ISSN 1 746-7233, England, UK
World Journal of Modelling and Simulation
Vol. 5 (2009) No. 4, pp. 278-286

Hyperbolic function solutions to the (3+1)-dimensional Burgers System*

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(*Received November 21 2008, Accepted May 1 2009*)

Abstract. The extended modified tanh-function method is developed to solve nonlinear mathematics physics equations. This method has been improved from tanh-function method in the form of solution and the solutions of constraint condition. By using the extended modified tanh-function method and with the aid of computer symbolism system Mathematica, abundant exact solutions of the (3+1)-dimensional Burgers System are obtained, including various hyperbolic function solutions as uniformly as possible, where the kink-shape solutions, rational fraction solutions, and periodic solutions, etc.

Keywords: (3+1)-dimensional Burgers System, extended modified tanh-function method, hyperbolic function solution, Riccati function equation

1 Introduction

Nonlinear partial differential equations (NLPDEs) are widely used to describe complex phenomena in various fields of science, especially in physics. The investigation of exact solutions of NLPDEs plays an important role in study of nonlinear physical phenomena. As is well known, solving a nonlinear physical system is much more difficult than solving the linear ones. Fortunately, with the development of soliton theory, a wealth of approaches for finding exact solutions of NLPDEs, such as homogeneous balance method[8], hyperbola function method[12], tanh-function method[5], Jacobi elliptic function method[9], extended F-expansion method[2, 3], homology analysis method[4], iterative method[1] and so on have been developed. In this paper, we use the extended modified tanh-function method to construct exact traveling solutions of the (3+1)-dimensional Burgers System. As a result, many new general exact hyperbolic function solutions are obtained, including the kink-shape solutions, rational fraction solutions, and periodic solutions and so on.

The rest of this paper is organized as follows: in section 2, we give the description of the extended modified tanh-function method; in section 3, we use this method to obtain new exact hyperbolic function solutions of the (3+1)-dimensional Burgers System; in section 4, some conclusions are given.

2 Description of the extended modified tanh-function method

In 2000, Fan[6] has proposed an extended tanh-function method and used it to construct multiple traveling wave solutions of nonlinear equations. Fan's method is to take full advantage of a Riccati equation:

$$\omega'(\xi) = b + \omega^2(\xi) \tag{1}$$

and use it's solutions:

* The work is supported by the National Natural Science Foundation of China (No. 70571030 and No. 90610031), and the Department of Education Foundation of Jiangsu Province (No. 03SJB790008), and the Advanced Talents' Foundation of Jiangsu University (No. 07JDG054). The authors wish to thank Jiangsu University for this support.

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$$\omega(\xi)=\begin{cases}-\sqrt{-b}\tanh\sqrt{-b}\xi\,, & \text{as } b<0\\ -\sqrt{-b}\coth\sqrt{-b}\xi\,, & \text{as } b<0\\ -\frac{1}{\xi}, & \text{as } b<0\\ \sqrt{b}\tan\sqrt{b}\xi, & \text{as } b>0\\ -\sqrt{b}\cot\sqrt{b}\xi, & \text{as } b>0\end{cases} \tag{2}$$

to replace $\tanh(kz)$ in the tanh-function method . But our study shows that Eq. (1) exists more solutions than Eq. (2) which can lead new traveling wave solutions for nonlinear equations. In this paper, we find twelve types of new exact solutions of the Eq. (1) as listed in Tab. 1 and these new solutions with the extended modified tanh-function method are used to solve the (3+1)-dimensional Burgers System, and obtain more new exact various hyperbolic function solutions including kink-shape solutions, rational fraction solutions, and periodic solutions, and so on.

The extended modified tanh-function method will be simply reviewed as follows:

For a given NLPDE with independent variables $x=(t,x_1,x_2,\cdots,x_m)$ and dependent variable u:

$$P(u,u_t,u_{x_1},u_{x_2},\cdots,u_{x_m},u_{x_1t},u_{x_2t},\cdots,u_{x_mt},u_{tt},u_{x_1x_1},u_{x_2x_2},\cdots,u_{x_mx_m},\cdots)=0. \tag{3}$$

We seek its solutions in a more general form:

$$u(\xi)=a_0+\sum_{i=1}^{n}\left\{a_i\,\omega^{-i}(\xi)+b_i\,\omega^{i}(\xi)+c_i\,\omega^{1-i}(\xi)\,\omega'(\xi)\,+d_i\,\omega^{-i}(\xi)\,\omega'(\xi)\right\}, \tag{4}$$

where $\xi=k(x+\alpha y+\beta z+\gamma t)(k\neq 0)$, and k, α, β, γ are constants. Here k denotes value of waves, α denotes speed of waves, a_o, a_i, b_i, c_i, $d_i(i=1,\cdots,n)$, k, α, β, γ are all constants to be determined, $\omega(\xi)$ and $\omega^{'}(\xi)$ in Eq. (4) satisfy the Eq. (1), where b is a parameter, the sign " ' " denotes $\mathrm{d}/\mathrm{d}\xi$. Solution $\omega(\xi)$ of Eq. (1) varies according to the value b (see Tab. 1).

To determine the explicit form of $u(\xi)$, we take the following four steps:

Step 1. Determine the integer n by balancing the highest order nonlinear term and the highest order partial derivative of $u(\xi)$ in Eq. (3);

Step 2. Substitute (4) along with Eq. (1) into Eq. (3) and collect all coefficients of $\omega^i(\xi)(i=1,\cdots,n)$, then set each coefficient to zero to derive a set of over-determined algebraic equations for a_0, a_i, b_i, c_i, $d_i(i=1,\cdots,n)$, k, α, β, γ;

Step 3. Solve the above over-determined algebraic equations in Step 2 for a_0, a_i, b_i, c_i, d_i, $(i=1,\cdots,n)$, k, α, β, γ;

Step 4. Select $\omega(\xi)$ from Tab. 1 and substitute it along with a_0, a_i, b_i, c_i, $d_i(i=1,\cdots,n)$, k, α, β, γ; into Eq. (4) to obtain the exact solutions $u(\xi)$ of Eq. (3).

Compared with the traditional tanh-function method[5], the extended modified tanh-function method improved in two ways.

