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Comparative study of automatic generation control in traditional and deregulated power environment

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Abstract. An optimal/sub-optimal load frequency controller design based on a gradient based iterative technique is discussed in the present work. The feasibility of the design technique is shown by considering a 2-identical area power system consisting of non-reheat/reheat thermal plants. With the regulators designed for different cases discussed in the present work, the study is carried out in wake of 1% step load disturbance in area-1, area-2 and both the areas. The study is carried out for Automatic Generation Control strategies in traditional and deregulated environment.

Keywords: AGC, p.s. deregulation, optimal/suboptimal load frequency controller

1 Introduction

In an interconnected power system, the load variations are initially accommodated in a random fashion by the governor regulated machines which are in operation at that instant. The operating objectives require that the load changes must be allocated to the specific areas in accordance with pre-arranged patterns and not in a random fashion. To accomplish this, it becomes necessary to automatically manipulate the operation of main steam valves of turbo-generators in accordance with a suitable control strategy with a supplementary signal to speed changer master in the governors. The problem of regulating the power output of electric generators within a prescribed control area in response to changes in system frequency and the established interchange with other areas within predetermined limits is known as Automatic Generation Control (AGC)[6].

The traditional power system industry has a "Vertically Integrated Utility" (VIU) structure. In the restructured or deregulated environment, Vertically Integrated Utilities no longer exist. The utilities no longer own generation, transmission, and distribution, instead, three different entities emerge, namely:

- GENCOs—independent power producer;
- TRANSCOs—transmission facility owners;
- DISCOs—distribution system companies.

The first step in deregulation is to separate the generation of power from the transmission and distribution. An agreement between DISCO and GENCO should be established to supply regulation. As there are several GENCOs and DISCOs in the deregulated structure, a DISCO has the freedom to have a contract with any GENCO for transaction of power. A DISCO may have a contract with a GENCO in another control area. Such transactions are called "bilateral transactions". With the emergence of the distinct identities of GENCOs, TRANSCOs, DISCOs and the ISO (Independent System Operator), many of the ancillary services of a vertically integrated utility will have a different role to play and hence have to be modeled differently. Among these ancillary services is the Automatic Generation Control (AGC)[3–5].

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The automatic generation control system of a control area is a feedback control whose task is to control net interchange and frequency at scheduled values[8]. The conventional Load Frequency Control method includes a regulator which generates control as a function of integral of Area Control Error such that the deviations in system frequency and tie-line powers become zero in steady state[1]. In formulating the optimal AGC problem, the power system dynamic behavior is represented by a set of non-linear differential equations and a quadratic performance index is defined in terms of system state and control variables which is to be minimized subjected to system dynamic constraints[9]. Finally, an optimal control feedback law is obtained by solving a resulting non-linear Riccati Equation using suitable computational technique[2, 7]. An optimal AGC strategy is based on linear state regulator theory which requires either the feedback of all the state variables of the system or state reconstruction procedure from the point of view of its implementation. A suboptimal control law is obtained as a function of smaller number of states of the system based on model order reduction technique[10]. In output feedback control techniques, accessible and easily measurable output variables are selected for control realization.

2 State variable model for 2-area power system

A. Traditional power environment

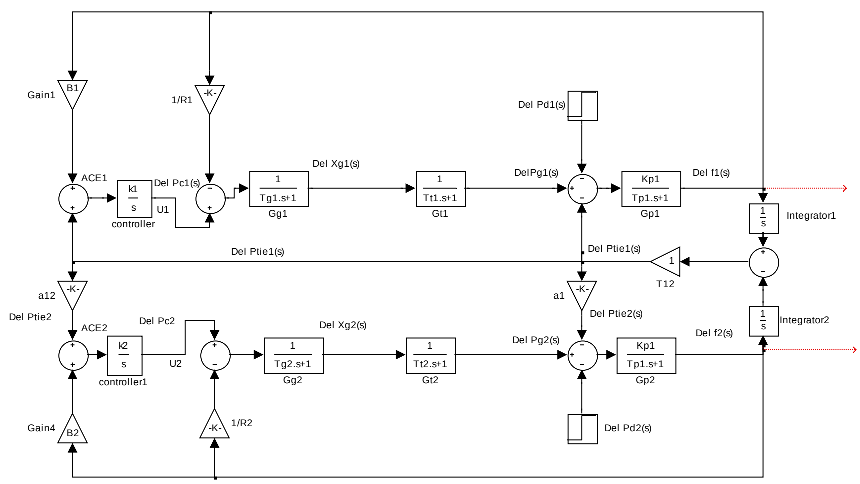

Fig. 1. Block diagram representation of 2-area power system with non-reheat turbines

