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A maple procedures for criticality of graphs

M. Matinfar* , S. Mirzamani

Department of Mathematics, University of Mazandaran, Babolsar 47416-1468, Iran

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Abstract. A subset of vertices D of a graph G is a dominating set for G if every vertex of G not in D is adjacent to one in D. A vertex v is critical in G if the domination number of $G - v$ is smaller than that of G. We have already had a chance to note that even most advanced computer-algebra systems have very limited capabilities with respect to the critically of graphs. Consider, for example, such a well-known system as Maple. In [3] we can see a procedure for calculate of $\gamma(G)$, moreover we can see the set of all dominating set of a graph. In this paper, we make, Algorithm of Maple procedure by use of adjacency matrix of any graph and find, all critical vertices of graphs and determined graph is critical or no? This is important that there is no any method in particular Maple procedures for determined critical vertices of graphs.

Keywords: critical, Minimum Dominating Set (MDS), adjacency matrix

1 Introduction

In a mathematician's terminology, a graph G is a collection of points $V(G)$, and lines connecting some (possibly empty) subset of them, such that denoted by $E(G)$. The points of a graph are most commonly known as graph vertices. Similarly, the lines connecting the vertices of a graph are most commonly known as graph edges. The most common type is graphs in which at most one edge (i.e., either one edge or no edges) may connect any two vertices. Such graphs are called simple graphs. If multiple edges are allowed between vertices, the graph is known as a multigraph. Vertices are usually not allowed to be self-connected, but this restriction is sometimes relaxed to allow such graph loops. Let G be a loopless graph with vertex set $V(G) = \{v_1, ..., v_n\}$ and edge set $E(G) = \{e_1, ..., e_m\}$. The *adjacency matrix* of G written $A(G)$, is the n-by-n matrix in which entry $a_{i,j}$ is the number of edges in G with endpoints $\{v_i, v_j\}$. For a graph G and a subset S of the vertex set $V(G)$, denote by $N_G[S]$ the set of vertices in G which are in S or adjacent to a vertex in S. If $N_G[S] = V(G)$, then S is said to be a dominating set (of vertices in G). The domination number of a graph G, denoted $\gamma(G)$, is the minimum size of a dominating set of vertices in G. Vizing conjectured that $\gamma(G)\gamma(H) \leq \gamma(G \times H)$, where $G \times H$ is the graph product [2, 3].While the full conjecture remains open, Clark and Suen[4] have proved the looser result $\gamma(G)\gamma(H) \leq 2\gamma(G \times H)$.

2 A maple procedure

This Algorithm finds $\gamma(G)$ and determined graph is critical or no? Moreover it determined all critical vertices of graphs. Examples are test in Maple environment

* Corresponding author. *E-mail address*: m.matinfar@umz.ac.ir.

```
1 > with(networks):
2 > with(combinat, choose):
3 > with(combinat, powerset) :
4 > with(linalg):
5 > G := graph({vertices}, {edges}) :
6 > MM := adjacency(G) :
7 > V := vertices(G) :
8 > f := nops(V) :
9 > ee := f : eee := f :
10 > for j from 1 to f do
11 > R := choose(f, j) : ff := nops(R) :
12 > for t from 1 to ff do
13 > flag := 5;
14 > L := R[t];
15 > M := V minus {L[]};
16 > nn := f − j;
17 > for p from 1 to nn do
18 > S := 0;
19 > k := 1 :
20 > while k <= j do
21 > S := S + MM[M[p], L[k]];
22 > k := k + 1;
23 > end do;
24 > pp := p;
25 > if S = 0 then p := nn + 1; end if;
26 > if pp = nn and S <> 0 then
      flag := 100; fflag := flag; end if;
27 > if flag = 100 then jj := j; bb := jj; end if;
28 > end do;
29 > if fflag = 100 then j := f + 1; end if;
30 > end do;
31 > end do:
32 > print ("gamma(G) = ", jj);
      o := 0 : for i from 1 to ee do
33 > K := delrows(MM, i..i) :
34 > LL := delcols(K, i..i) :
35 > V := vertices(G) minus eee :
36 > f := nops(V) :
37 > for j from 1to f do
38 > R := choose(f, j) :
39 > ff := nops(R) :
40 > for t from 1 to ff do
41 > flag := 5; L := R[t];
42 > M := V minus {L[]};
43 > nn := f − j; fflag := 3;
44 > for p from 1 to nn do
45 > S := 0; k := 1 :
46 > while k <= j do
47 > S := S + LL[M[p], L[k]];
48 > k := k + 1;
49 > end do :
50 > pp := p;
51 > if S = 0 then p := nn + 1; end if;
52 > if pp = nn and S <> 0 then
53 > flag := 100; fflag := flag;
54 > end if;
55 > if flag = 100 then
56 > jj := j; end if;
57 > end do:
58 > if fflag = 100 then j := f + 1; end if;
59 > end do:
60 > end do : b := jj;
61 > if b < bb then o := o + 1;
      print(i, "is critical"); end if;
62 > end do:
63 > if o = eee then
print ("G is critical graph"); else
64 > if o = 0 then print ("G have
not any vertex critical");
65 > else print ("G is not vertex
      critical graph"); end if;
66 > end if;
```