First, the form of $u(\xi)$ is expanded in the extended modified tanh-function method. When $a_i=0$, $c_i=0$, $d_i=0$, the method turns to be the traditional tanh-function method. The form of $u(\xi)$ and the calculation process are more complex than the traditional one, but by using the extended modified tanh-function method we can obtain more new solutions.

Second, as can be seen in Tab. 1, another difference is that the Riccati Eq. (1) has more solutions in the extended modified tanh-function method than in the traditional one.

As a result of these changes, the extended modified tanh-function method can be more effective in obtaining various hyperbolic function solutions of the NLPDEs, such as the kink-shape solutions, rational fraction solutions, and periodic solutions. It shows the extended modified tanh-function method is more powerful in construction exact solutions of the NLPDEs.

3 Hyperbolic function solutions for the (3+1)-dimensional Burgers System

Let us consider the well known (3+1)-dimensional Burgers System[10]:

Table 1. New solutions of the Riccati Eq. (1) with $\varepsilon = \pm 1$

$\omega(\xi)$	$\omega(\xi)$
(1) $-\sqrt{-b}\left[\tanh(2\sqrt{-b}\xi) + i\varepsilon\,\mathrm{sech}(2\sqrt{-b}\xi)\right]$, $b<0$	(7) $\frac{b-\sqrt{-b}\tanh(\sqrt{-b}\xi)}{1+\sqrt{-b}\tanh(\sqrt{-b}\xi)}$, $b<0$
(2) $\sqrt{b}\left[\tan(2\sqrt{b}\xi) + \varepsilon\sec(2\sqrt{b}\xi)\right]$, $b>0$	(8) $-\frac{\sqrt{b}(1-\tan(\sqrt{b}\xi))}{1+\tan(\sqrt{b}\xi)}$, $b>0$
(3) $-\sqrt{-b}\left[\coth(2\sqrt{-b}\xi) + \varepsilon\, csch(2\sqrt{-b}\xi)\right]$, $b<0$	(9) $\frac{\sqrt{-b}(5-4\cosh(2\sqrt{-b}\xi))}{3+4\sinh(2\sqrt{-b}\xi)}$, $b<0$
(4) $-\sqrt{b}\left[\cot(2\sqrt{b}\xi) + \varepsilon\csc(2\sqrt{b}\xi)\right]$, $b>0$	(10) $\frac{\sqrt{b}(4-5\cos(2\sqrt{b}\xi))}{3+5\sin(2\sqrt{b}\xi)}$, $b>0$
(5) $-\frac{\sqrt{-b}}{2}\left[\tanh(\frac{\sqrt{-b}}{2}\xi) + \coth(\frac{\sqrt{-b}}{2}\xi)\right]$, $b<0$	(11) $\frac{-2\sqrt{-b}}{\tanh(\frac{\sqrt{-b}}{2}\xi)+\coth(\frac{\sqrt{-b}}{2}\xi)}$, $b<0$
(6) $\frac{\sqrt{b}}{2}\left[\tan(\frac{\sqrt{b}}{2}\xi) - \cot(\frac{\sqrt{b}}{2}\xi)\right]$, $b>0$	(12) $\frac{-2\sqrt{b}}{\tan(\frac{\sqrt{b}}{2}\xi)+\cot(\frac{\sqrt{b}}{2}\xi)}$, $b>0$

$$U_t = 2UUy + 2VUx + 2QUz + Uxx + Uyy + Uz,\ Ux = Vy,\ Uz = Qy, \tag{5}$$

which is a generalized version in the (3+1)-dimensions of the Burgers system. Obviously, if $Q = U$, $z = x$, Eqs. (5) are degenerated to the (2+1)-dimensional Burgers equations[7]: $U_t = 2UU_y + 2VU_x + U_{xx} + U_{yy}$, $U_x = V_y$. Furthermore, if U is both z-independent and y-independent, Eqs. (5) become a usual (1+1)-dimensional Burgers equation[11]: $U_t = 2UU_x + U_{xx}$, which has been widely applied in many scientific fields.

Next, the extended modified tanh-function method will be applied to solve the (3+1)-dimensional Burgers System (5). Using the transformations $U(x, y, z, t) = U(\xi)$, $V(x, y, z, t) = V(\xi)$, $Q(x, y, z, t) = Q(\xi)$ with $\xi = k(x + \alpha y + \beta z + \gamma t)$, Eqs. (5) are transformed into the following form:

$$\begin{aligned}&\gamma U'(\xi) - 2\alpha U(\xi)U'(\xi) - 2V(\xi)U'(\xi) - 2\beta Q(\xi)U'(\xi) - kU''(\xi) - k\alpha^2 U''(\xi) - k\beta^2 U''(\xi) = 0\\ &U'(\xi) - \alpha V'(\xi) = 0, \quad \beta U'(\xi) - \alpha Q'(\xi) = 0.\end{aligned} \tag{6}$$

According to Step 1, we get $n = 1$ for U, V and Q. Suppose the solutions of Eqs. (5) have the following forms:

$$U(\xi) = a_0 + a_1\omega^{-1}(\xi) + a_2\omega(\xi) + a_3\omega'(\xi) + a_4\omega^{-1}(\xi)\omega'(\xi), \tag{7}$$

$$V(\xi) = b_0 + b_1\omega^{-1}(\xi) + b_2\omega(\xi) + b_3\omega'(\xi) + b_4\omega^{-1}(\xi)\omega'(\xi), \tag{8}$$

$$Q(\xi) = c_0 + c_1\omega^{-1}(\xi) + c_2\omega(\xi) + c_3\omega'(\xi) + c_4\omega^{-1}(\xi)\omega'(\xi). \tag{9}$$

With the aid of Mathematica, substituting (7), (8) and (9) along with Eq. (1) into Eqs. (5), the left-hand sides of (6), are converted into three polynomials of $\omega^i(\xi)$ $(i = 1, \cdots, n)$. Setting each coefficient of polynomial to zero, we get a set of over-determined algebraic equations for a_0, a_1, a_2, a_3, a_4, b_0, b_1, b_2, b_3, b_4, c_0, c_1, c_2, c_3, c_4, k, α, β and γ.