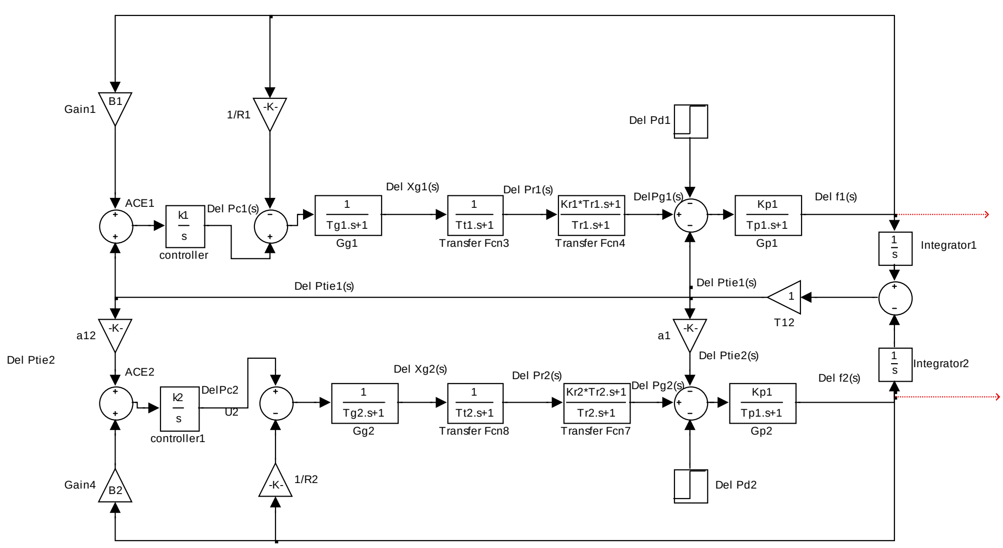

Fig. 2. Block diagram representation of 2-area power system with reheat turbines

The 2-area power system, shown in Fig. 1 and Fig. 2, can be described by the following controllable and observable linear time-invariant state space representation: $\dot{X}(t) = AX(t) + Bu(t) + F_d P(t), \quad Y(t) = CX(t)$, where

- State vector:

$$X(9 \times 1) = [\Delta f_1, \Delta P_{g1}, \Delta X_{g1}, \Delta f_2, \Delta P_{g2}, \Delta X_{g2}, \Delta P_{\text{tie},1}, \text{IACE}_1, \text{IACE}_2]^T$$
—(Non-reheat turbine model),
$$X(11 \times 1) = [\Delta f_1, \Delta P_{g1}, \Delta P_{r1}, \Delta X_{g1}, \Delta f_2, \Delta P_{g2}, \Delta P_{r2}, \Delta X_{g2}, \Delta P_{\text{tie},1}, \text{IACE}_1, \text{IACE}_2]^T$$
—(Reheat turbine model).

- Control vector: $u(2 \times 1) = [\Delta P_{c1}, \Delta P_{c2}]^T$
- Disturbance vector: $p(2 \times 1) = [\Delta P_{d1}, \Delta P_{d2}]^T$
- System matrix (non-zero elements):
  Non-reheat turbine model,
  $A_{11} = -1/T_{p1}$, $A_{12} = K_{p1}/T_{p1}$, $A_{17} = -K_{p1}/T_{p1}$, $A_{22} = -1/T_{t1}$, $A_{23} = 1/T_{t1}$, $A_{31} = -1/R_1 T_{g1}$,
  $A_{33} = -1/T_{g1}$, $A_{44} = -1/T_{p2}$, $A_{45} = K_{p2}/T_{p2}$, $A_{47} = -a_{12}K_{p2}/T_{p2}$, $A_{55} = -1/T_{t2}$, $A_{56} = 1/T_{t2}$,
  $A_{64} = -1/R_2 T_{g2}$, $A_{66} = -1/T_{g2}$, $A_{71} = T_{12}$, $A_{74} = -T_{12}$, $A_{81} = b_1$, $A_{87} = 1$, $A_{94} = b_2$, $A_{97} = a_{12}$.
  Reheat turbine model,
  $A_{11} = -1/T_{p1}$, $A_{12} = K_{p1}/T_{p1}$, $A_{19} = -K_{p1}/T_{p1}$, $A_{22} = -1/T_{r1}$, $A_{23} = 1/T_{r1}$, $A_{24} = -K_{r1}/T_{t1}$,
  $A_{25} = K_{r1}/T_{t1}$, $A_{33} = -1/T_{t1}$, $A_{34} = 1/T_{t1}$, $A_{41} = -1/R_1 T_{g1}$, $A_{44} = -1/T_{g1}$, $A_{55} = -1/T_{p2}$,
  $A_{56} = K_{p2}/T_{p2}$, $A_{59} = -a_{12}K_{p2}/T_{p2}$, $A_{66} = -1/T_{r2}$, $A_{67} = 1/T_{r2}$, $A_{68} = -K_{r2}/T_{t2}$,
  $A_{69} = K_{r2}/T_{t2}$, $A_{77} = -1/T_{t2}$, $A_{78} = 1/T_{t2}$, $A_{85} = -1/R_2 T_{g2}$, $A_{88} = -1/T_{g2}$, $A_{91} = T_{12}$,
  $A_{95} = -T_{12}$, $A_{10,1} = b_1$, $A_{10,9} = 1$, $A_{11,5} = b_2$, $A_{11,9} = a_{12}$.
- Control distribution matrix:
  Non-reheat turbine model, $B_{31} = 1/T_{g1}$, $B_{62} = 1/T_{g2}$.
  Reheat turbine model, $B_{41} = 1/T_{g1}$, $B_{82} = 1/T_{g2}$.
- Disturbance matrix:
  Non-reheat turbine model, $F_{d11} = -K_{p1}/T_{p1}$, $F_{d42} = -K_{p2}/T_{p2}$.
  Reheat turbine model, $F_{d11} = -K_{p1}/T_{p1}$, $F_{d52} = -K_{p2}/T_{p2}$,

B. Deregulated environment

The DISCO Participation Matrix concept is used as described by V.Donde, M.A.Pai and I.A.Hiskens[5] considering a 2-area system in which each area has two GENCOs and two DISCOs in it, as shown in Fig. 3.

