs several seconds.

• Good visualization: the models are graph-based and easy to be understood and operated; both the halfway results and the final results can be clearly displayed, as shown in Figs. 5∼10.

• Good modularization: one model is composed by a number of modules; it is easy to be modified for different tasks.

• High concision: the main solvers are provided by the software package; the stress uniformity results perfectly agree with the theoretical analyses in [13], as shown in Figs. 6 (a), 8 (a) and 10 (a); also, our method can give full stress uniformity degree curves but the theoretical method in [13] can only give finite data points.

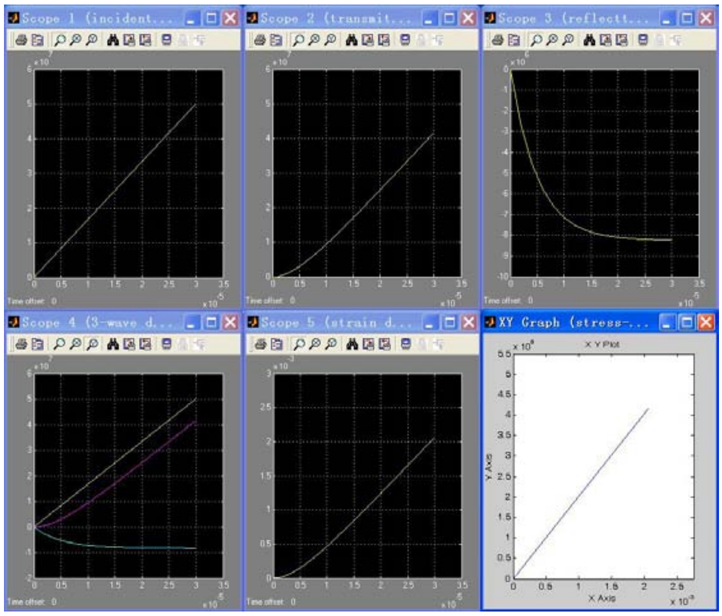
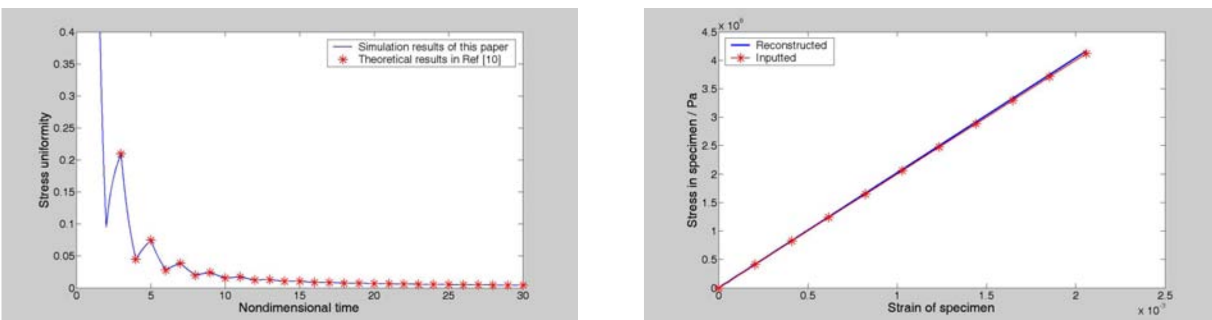


Fig. 9. Originally displayed simulation results (with a sloping incident wave). The upper three windows display the incident, transmitted and reflected stress waves, respectively. The window Scope 4 displays all the three. Scope 5 displays the strain history of the specimen. The last window shows the reconstructed stress-train curve



(a) (b)

Fig. 10. Post-processed results of the simulation (with a sloping incident wave). (a) Stress uniformity degree calculated by this paper and [13]. (b) Reconstructed stress-strain relationship and the inputted curve

Furthermore, comparisons between the reconstructed stress-strain relations and the inputted curves show that there are errors with different extents for different incident waves, as shown in Figs. 6 (b), 8 (b) and 10 (b). Those just come from the non-uniformity of the stress in specimen. The results can explain why it is not always reliable to use an SHB test to determine Young's modulus of a specimen. Here we just give some examples. More discussions about this topic can be found in [2, 4, 9–11, 13].

The above three simulations only consider the incident waves with simple shapes. Here a From Workspace block is used to input an actual incident wave with signal length $95.8E - 6\ s$. The simulation results are given in Figs. 11 and 12. This simulation and its results show that our method not only can be used in theoretical studies but also has significantly practical values. So it provides a good tool for SHB test design, analysis and its data validation in engineering applications.