Solving these over-determined algebraic equations by using Mathematica, we get the following results:

Case 1.

$$a_1 = -ba_4, b_0 = \frac{1}{2}\gamma - a_0\sqrt{-1-\beta^2} - \beta c_0, b_1 = -bb_4, b_2 = -\frac{(a_2+a_4)\sqrt{-1-\beta^2}}{1+\beta^2} - b_4,$$

$$c_1 = -bc_4, c_2 = -\frac{\beta(a_2+a_4)\sqrt{-1-\beta^2}}{1+\beta^2} - c_4, a_3 = 0, b_3 = 0, c_3 = 0, \alpha = -\sqrt{-1-\beta^2}.$$

Case 2.

$$a_1 = -ba_4, b_0 = \frac{1}{2}\gamma + a_0\sqrt{-1-\beta^2} - \beta c_0, b_1 = -bb_4, b_2 = \frac{(a_2+a_4)\sqrt{-1-\beta^2}}{1+\beta^2} - b_4,$$

$$c_1 = -bc_4, c_2 = \frac{\beta(a_2+a_4)\sqrt{-1-\beta^2}}{1+\beta^2} - c_4, a_3 = 0, b_3 = 0, c_3 = 0, \alpha = -\sqrt{-1-\beta^2}.$$

Case 3.

$$a_1 = -ba_4, b_0 = \frac{1}{2}\gamma - a_0\sqrt{-1-\beta^2} - \beta c_0, b_1 = -bb_4, b_2 = -\frac{(a_2+a_4)\sqrt{-1-\beta^2}}{1+\beta^2} - b_4,$$
$$b_3 = \frac{a_3}{\sqrt{-1-\beta^2}}, c_1 = -bc_4, c_2 = -\frac{\beta(a_2+a_4)\sqrt{-1-\beta^2}}{1+\beta^2} - c_4, c_3 = \frac{\beta a_3}{\sqrt{-1-\beta^2}}, \alpha = \sqrt{-1-\beta^2}.$$

Case 4.

$$a_1 = -ba_4, b_0 = \frac{1}{2}\gamma + a_0\sqrt{-1-\beta^2} - \beta c_0, b_1 = -bb_4, b_2 = \frac{(a_2+a_4)\sqrt{-1-\beta^2}}{1+\beta^2} - b_4,$$
$$b_3 = \frac{a_3\sqrt{-1-\beta^2}}{1+\beta^2}, c_1 = -bc_4, c_2 = \frac{\beta(a_2+a_4)\sqrt{-1-\beta^2}}{1+\beta^2} - c_4, c_3 = \frac{\beta a_3\sqrt{-1-\beta^2}}{1+\beta^2},$$
$$\alpha = \sqrt{-1-\beta^2}.$$

Case 5.

$$b_0 = \frac{1}{2}\gamma + a_0\sqrt{-1-\beta^2} - \beta c_0, b_1 = -\frac{(a_1+a_4)\sqrt{-1-\beta^2}}{1+\beta^2} - bb_4, b_2 = -\frac{(a_2+a_4)\sqrt{-1-\beta^2}}{1+\beta^2} - b_4,$$
$$b_3 = \frac{a_3}{\sqrt{-1-\beta^2}}, c_1 = -\frac{\beta(a_1+a_4)\sqrt{-1-\beta^2}}{1+\beta^2} - bc_4, c_2 = -\frac{\beta(a_2+a_4)\sqrt{-1-\beta^2}}{1+\beta^2} - c_4,$$
$$c_3 = \frac{\beta a_3}{\sqrt{-1-\beta^2}}, \alpha = \sqrt{-1-\beta^2}.$$

Case 6.

$$b_0 = \frac{1}{2}\gamma + a_0\sqrt{-1-\beta^2} - \beta c_0, b_1 = \frac{(a_1+ba_4)\sqrt{-1-\beta^2}}{1+\beta^2} - bb_4, b_2 = \frac{(a_2+a_4)\sqrt{-1-\beta^2}}{1+\beta^2} - b_4,$$
$$b_3 = \frac{a_3\sqrt{-1-\beta^2}}{1+\beta^2}, c_1 = \frac{\beta(a_1+ba_4)\sqrt{-1-\beta^2}}{1+\beta^2} - bc_4, c_2 = \frac{\beta(a_2+a_4)\sqrt{-1-\beta^2}}{1+\beta^2} - c_4,$$
$$c_3 = \frac{\beta a_3\sqrt{-1-\beta^2}}{1+\beta^2}, \alpha = -\sqrt{-1-\beta^2}.$$

Case 7.

$$b_0 = \frac{1}{2}\gamma - a_0\sqrt{-1-\beta^2} - \beta c_0, b_1 = -\frac{(a_1+ba_4)\sqrt{-1-\beta^2}}{1+\beta^2} - bb_4, b_2 = -\frac{(a_2+a_4)\sqrt{-1-\beta^2}}{1+\beta^2} - b_4,$$
$$c_1 = -\frac{\beta(a_1+ba_4)\sqrt{-1-\beta^2}}{1+\beta^2} - bc_4, c_2 = -\frac{\beta(a_2+a_4)\sqrt{-1-\beta^2}}{1+\beta^2} - c_4, a_3 = 0, b_3 = 0, c_3 = 0,$$
$$\alpha = \sqrt{-1-\beta^2}.$$

Case 8.

$$b_0 = \frac{1}{2}\gamma + a_0\sqrt{-1-\beta^2} - \beta c_0, b_1 = -\frac{(a_1+ba_4)\sqrt{-1-\beta^2}}{1+\beta^2} - bb_4, b_2 = \frac{(a_2+a_4)\sqrt{-1-\beta^2}}{1+\beta^2} - b_4,$$
$$c_1 = \frac{\beta(a_1+ba_4)\sqrt{-1-\beta^2}}{1+\beta^2} - bc_4, c_2 = \frac{\beta(a_2+a_4)\sqrt{-1-\beta^2}}{1+\beta^2} - c_4, a_3 = 0, b_3 = 0, c_3 = 0,$$
$$\alpha = -\sqrt{-1-\beta^2}.$$

Case 9.

$$a_1 = -ba_2 - 2ba_4, b_0 = \frac{1}{2}\gamma - \alpha a_0 - \beta c_0, b_1 = \frac{-ba_2 - ba_4 - \alpha bb_4}{\alpha}, b_2 = \frac{a_2 + a_4 - \alpha b_4}{\alpha},$$
$$c_1 = \frac{-b\beta a_2 - b\beta a_4 - \alpha bc_4}{\alpha}, c_2 = \frac{\beta a_2 + \beta a_4 - \alpha c_4}{\alpha}, a_3 = 0, b_3 = 0, k = \frac{-a_2 - a_4}{\alpha}.$$

Case 10.

$$a_1 = -ba_4, b_0 = \frac{1}{2}\gamma - \alpha a_0 - \beta c_0, b_1 = -bb_4, b_2 = \frac{a_2 + a_4 - \alpha b_4}{\alpha}, c_1 = -bc_4, c_2 = \frac{\beta a_2 + \beta a_4 - \alpha c_4}{\alpha}$$
$$a_3 = 0, b_3 = 0, c_3 = 0, k = \frac{-a_2 - a_4}{\alpha}.$$

Substituting coefficients in Case 1 to Case 10 into (7), (8) and (9) respectively, we have ten kinds of solutions of (7), (8) and (9).