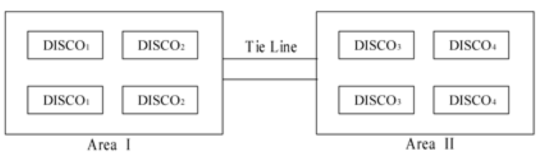

Fig. 3. Schematic of two area system in restructured environment

The corresponding Disco Participation matrix (DPM) is:

$$\text{DPM} = \begin{bmatrix} \text{cpf}_{11} & \text{cpf}_{12} & \text{cpf}_{13} & \text{cpf}_{14} \\ \text{cpf}_{21} & \text{cpf}_{22} & \text{cpf}_{23} & \text{cpf}_{24} \\ \text{cpf}_{31} & \text{cpf}_{32} & \text{cpf}_{33} & \text{cpf}_{34} \\ \text{cpf}_{41} & \text{cpf}_{42} & \text{cpf}_{43} & \text{cpf}_{44} \end{bmatrix}$$

where cpf refers to “contract participation factor”.

The block diagram for a two-area AGC system in the deregulated scenario is formulated. Whenever a load demanded by a DISCO changes, these demands are specified by cpf_s (elements of DPM) and the pu MW load of a DISCO. The 2-area power system, as shown in Fig. 4 and Fig. 5, can be described by the following

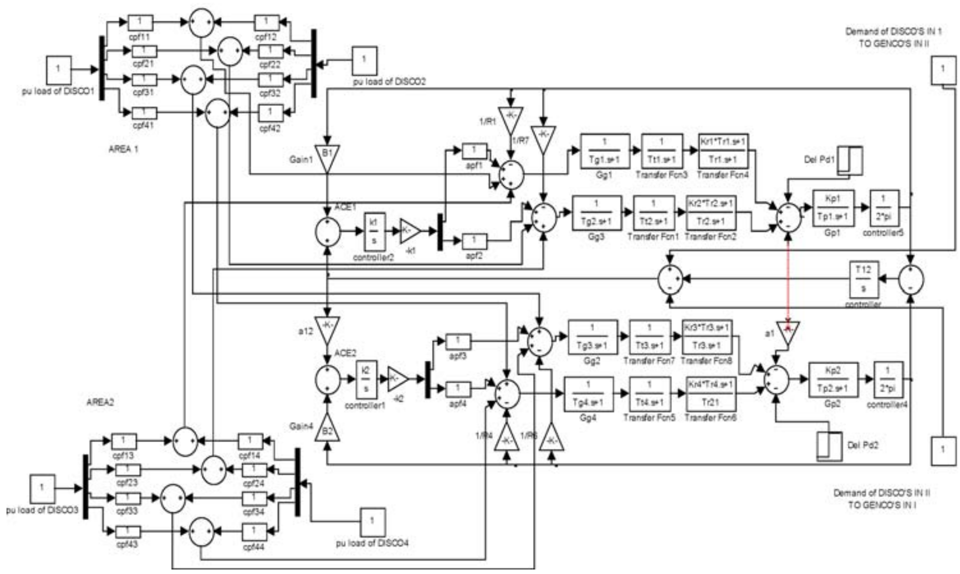

Fig. 4. Two-area AGC system block diagram for non-reheat turbines in deregulated environment

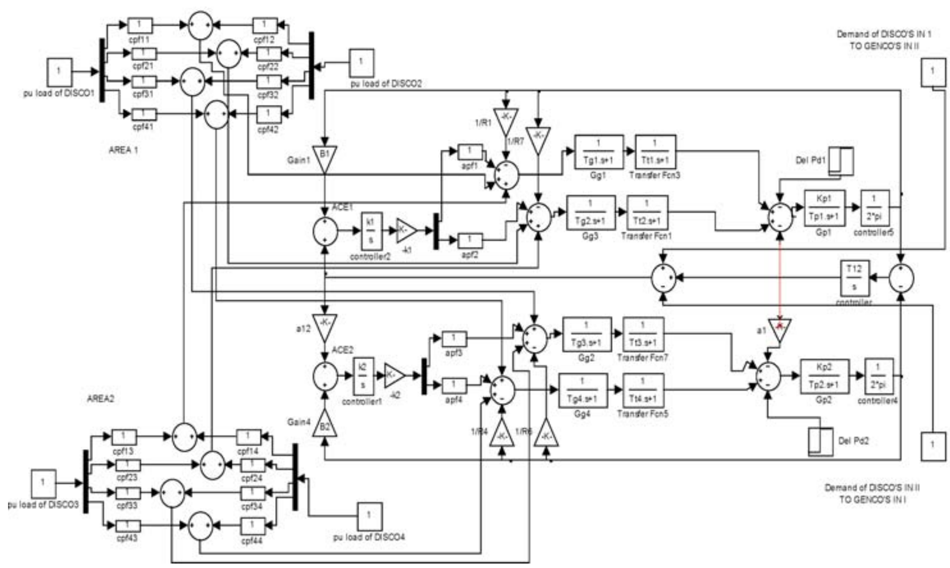

Fig. 5. Two-area AGC system block diagram for reheat turbines in deregulated environment