3 Main results

Example 1. A Hamiltonian circuit, is a graph cycle through a graph that visits each node exactly once. A graph possessing a Hamiltonian circuit is said to be a Hamiltonian graph and A graph that is not Hamiltonian is said to be nonhamiltonian. An $n-$ polyhedral graph is a 3-connected simple planar graph on nodes. The Herschel graph is the smallest nonhamiltonian polyhedral graph (shown in Fig. 1). It is the unique such graph on 11 nodes, and has 18 edges. In this graph, we can see $\gamma(G) = 3$ and and it has not any vertex critical. So Hershel graph is not vertex critical graph.

Example 2. The Petersen graph is the graph, (illustrated in Fig. 2) in several embeddings possessing ten nodes, all of whose nodes have degree three. In Petersen graph, we can see $\gamma(G) = 3$ and it has not any vertex critical. So Petersen graph is not vertex critical graph.

Example 3. Harary graph $H_{k,n}$ is the smallest k-connected graph with n graph vertices, having $\lceil kn/2 \rceil$ edges, where $\lceil x \rceil$ is the ceiling function. In $H_{3,18}$, (Fig. 3) $\gamma(H_{3,18}) = 5$ and it has not any vertex critical. So Fig. 3 is not vertex critical graph.

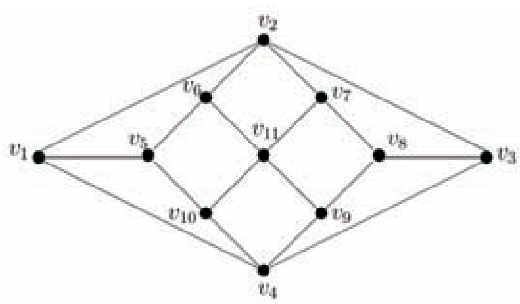

Fig. 1. The Hershel graph

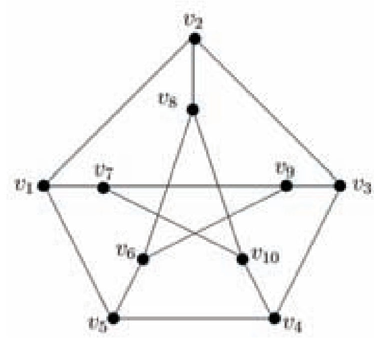

Fig. 2. The Petersen graph

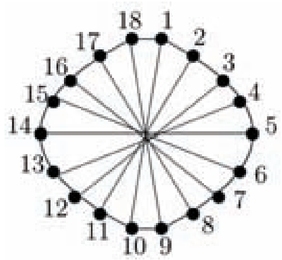

Fig. 3.

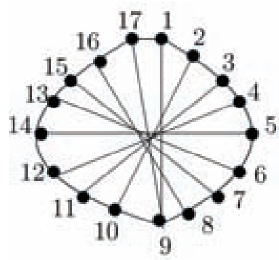

Fig. 4.

Moreover in $H_{3,17}$ (Fig. 4), $\gamma(H_{3,17}) = 5$ and vertices 2, 6, 8, 9, 10, 12, and 16 are critical. And $H_{3,17}$ is not vertex critical graph.