Now we give the exact solutions of the (3+1)-dimensional Burgers System in Case 1. In the coming text, where $\xi = k(x + \alpha y + \beta z + \gamma t), \alpha = \sqrt{-1 - \beta^2}, k, a_0, a_2, a_3, a_4, b_0, b_4, c_0, c_4, \beta$ and γ are arbitrary constants.

Family 1. From Tab. 1, choosing $\omega(\xi) = -\sqrt{-b}\left[\tanh(2\sqrt{-b}\xi) + i\varepsilon \mathrm{sech}(2\sqrt{-b})\xi\right]$, inserting them into (7), (8) and (9), we obtain:

$$\begin{aligned}
U_1(\xi) &= a_0 - \sqrt{-b}a_2[\tanh(2\sqrt{-b}\xi) + i\varepsilon \mathrm{sech}(2\sqrt{-b}\xi)] \\
&\quad + \frac{\sqrt{-b}a_4[2\mathrm{sech}^2(2\sqrt{-b}\xi) + 2i\varepsilon \mathrm{sech}(2\sqrt{-b}\xi)\tanh(2\sqrt{-b}\xi) - 1]}{\tanh(2\sqrt{-b}\xi) + i\varepsilon \mathrm{sech}(2\sqrt{-b}\xi)}, \\
V_1(\xi) &= \frac{1}{2}\gamma - a_0\sqrt{-1-\beta^2} - \beta c_0 + \frac{(a_2 + a_4)\sqrt{b + \beta^2}[\tanh(2\sqrt{-b}\xi) + i\varepsilon \mathrm{sech}(2\sqrt{-b}\xi)]}{1 + \beta^2} \\
&\quad + \sqrt{-b}b_4[\tanh(2\sqrt{-b}\xi) + i\varepsilon \mathrm{sech}(2\sqrt{-b}\xi)] \\
&\quad + \frac{\sqrt{-b}b_4[2\mathrm{sech}^2(2\sqrt{-b}\xi) + 2i\varepsilon \mathrm{sech}(2\sqrt{-b}\xi)\tanh(2\sqrt{-b}\xi) - 1]}{\tanh(2\sqrt{-b}\xi) + i\varepsilon \mathrm{sech}(2\sqrt{-b}\xi)}, \\
Q_1(\xi) &= c_0 + \frac{\beta(a_2 + a_4)\sqrt{b + \beta^2}[\tanh(2\sqrt{-b}\xi) + i\varepsilon \mathrm{sech}(2\sqrt{-b}\xi)]}{1 + \beta^2} \\
&\quad + \sqrt{-b}c_4[\tanh(2\sqrt{-b}\xi) + i\varepsilon \mathrm{sech}(2\sqrt{-b}\xi)] \\
&\quad + \frac{\sqrt{-b}c_4[2\mathrm{sech}^2(2\sqrt{-b}\xi) + 2i\varepsilon \mathrm{sech}(2\sqrt{-b}\xi)\tanh(2\sqrt{-b}\xi) - 1]}{\tanh(2\sqrt{-b}\xi) + i\varepsilon \mathrm{sech}(2\sqrt{-b}\xi)}.
\end{aligned}$$

Family 2. Choosing $\omega(\xi) = \sqrt{b}\left[\tan(2\sqrt{b}\xi) + \varepsilon \sec(2\sqrt{b}\xi)\right]$, inserting them into (7), (8) and (9), we obtain:

$$\begin{aligned}
U_2(\xi) &= a_0 + \sqrt{b}a_2[\tan(2\sqrt{b}\xi) + \varepsilon \sec(2\sqrt{b}\xi)] + \frac{\sqrt{b}a_4[2\sec^2(2\sqrt{b}\xi) + 2i\varepsilon \sec(2\sqrt{b}\xi)\tan(2\sqrt{b}\xi) - 1]}{\tan(2\sqrt{b}\xi) + \varepsilon \sec(2\sqrt{b}\xi)}, \\
V_2(\xi) &= \frac{1}{2}\gamma - a_0\sqrt{-1-\beta^2} - \beta c_0 + \frac{\sqrt{b}b_4[2\sec^2(2\sqrt{b\xi}) + 2i\varepsilon \sec(2\sqrt{b\varepsilon})\tan(2\sqrt{b\varepsilon}) - 1]}{\tan(2\sqrt{b\varepsilon}) + \varepsilon \sec(2\sqrt{b})\xi} \\
&\quad - \frac{(a_2 + a_4)\sqrt{-b - b\beta^2}[\tan(2\sqrt{b}\xi) + \varepsilon \sec(2\sqrt{b}\xi)]}{1 + \beta^2} - \sqrt{b}b_4[\tan(2\sqrt{b}\xi) + \varepsilon \sec(2\sqrt{b}\xi)], \\
Q_2(\xi) &= c_0 - \frac{\beta(a_2 + a_4)\sqrt{-b - b\beta^2}[\tan(2\sqrt{b}\xi) + \varepsilon \sec(2\sqrt{b}\xi)]}{1 + \beta^2} - \sqrt{b}c_4[\tan(2\sqrt{b}\xi) + \varepsilon \sec(2\sqrt{b}\xi)] \\
&\quad + \frac{\sqrt{b}c_4[2\sec^2(2\sqrt{b}\xi) + 2i\varepsilon \sec(2\sqrt{b}\xi)\tan(2\sqrt{b}\xi) - 1]}{\tan(2\sqrt{b}\xi) + \varepsilon \sec(2\sqrt{b}\xi)}.
\end{aligned}$$