controllable and observable linear time-invariant state space representation:

$$\dot{X}(t) = AX(t) + Bu(t),\; Y(t) = CX(t),$$

where

- State vector:

$$X(13\times 1) = [\Delta f_1, \Delta f_2, \Delta P_{g1}, \Delta P_{g2}, \Delta P_{g3}, \Delta P_{g4}, \Delta P_{m1}, \Delta P_{m2}, \Delta P_{m3}, \Delta P_{m4}, \Delta P_{\text{tie},1}, \text{IACE}_1, \text{IACE}_2]^T\text{—(NR T model)}$$

$$X(17\times 1) = [\Delta f_1, \Delta f_2, \Delta P_{g1}, \Delta P_{g2}, \Delta P_{g3}, \Delta P_{g4}, \Delta P_{r1}, \Delta P_{r2}, \Delta P_{r3}, \Delta P_{r4}, \Delta P_{m1}, \Delta P_{m2}, \Delta P_{m3}, \Delta P_{m4}, \Delta P_{\text{tie},1}, \text{IACE}_1, \text{IACE}_2]^T]^T\text{—(R T model)}$$

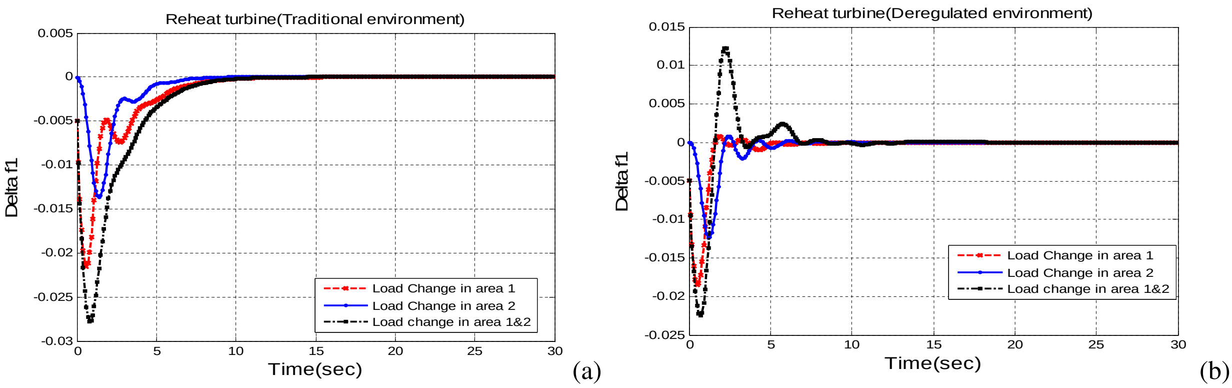

Fig. 6. Frequency deviation in area-1 ((a) traditional (b) and deregulated)

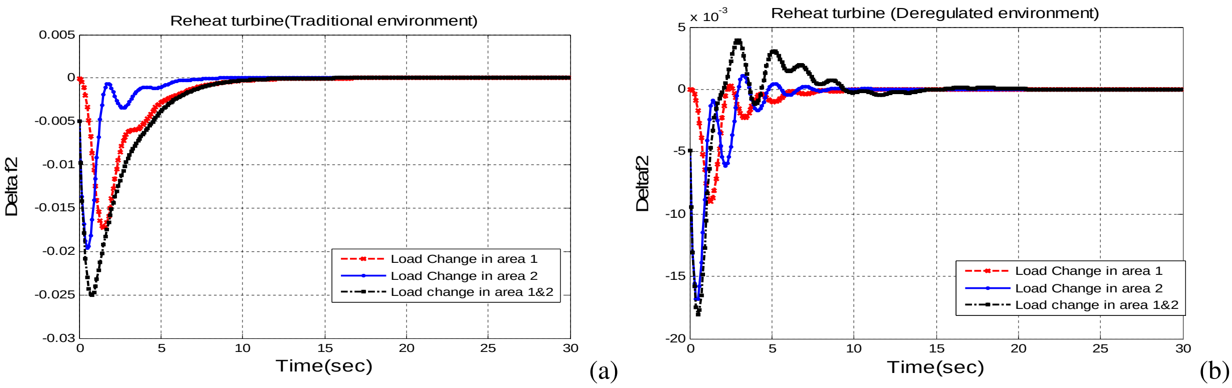

Fig. 7. Frequency deviation in area-2 ((a) traditional (b) and deregulated)