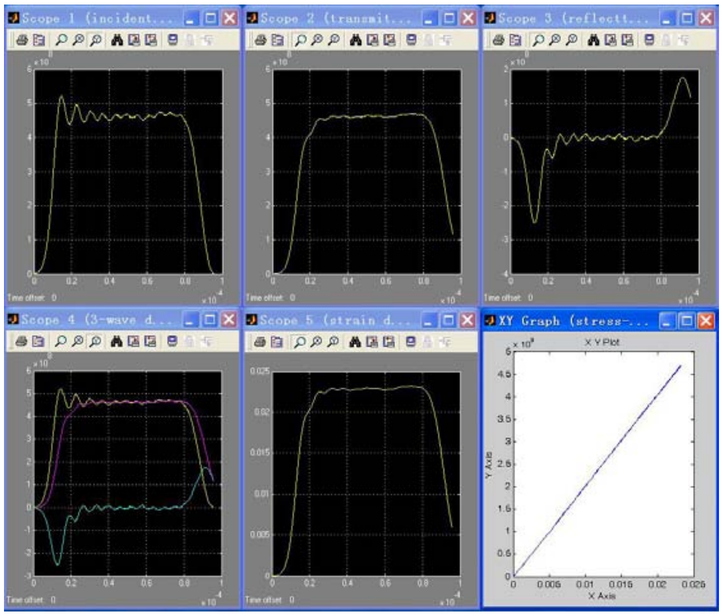


Fig. 11. Originally displayed simulation results (with an actual incident wave). The upper three windows display the incident, transmitted and reflected stress waves, respectively. The window Scope 4 displays all the three. Scope 5 displays the strain history of the specimen. The last window shows the reconstructed stress-train curve

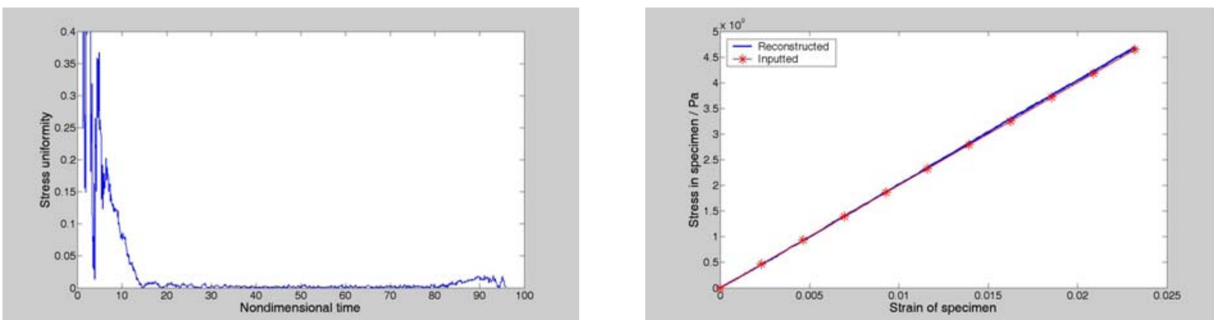


Fig. 12. Post-processed results of the simulation (with an actual incident wave). (a) Stress uniformity degree curve. (b) Reconstructed stress-strain relationship and the inputted curve

From the above four simulations, we can also find that the test accuracy does not rely on the time duration for the stress uniformity to be obtained, as shown in Figs. 6, 8, 10 and 12. This is because: (1) not the time, but the strain value for the stress uniformity to be obtained significantly influences the accuracy of the tested stress-strain curve, and (2) the accumulated error of the strain also affects the accuracy, although the stress uniformity has been obtained. So the criteria for evaluating the influence of stress uniformity on the SHB results should not be limited in the time-domain. Here we just gives several examples, the details of this finding will be reported in our further publications.

5 Conclusions

This paper develops a brief method based on Matlab/Simulink for one-dimensionally modelling and simulating SHB tests. The results of the examples are very agreeable with the theoretical analyses in [13]. This shows the validity and high accuracy of our method. Also according to the examples, it can be seen that this method has the features of good visualization, good modularization and high efficiency. Using the method, simulations and analyses can be easily performed, for both simple and actual incident pulses. So this paper provides an effective and practical tool for SHB test design, analysis and its data validation, not only for theoretical studies, but also for engineering applications.

Furthermore, this paper, for the first time, employs the software package Matlab/Simulink into the field of impact dynamics and successfully models and simulates the behaviours of 1-D stress wave. This kind of extended application would be meaningful for further theoretical studies on stress wave. So the research in this paper would have a theoretical contribution for modelling and simulation of stress wave using Matlab/Simulink.

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