Family 3. Choosing $\omega(\xi) = -\sqrt{-b}[\coth(2\sqrt{-b}\xi) + \varepsilon \csc h(2\sqrt{-b}\xi)]$, inserting them into (7), (8) and (9), we obtain:

$$U_3(\xi) = a_0 - \sqrt{-b}a_2[\coth(2\sqrt{-b}\xi) + \varepsilon\, csch(2\sqrt{-b}\xi)] - \frac{\sqrt{-b}a_4[2csch^2(2\sqrt{-b}\xi) + 2\varepsilon \coth(2\sqrt{-b}\xi)csch(2\sqrt{-b}\xi) + 1]}{\coth(2\sqrt{-b}\xi) + \varepsilon\, csch(2\sqrt{-b}\xi)},$$

$$V_3(\xi) = \frac{1}{2}\gamma - a_0\sqrt{-1-\beta^2} - \beta c_0 + \frac{(a_2+a_4)\sqrt{b+b\beta^2}[\coth(2\sqrt{-b}\xi) + \varepsilon\, csch(2\sqrt{-b}\xi)]}{1+\beta^2} + \sqrt{-b}b_4[\coth(2\sqrt{-b}\xi) + \varepsilon\, csch(2\sqrt{-b}\xi)] - \frac{\sqrt{-b}b_4[2csch^2(2\sqrt{-b}\xi) + 2\varepsilon \coth(2\sqrt{-b}\xi)csch(2\sqrt{-b}\xi) + 1]}{\coth(2\sqrt{-b}\xi) + \varepsilon\, csch(2\sqrt{-b}\xi)},$$

$$Q_3(\xi) = +\frac{\beta(a_2+a_4)\sqrt{b+b\beta^2}[\coth(2\sqrt{-b}\xi) + \varepsilon\, csch(2\sqrt{-b}\xi)]}{1+\beta^2} + \sqrt{-b}c_4[\coth(2\sqrt{-b}\xi) + \varepsilon\, csch(2\sqrt{-b}\xi)] - \frac{\sqrt{-b}c_4[2csch^2(2\sqrt{-b}\xi) + 2\varepsilon \coth(2\sqrt{-b}\xi)csch(2\sqrt{-b}\xi) + 1]}{\coth(2\sqrt{-b}\xi) + \varepsilon\, csch(2\sqrt{-b}\xi)}.$$

Family 4. Choosing $\omega(\xi) = -\sqrt{b}[\cot(2\sqrt{b}\xi) + \varepsilon \csc(2\sqrt{b}\xi)]$, inserting them into (7), (8) and (9), we obtain:

$$U_4(\xi) = a_0 - \sqrt{b}a_2[\cot(2\sqrt{b}\xi) + \varepsilon \csc(2\sqrt{b}\xi)] - \frac{\sqrt{b}a_4[2\csc^2(2\sqrt{b}\xi) + 2\varepsilon\cot(2\sqrt{b}\xi)\csc(2\sqrt{b}\xi) - 1]}{\cot(2\sqrt{b}\xi) + \varepsilon \csc(2\sqrt{b}\xi)},$$

$$V_4(\xi) = \frac{1}{2}\gamma - a_0\sqrt{-1-\beta^2} - \beta c_0 + \frac{(a_2+a_4)\sqrt{-b-b\beta^2}[\cot(2\sqrt{b}\xi) + \varepsilon \csc(2\sqrt{b}\xi)]}{1+\beta^2} + \sqrt{b}b_4[\cot(2\sqrt{b}\xi) + \varepsilon \csc(2\sqrt{b}\xi)] - \frac{\sqrt{b}b_4[2\csc^2(2\sqrt{b}\xi) + 2\varepsilon\cot(2\sqrt{b}\xi)\csc(2\sqrt{b}\xi) - 1]}{\cot(2\sqrt{b}\xi) + \varepsilon \csc(2\sqrt{b}\xi)},$$

$$Q_4(\xi) = c_0 + \frac{\beta(a_2+a_4)\sqrt{-b-b\beta^2}[\cot(2\sqrt{b}\xi) + \varepsilon \csc(2\sqrt{b}\xi)]}{1+\beta^2} + \sqrt{b}c_4[\cot(2\sqrt{b}\xi) + \varepsilon \csc(2\sqrt{b}\xi)] - \frac{\sqrt{b}c_4[2\csc^2(2\sqrt{b}\xi) + 2\varepsilon\cot(2\sqrt{b}\xi)\csc(2\sqrt{b}\xi) - 1]}{\cot(2\sqrt{b}\xi) + \varepsilon \csc(2\sqrt{b}\xi)}.$$

Family 5. Choosing $\omega(\xi) = -\frac{\sqrt{-b}}{2}[\tanh(\frac{\sqrt{-b}}{2}\xi) + \coth(\frac{\sqrt{-b}}{2}\xi)]$, inserting them into (7), (8) and (9), we obtain:

$$U_5(\xi) = a_0 - \frac{\sqrt{-b}a_2}{2}[\tanh(\frac{\sqrt{-b}}{2}\xi) + \coth(\frac{\sqrt{-b}}{2}\xi)] + \frac{\sqrt{-b}a_4[\mathrm{sech}^2(\frac{\sqrt{-b}}{2}\xi) - \csc h^2(\frac{\sqrt{-b}}{2}\xi) - 4]}{2[\tanh(\frac{\sqrt{-b}}{2}\xi) + \coth(\frac{\sqrt{-b}}{2}\xi)]},$$

$$V_5(\xi) = \frac{1}{2}\gamma - a_0\sqrt{-1-\beta^2} - \beta c_0 + \frac{(a_2+a_4)\sqrt{b+b\beta^2}}{2+2\beta^2}[\tanh(\frac{\sqrt{-b}}{2}\xi) + \coth(\frac{\sqrt{-b}}{2}\xi)] + \frac{\sqrt{-b}b_4}{2}[\tanh(\frac{\sqrt{-b}}{2}\xi) + \coth(\frac{\sqrt{-b}}{2}\xi)] + \frac{\sqrt{-b}b_4[\mathrm{sech}^2(\frac{\sqrt{-b}}{2}\xi) - \csc h^2(\frac{\sqrt{-b}}{2}\xi) - 4]}{2[\tanh(\frac{\sqrt{-b}}{2}\xi) + \coth(\frac{\sqrt{-b}}{2}\xi)]},$$