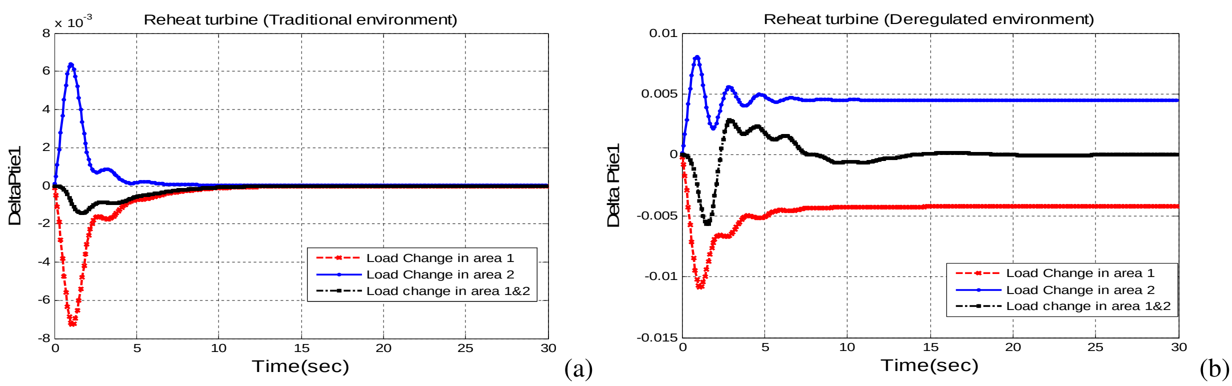

Fig. 8. Tie-line power deviation ((a) traditional (b) and deregulated)

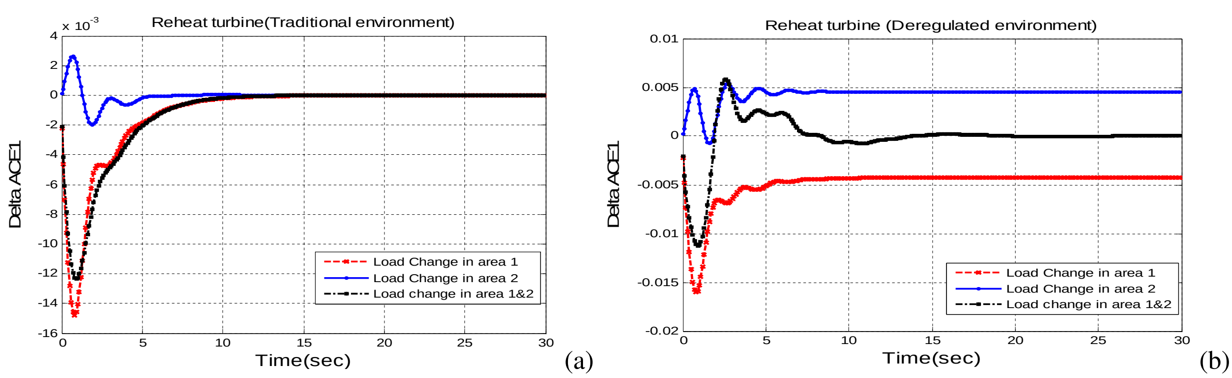

Fig. 9. Area Control Error in area-1 ((a) traditional and (b) deregulated)

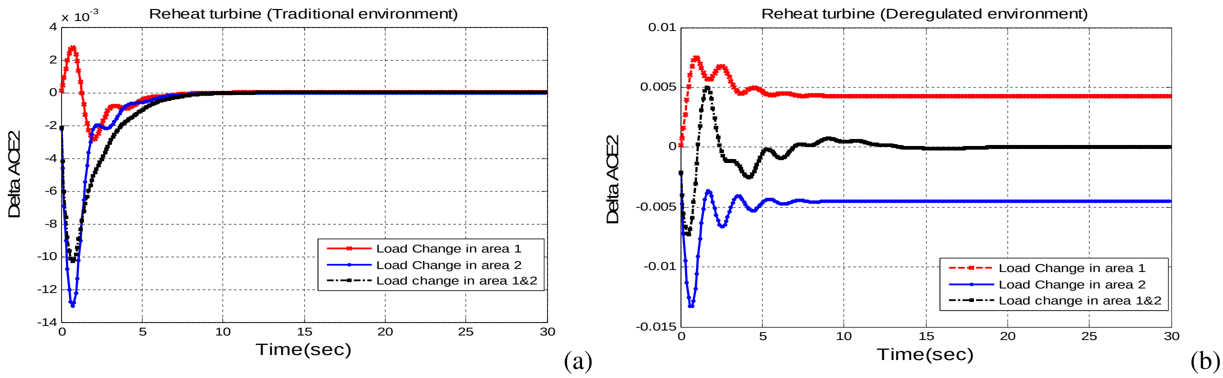

Fig. 10. Area control in area-2((a) traditional and (b) deregulated)