$$Q_5(\xi) = c_0 + \frac{\beta(a_2+a_4)\sqrt{b+b\beta^2}}{2+2\beta^2}[\tanh(\frac{\sqrt{-b}}{2}\xi) + \coth(\frac{\sqrt{-b}}{2}\xi)] + \frac{\sqrt{-b}c_4}{2}[\tanh(\frac{\sqrt{-b}}{2}\xi) + \coth(\frac{\sqrt{-b}}{2}\xi)] + \frac{\sqrt{-b}c_4[\mathrm{sech}^2(\frac{\sqrt{-b}}{2}\xi) - \csc h^2(\frac{\sqrt{-b}}{2}\xi) - 4]}{2[\tanh(\frac{\sqrt{-b}}{2}\xi) + \coth(\frac{\sqrt{-b}}{2}\xi)]}.$$

Family 6. Choosing $\omega(\xi) = \frac{\sqrt{b}}{2}[\tan(\frac{\sqrt{b}}{2}\xi) - \cot(\frac{\sqrt{b}}{2}\xi)]$, inserting them into (7), (8) and (9), we obtain:

$$U_6(\xi) = a_0 + \frac{\sqrt{b}a_2}{2}[\tan(\frac{\sqrt{b}}{2}\xi) - \cot(\frac{\sqrt{b}}{2}\xi)] + \frac{\sqrt{b}a_4[\sec^2(\frac{\sqrt{b}}{2}\xi) + \csc^2(\frac{\sqrt{b}}{2}\xi) - 4]}{2[\tan(\frac{\sqrt{b}}{2}\xi) - \cot(\frac{\sqrt{b}}{2}\xi)]},$$

$$V_6(\xi) = \frac{1}{2}\gamma - a_0\sqrt{-1-\beta^2} - \beta c_0 - \frac{(a_2+a_4)\sqrt{-b-b\beta^2}}{2+2\beta^2}[\tan(\frac{\sqrt{b}}{2}\xi) - \cot(\frac{\sqrt{b}}{2}\xi)]$$
$$- \frac{\sqrt{b}b_4}{2}[\tan(\frac{\sqrt{b}}{2}\xi) - \cot(\frac{\sqrt{b}}{2}\xi)] + \frac{\sqrt{b}b_4[\sec^2(\frac{\sqrt{b}}{2}\xi) + \csc^2(\frac{\sqrt{b}}{2}\xi) - 4]}{2[\tan(\frac{\sqrt{b}}{2}\xi) - \cot(\frac{\sqrt{b}}{2}\xi)]},$$

$$Q_6(\xi) = c_0 - \frac{\beta(a_2+a_4)\sqrt{-b-b\beta^2}}{2+2\beta^2}[\tan(\frac{\sqrt{b}}{2}\xi) - \cot(\frac{\sqrt{b}}{2}\xi)] - \frac{\sqrt{b}c_4}{2}[\tan(\frac{\sqrt{b}}{2}\xi) - \cot(\frac{\sqrt{b}}{2}\xi)]$$
$$+ \frac{\sqrt{b}c_4[\sec^2(\frac{\sqrt{b}}{2}\xi) + \csc^2(\frac{\sqrt{b}}{2}\xi) - 4]}{2[\tan(\frac{\sqrt{b}}{2}\xi) - \cot(\frac{\sqrt{b}}{2}\xi)]}.$$

Family 7. Choosing $\omega(\xi) = \frac{b-\sqrt{-b}\tanh(\sqrt{-b}\xi)}{1+\sqrt{-b}\tanh(\sqrt{-b}\xi)}$, inserting them into (7), (8) and (9), we obtain:

$$U_7(\xi) = a_0 - \frac{ba_4[1+\sqrt{-b}\tanh(\sqrt{-b}\xi) - \text{sech}^2(\sqrt{-b}\xi)]}{b-\sqrt{-b}\tanh(\sqrt{-b}\xi)} + \frac{ba_4 - \sqrt{-b}a_2\tanh(\sqrt{-b}\xi)}{1+\sqrt{-b}\tanh(\sqrt{-b}\xi)}$$
$$+ \frac{ba_4[b-\sqrt{-b}\tanh(\sqrt{-b}\xi)\text{sech}^2(\sqrt{-b}\xi)]}{[(1+\sqrt{-b}\tanh(\sqrt{-b}\xi)][b-\sqrt{-b}\tanh(\sqrt{-b}\xi)]},$$

$$V_7(\xi) = \frac{1}{2}\gamma - a_0\sqrt{-1-\beta^2} - \beta c_0 - \frac{bb_4[1+\sqrt{-b}\tanh(\sqrt{-b}\xi) - \text{sech}^2(\sqrt{-b}\xi)]}{b-\sqrt{-b}\tanh(\sqrt{-b}\xi)}$$
$$- \frac{(a_2+a_4)\sqrt{-1-\beta^2}[b-\sqrt{-b}\tanh(\sqrt{-b}\xi)]}{(1+\beta^2)[(1+\sqrt{-b}\tanh(\sqrt{-b}\xi)]} - \frac{bb_4 - \sqrt{-b}b_4\tanh(\sqrt{-b}\xi)}{(1+\beta^2)[(1+\sqrt{-b})\tanh(\sqrt{-b})]}$$
$$+ \frac{bb_4[b-\sqrt{-b}\tanh(\sqrt{-b}\xi)\text{sech}^2(\sqrt{-b}\xi)]}{[(1+\sqrt{-b}\tanh(\sqrt{-b}\xi)][b-\sqrt{-b}\tanh(\sqrt{-b}\xi)]},$$

$$Q_7(\xi) = c_0 - \frac{bc_4[1+\sqrt{-b}\tanh(\sqrt{-b}\xi) - \text{sech}^2(\sqrt{-b}\xi)]}{b-\sqrt{-b}\tanh(\sqrt{-b}\xi)}$$
$$- \frac{\beta(a_2+a_4)\sqrt{-1-\beta^2}[b-\sqrt{-b}\tanh(\sqrt{-b}\xi)]}{(1+\beta^2)[(1+\sqrt{-b}\tanh(\sqrt{-b}\xi)]} - \frac{bc_4 - \sqrt{-b}c_4\tanh(\sqrt{-b}\xi)}{(1+\beta^2)[1+\sqrt{-b}\tanh(\sqrt{-b}\xi)]}$$
$$+ \frac{bc_4[b-\sqrt{-b}\tanh(\sqrt{-b}\xi)\text{sech}^2(\sqrt{-b}\xi)]}{[(1+\sqrt{-b}\tanh(\sqrt{-b}\xi)][b-\sqrt{-b}\tanh(\sqrt{-b}\xi)]}.$$

Family 8. Choosing $\omega(\xi) = -\frac{\sqrt{b}[1-\tan(\sqrt{b}\xi)]}{1+\tan(\sqrt{b}\xi)}$, inserting them into (7), (8) and (9), we obtain:

$$U_8(\xi) = a_0 + \frac{\sqrt{b}[a_4 + a_4\tan(\sqrt{b}\xi) - \sec^2(\sqrt{b}\xi)]}{1-\tan(\sqrt{b}\xi)} - \frac{\sqrt{b}[a_2 - a_2\tan(\sqrt{b}\xi) - a_4\sec^2(\sqrt{b}\xi)]}{1+\tan(\sqrt{b}\xi)},$$

$$V_8(\xi) = \frac{1}{2}\gamma - a_0\sqrt{-1-\beta^2} - \beta c_0 + \frac{\sqrt{b}b_4[1+\tan(\sqrt{b}\xi) - \sec^2(\sqrt{b}\xi)]}{1-\tan(\sqrt{b}\xi)}$$
$$+ \frac{(a_2+a_4)\sqrt{-b-b\beta^2}[1-\tan(\sqrt{b}\xi)]}{(1+\beta^2)[1+\tan(\sqrt{b}\xi)]} + \frac{\sqrt{b}b_4[1-\tan(\sqrt{b}\xi) - \sec^2(\sqrt{b}\xi)]}{1+\tan(\sqrt{b}\xi)},$$

$$Q_8(\xi) = c_0 + \frac{\sqrt{b}c_4[1+\tan(\sqrt{b}\xi) - \sec^2(\sqrt{b}\xi)]}{1-\tan(\sqrt{b}\xi)} + \frac{\beta(a_2+a_4)\sqrt{-b-b\beta^2}[1-\tan(\sqrt{b}\xi)]}{(1+\beta^2)[1+\tan(\sqrt{b}\xi)]}$$
$$+ \frac{\sqrt{b}c_4[1-\tan(\sqrt{b}\xi) - \sec^2(\sqrt{b}\xi)]}{1+\tan(\sqrt{b}\xi)}.$$

Family 9. Choosing $\omega(\xi) = \frac{\sqrt{-b}[5-4\cosh(2\sqrt{-b}\xi)]}{3+4\sinh(2\sqrt{-b}\xi)}$, inserting them into (7), (8) and (9), we obtain:

$$U_9(\xi) = a_0 + \frac{\sqrt{-b}a_4[3+12\sinh(2\sqrt{-b}\xi)]}{5-4\cosh(2\sqrt{-b}\xi)} + \frac{\sqrt{-b}a_2[5-12\cosh(2\sqrt{-b}\xi)]}{3+4\sinh(2\sqrt{-b}\xi)},$$

$$V_9(\xi) = \frac{1}{2}\gamma - a_0\sqrt{-1-\beta^2} - \beta c_0 + \frac{\sqrt{b}b_4[3+12\sinh(2\sqrt{-b}\xi)]}{5-4\cosh(2\sqrt{-b}\xi)} - \frac{(a_2+a_4)\sqrt{b+b\beta^2}[5-4\cosh(2\sqrt{-b}\xi)]}{(1+\beta^2)[3+4\sinh(2\sqrt{-b}\xi)]} - \frac{\sqrt{-b}b_4[5+4\cosh(2\sqrt{-b}\xi)]}{3+4\sinh(2\sqrt{-b}\xi)},$$

$$Q_9(\xi) = c_0 + + \frac{\sqrt{-b}c_4[3+12\sinh(2\sqrt{-b}\xi)]}{5-4\cosh(2\sqrt{-b}\xi)} - \frac{\beta(a_2+a_4)\sqrt{b+b\beta^2}[5-4\cosh(2\sqrt{-b}\xi)]}{(1+\beta^2)[3+4\sinh(2\sqrt{-b}\xi)]} - \frac{\sqrt{-b}c_4[5+4\cosh(2\sqrt{-b}\xi)]}{3+4\sinh(2\sqrt{-b}\xi)}.$$

Family 10. Choosing $\omega(\xi) = \frac{\sqrt{b}[4-5\cos(2\sqrt{b}\xi)]}{3+5\sin(2\sqrt{b}\xi)}$, inserting them into (7), (8) and (9), we obtain:

$$U_{10}(\xi) = a_0 - \frac{\sqrt{b}a_4[3-5\sin(2\sqrt{b}\xi)]}{4-5\cos(2\sqrt{b}\xi)} + \frac{\sqrt{b}[4a_2-5a_2\cos(2\sqrt{b}\xi)-10\sqrt{b}\cos(2\sqrt{b}\xi)]}{3+5\sin(2\sqrt{b}\xi)},$$

$$V_{10}(\xi) = \frac{1}{2}\gamma - a_0\sqrt{-1-\beta^2} - \beta c_0 - \frac{\sqrt{b}b_4[3-5\sin(2\sqrt{b}\xi)]}{4-5\cos(2\sqrt{b}\xi)} - \frac{(a_2+a_4)\sqrt{-b-b\beta^2}[4-5\cos(2\sqrt{b}\xi)]}{(1+\beta^2)[3+5\sin(2\sqrt{b}\xi)]} - \frac{\sqrt{b}b_4[4+5\cos(2\sqrt{b}\xi)]}{3+5\sin(2\sqrt{b}\xi)},$$

$$Q_{10}(\xi) = c_0 - \frac{\sqrt{b}c_4[3-5\sin(2\sqrt{b}\xi)]}{4-5\cos(2\sqrt{b}\xi)} - \frac{\beta(a_2+a_4)\sqrt{-b-b\beta^2}[4-5\cos(2\sqrt{b}\xi)]}{(1+\beta^2)[3+5\sin(2\sqrt{b}\xi)]} - \frac{\sqrt{b}c_4[4+5\cos(2\sqrt{b}\xi)]}{3+5\sin(2\sqrt{b}\xi)}.$$