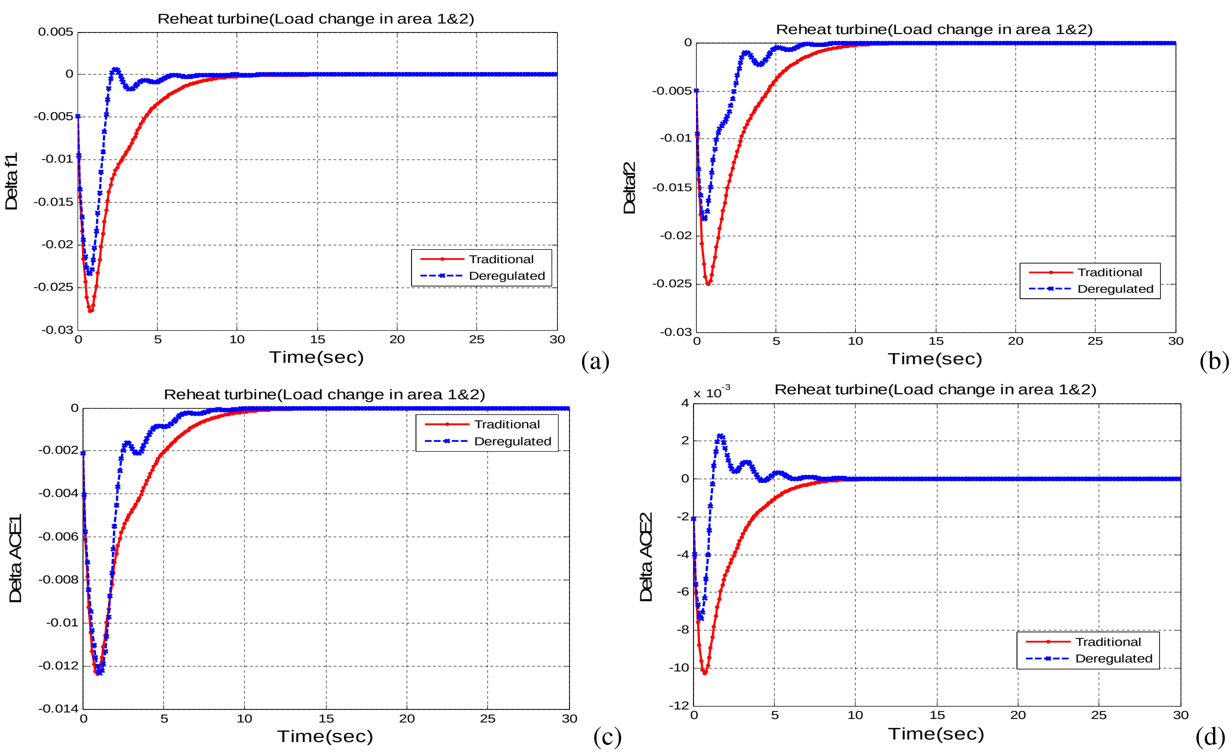

Fig. 11. Frequency deviations and Area Control Error for load change in both areas 1 and 2 in traditional and deregulated environment

- Control vector: $u(2\times 1) = [\Delta P_{c1}, \Delta P_{c2}]^T$
- Disturbance vector: $p(4\times 1) = [\Delta P_{L1}, \Delta P_{L2}, \Delta P_{L3}, \Delta P_{L4}]^T$
- System matrix (non-zero elements):

  Non-reheat turbine model,

  $A_{11} = -1/T_{p1},\ A_{13} = K_{p1}/T_{p1},\ A_{14} = K_{p1}/T_{p1},\ A_{1,11} = -K_{p1}/T_{p1},\ A_{22} = -1/T_{p2},$
  $A_{25} = K_{p2}/T_{p2},\ A_{26} = K_{p2}/T_{p2},\ A_{2,11} = K_{p2}/T_{p2},\ A_{33} = -1/T_{t1},\ A_{37} = 1/T_{t1},\ A_{44} = -1/T_{t2},$
  $A_{48} = 1/T_{t2},\ A_{55} = -1/T_{t3},\ A_{59} = 1/T_{t2},\ A_{66} = -1/T_{t4},\ A_{6,10} = 1/T_{t4},\ A_{71} = -1/2\pi R_1 T_{g1},$
  $A_{77} = -1/T_{g1},\ A_{81} = -1/2\pi R_2 T_{g2},\ A_{88} = -1/T_{g2},\ A_{92} = -1/2\pi R_3 T_{g3},\ A_{99} = -1/T_{g3},$
  $A_{10,2} = -1/2\pi R_4 T_{g4},\ A_{10,10} = -1/T_{g4},\ A_{11,1} = T_{12}/2\pi,\ A_{11,2} = -T_{12}/2\pi,\ A_{12,1} = b_1/2\pi,$
  $A_{12,11} = 1,\ A_{13,2} = b_2/2\pi,\ A_{13,11} = -1$

  Reheat turbine model,

  $A_{11} = -1/T_{p1},\ A_{13} = K_{p1}/T_{p1},\ A_{14} = K_{p1}/T_{p1},\ A_{1,15} = -K_{p1}/T_{p1},\ A_{22} = -1/T_{p2},$
  $A_{25} = K_{p2}/T_{p2},\ A_{26} = K_{p2}/T_{p2},\ A_{2,11} = K_{p2}/T_{p2},\ A_{33} = -1/T_{r1},\ A_{37} = 1/T_{r1} - K_{r1}/T_{t1},$
  $A_{3,11} = K_{r1}/T_{t1},\ A_{44} = -1/T_{r2},\ A_{48} = 1/T_{r2} - K_{r2}/T_{t2},\ A_{4,12} = K_{r2}/T_{t2},\ A_{55} = -1/T_{r3},$
  $A_{59} = 1/T_{r3} - K_{r3}/T_{t3},\ A_{5,13} = K_{r3}/T_{t3},\ A_{66} = -1/T_{r4},\ A_{6,10} = 1/T_{r4} - K_{r4}/T_{t4},$

$A_{6,14} = K_{r4}/T_{t4},\ A_{77} = -1/T_{t1},\ A_{7,11} = 1/T_{t1},\ A_{88} = -1/T_{t2},\ A_{8,12} = 1/T_{t2},\ A_{99} = -1/T_{t3},$
$A_{9,13} = 1/T_{t2},\ A_{10,10} = -1/T_{t4},\ A_{10,14} = 1/T_{t4},\ A_{11,1} = -1/2\pi R_1 T_{g1},\ A_{11,11} = -1/T_{g1},$
$A_{12,1} = -1/2\pi R_2 T_{g2},\ A_{12,12} = -1/T_{g2},\ A_{13,2} = -1/2\pi R_3 T_{g3},\ A_{13,13} = -1/T_{g3},$
$A_{14,2} = -1/2\pi R_4 T_{g4},\ A_{14,14} = -1/T_{g4},\ A_{15,1} = T_{12}/2\pi,\ A_{15,2} = -T_{12}/2\pi,\ A_{16,1} = b_1/2\pi,$
$A_{16,15} = 1,\ A_{16,2} = b_2/2\pi,\ A_{16,15} = -1$

- Control distribution matrix:
  Non-reheat turbine model, $B_{71} = apf_1/T_{g1}, B_{81} = apf_2/T_{g2}, B_{92} = apf_3/T_{g3}, B_{10,2} = apf_4/T_{g4}$,
  Reheat turbine model, $B_{11,1} = apf_1/T_{g1}, B_{12,1} = apf_2/T_{g2}, B_{13,2} = apf_3/T_{g3}, B_{14,2} = apf_4/T_{g4}$.
- Disturbance matrix,
  Non-reheat turbine model,
  $F_{d11} = -K_{p1}/T_{p1}, F_{d12} = -K_{p1}/T_{p1}, F_{d23} = -K_{p2}/T_{p2}, F_{d24} = -K_{p2}/T_{p2}, F_{d71} = cpf_{11}/T_{g1},$
  $F_{d72} = cpf_{12}/T_{g1}, F_{d73} = cpf_{13}/T_{g1}, F_{d74} = cpf_{14}/T_{g1}, F_{d81} = cpf_{21}/T_{g2}, F_{d82} = cpf_{22}/T_{g2},$
  $F_{d83} = cpf_{23}/T_{g2}, F_{d84} = cpf_{24}/T_{g2}, F_{d91} = cpf_{31}/T_{g3}, F_{d92} = cpf_{32}/T_{g3}, F_{d93} = cpf_{33}/T_{g3},$
  $F_{d94} = cpf_{34}/T_{g3}, F_{d10,1} = cpf_{41}/T_{g4}, F_{d10,2} = cpf_{42}/T_{g4}, F_{d10,3} = cpf_{43}/T_{g4}, F_{d10,4} = cpf_{44}/T_{g4},$
  $F_{d12,1} = cpf_{31}+cpf_{41}, F_{d12,2} = cpf_{32}+cpf_{42}, F_{d12,3} = -(cpf_{11}+cpf_{23}), F_{d12,4} = -(cpf_{14}+cpf_{24}),$
  $F_{d13,1} = -(cpf_{31} + cpf_{41}), F_{d13,2} = -(cpf_{32} + cpf_{42}), F_{d13,3} = cpf_{11} + cpf_{23},$
  $F_{d13,4} = cpf_{14} + cpf_{24}.$
  Reheat turbine model,
  $F_{d11} = -K_{p1}/T_{p1}, F_{d12} = -K_{p1}/T_{p1}, F_{d23} = -K_{p2}/T_{p2}, F_{d24} = -K_{p2}/T_{p2}, F_{d11,1} = cpf_{11}/T_{g1},$
  $F_{d11,2} = cpf_{12}/T_{g1}, F_{d11,3} = cpf_{13}/T_{g1}, F_{d11,4} = cpf_{14}/T_{g1}, F_{d12,1} = cpf_{21}/T_{g2},$
  $F_{d12,2} = cpf_{22}/T_{g2}, F_{d12,3} = cpf_{23}/T_{g2}, F_{d12,4} = cpf_{24}/T_{g2}, F_{d13,1} = cpf_{31}/T_{g3},$
  $F_{d13,2} = cpf_{32}/T_{g3}, F_{d13,3} = cpf_{33}/T_{g3}, F_{d13,4} = cpf_{34}/T_{g3}, F_{d14,1} = cpf_{41}/T_{g4},$
  $F_{d14,2} = cpf_{42}/T_{g4}, F_{d14,3} = cpf_{43}/T_{g4}, F_{d14,4} = cpf_{44}/T_{g4}, F_{d16,1} = cpf_{31} + cpf_{41},$
  $F_{d16,2} = cpf_{32} + cpf_{42}, F_{d16,3} = -(cpf_{11} + cpf_{23}), F_{d16,4} = -(cpf_{14} + cpf_{24}),$
  $F_{d17,1} = -(cpf_{31}+cpf_{41}), F_{d17,2} = -(cpf_{32}+cpf_{42}), F_{d17,3} = cpf_{11}+cpf_{23}, F_{d17,4} = cpf_{14}+cpf_{24}.$

An Algorithm based on steepest descent iterative gradient technique[9] for the solution of the problem is reported in Appendix.