Family 11. Choosing $\omega(\xi) = \frac{-2\sqrt{-b}}{\tanh(\frac{\sqrt{-b}}{2}\xi)+\coth(\frac{\sqrt{-b}}{2}\xi)}$, inserting them into (7), (8) and (9), we obtain:

$$U_{11}(\xi) = a_0 - \frac{1}{2}\sqrt{-b}a_4[\tanh(\frac{\sqrt{-b}}{2}\xi) + \coth(\frac{\sqrt{-b}}{2}\xi) + \frac{\sqrt{-b}[\frac{1}{2}a_4\csc h^2(\frac{\sqrt{-b}}{2}\xi) - \frac{1}{2}a_4\text{sech}^2(\frac{\sqrt{-b}}{2}\xi) - 2a_2]}{\tanh(\frac{\sqrt{-b}}{2}\xi)+\coth(\frac{\sqrt{-b}}{2}\xi)},$$

$$V_{11}(\xi) = \frac{1}{2}\gamma - a_0\sqrt{-1-\beta^2} - \beta c_0 - \frac{1}{2}\sqrt{-b}b_4[\tanh(\frac{\sqrt{-b}}{2}\xi)+\coth(\frac{\sqrt{-b}}{2}\xi)] + \frac{2(a_2+a_4)\sqrt{b+b\beta^2}}{(1+\beta^2)[\tanh(\frac{\sqrt{-b}}{2}\xi)+\coth(\frac{\sqrt{-b}}{2}\xi)]} + \frac{\sqrt{-b}b_4[\frac{1}{2}\csc h^2(\frac{\sqrt{-b}}{2}\xi) - \frac{1}{2}\text{sech}^2(\frac{\sqrt{-b}}{2}\xi) + 2]}{\tanh(\frac{\sqrt{-b}}{2}\xi)+\coth(\frac{\sqrt{-b}}{2}\xi)},$$

$$Q_{11}(\xi) = c_0 - \frac{1}{2}\sqrt{-b}c_4[\tanh(\frac{\sqrt{-b}}{2}\xi)+\coth(\frac{\sqrt{-b}}{2}\xi)] + \frac{2\beta(a_2+a_4)\sqrt{b+b\beta^2}}{(1+\beta^2)[\tanh(\frac{\sqrt{-b}}{2}\xi)+\coth(\frac{\sqrt{-b}}{2})\xi]} + \frac{\sqrt{-b}c_4[\frac{1}{2}csch^2(\frac{\sqrt{-b}}{2}\xi) - \frac{1}{2}\text{sech}^2(\frac{\sqrt{-b}}{2}\xi) + 2]}{\tanh(\frac{\sqrt{-b}}{2}\xi)+\coth(\frac{\sqrt{-b}}{2}\xi)}.$$

Family 12. Choosing $\omega(\xi) = \frac{-2\sqrt{b}}{\tan(\frac{\sqrt{b}}{2}\xi)-\cot(\frac{\sqrt{b}}{2}\xi)}$, inserting them into (7), (8) and (9), we obtain:

$$U_{12}(\xi) = a_0 + \frac{1}{2}\sqrt{b}a_4[\tan(\frac{\sqrt{b}}{2}\xi) - \cot(\frac{\sqrt{b}}{2}\xi)] - \frac{\sqrt{b}[\frac{1}{2}a_4\sec^2(\frac{\sqrt{b}}{2}\xi) - \frac{1}{2}a_4\csc^2(\frac{\sqrt{b}}{2}\xi) + 2a_2]}{\tan(\frac{\sqrt{b}}{2}\xi) - \cot(\frac{\sqrt{b}}{2}\xi)},$$

$$V_{12}(\xi) = \frac{1}{2}\gamma - a_0\sqrt{-1-\beta^2} - \beta c_0 + \frac{1}{2}\sqrt{b}b_4[\tan(\frac{\sqrt{b}}{2}\xi) - \cot(\frac{\sqrt{b}}{2}\xi)] + \frac{2(a_2+a_4)\sqrt{-b-b\beta^2}}{(1+\beta^2)[\tan(\frac{\sqrt{b}}{2}\xi) - \cot(\frac{\sqrt{b}}{2}\xi)]} - \frac{\sqrt{b}b_4[\frac{1}{2}\sec^2(\frac{\sqrt{b}}{2}\xi) - \frac{1}{2}\csc^2(\frac{\sqrt{b}}{2}\xi) - 2]}{\tan(\frac{\sqrt{b}}{2}\xi) - \cot(\frac{\sqrt{b}}{2}\xi)},$$

$$Q_{12}(\xi) = c_0 + \frac{1}{2}\sqrt{b}c_4[\tan(\frac{\sqrt{b}}{2}\xi) - \cot(\frac{\sqrt{b}}{2}\xi)] + \frac{2\beta(a_2+a_4)\sqrt{-b-n\beta^2}}{(1+\beta^2)[\tan(\frac{\sqrt{b}}{2}\xi) - \cot(\frac{\sqrt{b}}{2}\xi)]} - \frac{\sqrt{b}c_4[\frac{1}{2}\sec^2(\frac{\sqrt{b}}{2}\xi) - \frac{1}{2}\csc^2(\frac{\sqrt{b}}{2}\xi) - 2]}{\tan(\frac{\sqrt{b}}{2}\xi) - \cot(\frac{\sqrt{b}}{2}\xi)}.$$

We may give other exact solutions of the (3+1)-dimensional Burgers System for Case 2 to Case 10 in the same way. It is clear that we may get more new exact solutions for Eqs. (5).

4 Summary and conclusion

In this paper, we have proposed an extended modified tanh-function method to construct exact solutions of the (3+1)-dimensional Burgers System. With the aid of Mathematica, the method provides a powerful mathematical tool to obtain more new general exact solutions of a great many NLPDEs in mathematical physics. The advantage of the method is that it can be used to obtain more exact solutions which cannot be fully obtained by other methods. In this paper, we have obtained more new exact various hyperbolic function solutions including kink-shape solutions, rational fraction solutions, and periodic solutions and so on. Most of those results we obtained are newly found in present papers.

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