3 Simulation results

Simulation results are shown in Tab. 1.

4 Discussion

Characteristic plots are shown in Fig. 6 to Fig. 10 and Fig. 11.

The stability of closed loop system is ensured by investigating the nature of all eigenvalues of the closed loop system matrix. The appreciable shifting towards the left of jw-axis of few eigenvalues obtained for case-3 has led to increased stability margins as compared to case-1, 2 in case of traditional scenario. The eigenvalues for case-3 have substantially small magnitude of imaginary part which may yield a smooth system dynamic response as compared to those obtained in other case studies.

The system dynamic responses of Δf_1, Δf_2, ΔP_{tie}, obtained for case 3 (i) as shown in (a) part of Figs. 6 $\sim$ 8 have lesser number of oscillatory modes but with higher magnitudes than those obtained for case studies 3 (ii) as shown in part (b) of Figs. 6 $\sim$ 8.The system offers a substantial improvement in dynamic performance when full state variable feedback is given in case 3 (i) i.e. as shown in (a) part of Figs. 6 $\sim$ 8.

The investigations can be further extended using Fuzzy Logic Controllers which improves the dynamic performance of the power system[4].

5 Conclusion

In the present work, a comparative study is made to propose Automatic Generation Control strategies for 2-area non-reheat/reheat thermal power system. Different cases are studied and a comparative study is made between traditional and deregulated power environment. From the study, it has been inferred that although the controllers designed for deregulated power environment yield a degraded performance compared to traditional environment, however, the performance obtained is within the acceptable limits and may be implemented with lesser complexity.

Table 1. Eigen values

Eigen Values Of The Closed Loop System Matrix	Traditional Environment	Deregulated Environment
Non-reheat/Optimal Case (1)	-23.7165 -13.1477 -13.2431 -4.1005 $-1.2240+2.9891i$ $-1.2240-2.9891i$ $-1.3736+1.8200i$ $-1.3736-1.8200i$ -0.2868	$-0.6067+4.4209i$ $-0.6067-4.4209i$ $-1.1614+3.6106i$ $-1.1614-3.6106i$ -0.3479 -9.0563 -14.3014 -14.0562 -12.8961 -12.5000 -12.5000 -3.3333 -3.3333
Non-reheat/Suboptimal Case (2)	-13.5925 -13.3126 $-0.7936+3.8081i$ $-0.7936-3.8081i$ $-1.0565+2.4796i$ $-1.0565-2.4796i$ $-0.3476+0.6291i$ $-0.3476-0.6291i$ -0.4493	-14.0607 -13.8194 $-0.4739+4.8616i$ $-0.4739-4.8616i$ $-0.8766+3.5525i$ $-0.8766-3.5525i$ $-0.4333+0.7095i$ $-0.4333-0.7095i$ -0.3024 -12.5000 -12.5000 $-3.3333+0.0000i$ $-3.3333-0.0000i$
Reheat/Optimal Case (3)	3(i) -27.0638 -11.9528 -9.6634 $-3.1380+2.0073i$ $-3.1380-2.0073i$ -1.6979 -0.1651 $-0.9327+0.5395i$ $-0.9327-0.5395i$ $-0.6860+0.2582i$ $-0.6860-0.2582i$	3(ii) $-12.6058+0.0433i$ $-12.6058-0.0433i$ $-0.5510+2.9928i$ $-0.5510-2.9928i$ $-2.2765+0.5359i$ $-2.2765-0.5359i$ -2.6841 $-0.6468+0.2094i$ $-0.6468-0.2094i$ -0.7285 -0.2675 -12.5000 -12.5000 -3.3333 -3.3333 -0.2000 -0.2000

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Appendix

Step 1. Read data N, M, L, TOL, alpha, A, B, C, F_d, R, Q and initialize F;

Step 2. Set iteration count $k = 1$;

Step 3. Compute the closed loop system matrix $Z = A - BFC$;

Step 4. Solve Lyapunov equation for $n \times n$ matrix 'S', $Z^T S + SZ + Q + C^T F^T RFC = 0$;

Step 5. Compute trace of 'S' as, $tr_i[S] = \sum_i S_{ii}$;

Step 6. Solve Lyapunov equation for matrix 'P', $ZP + PZ^T + I = 0$;

Step 7. Determine gradient, $DELH = 2[RFCPC^T - B^T SPC^T]$;

Step 8. Modify the feedback gain matrix, $F_I = F - alpha * DELH$, where alpha is step size;

Step 9. Compute the new value of 'S' and tr_2 i.e. trace of 'S';

Step 10. Check the convergence. If converged, then stop and print optimum F and go to step 12. Otherwise, proceed to next step;

Step 11. Increase iteration count by 1. Set $F = F_1$ and $tr_1 = tr_2$. Repeat from step-3 onwards;

Step 12. Set $F =$ optimum F;

Step 13. Set iteration count as 1;

Step 14. Initialize $x = 0, t = 0$ and $\Delta t = 0.1$;

Step 15. Apply Runga-Kutta routine to find new value of x;

Step 16. Check if $t \geq T_{\max}$. If yes, plot the characteristics, else go to next step;

Step 17. Increase iteration count by 1 and t by Δt.