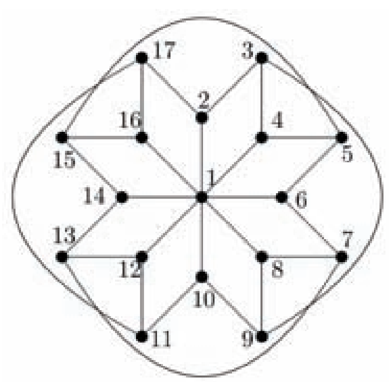

Fig. 5.

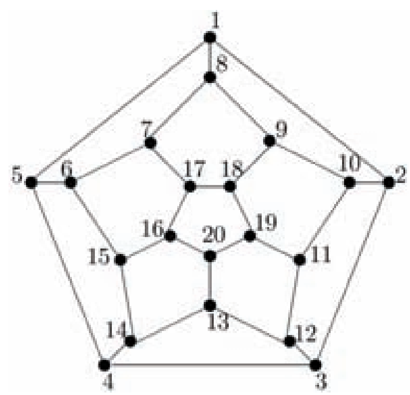

Fig. 6.

Example 4. In the particular graph G, that illustrated in (Fig. 5) we have $\gamma(G) = 5$ and only vertex 1 is critical. So Fig. 5 is not vertex critical graph.

Example 5. The degree of a graph vertex of a graph is the number of graph edges which touch the graph vertex, also called the local degree. A graph is said to be regular of degree r if all local degrees are the same number r. In Fig. 6, that is regular of degree 3, we have $\gamma(H) = 6$ and it has not any vertex critical. So Fig. 6 is not vertex critical graph.

Example 6. A circulant graph is a graph of n graph vertices in which the ith graph vertex is adjacent to the $(i + j)$th and $(i - j)th$ graph vertices for each j in a list l. The circulant graph on n vertices on a list of nodes l is denoted by $C_n\langle l\rangle$. Thus, in Circulant graph $C_8\langle 1, 4\rangle$, (shown in Fig. 7), we can see $\gamma(C_8\langle 1, 4\rangle) = 3$. For this example, we write for relation (1) in the Algorithm

$$G := graph(\{1,2,3,4,5,6,7,8\}, \{\{1,2\}, \{2,3\}, \{3,4\}, \{4,5\}, \{5,6\}\{6,7\}, \{8,7\}, \{8,1\},$$
$$\{1,5\}, \{2,6\}, \{3,7\}, \{4,8\}\}):$$

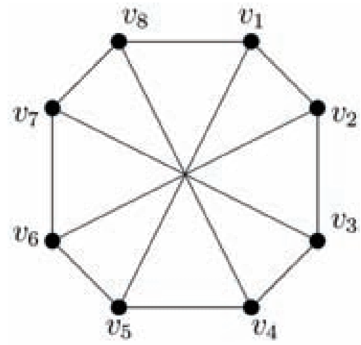

Fig. 7. The circulant graph

we obtain: $\gamma(H) = 3$ and any vertex is critical. So Circulant graph is vertex critical graph.

In final, we apply this Algorithm for P_n, C_n, $H_{2m+1,2n+1}$ and $H_{2m,n}$ and we obtain resultant in the following table:

Table 1.

P_n	$\gamma(P_n)$	position of criticality	
P_3	1	has not any vertex critical	P_3 is not vertex critical
P_4	2	1 and 4 are vertices critical	P_4 is not vertex critical
P_5	2	has not any vertex critical	P_5 is not vertex critical
P_6	2	has not any vertex critical	P_6 is not vertex critical
P_7	3	1, 4, 7 are vertices critical	P_7 is not vertex critical
		⋮	
P_{10}	4	1, 4, 7, 10 are vertices critical	P_{10} is not vertex critical
		⋮	
P_{13}	5	1, 4, 7, 10, 13 are vertices critical	P_{13} is not vertex critical
		⋮	

This Algorithm for C_n is:

Table 2.

C_n	$\gamma(C_n)$	position of criticality	
C_3	1	has not any vertex critical	C_3 is not vertex critical
C_4	2	1, 2, 3, 4 are vertices critical	C_4 is vertex critical
C_5	2	has not any vertex critical	C_5 is not vertex critical
C_6	2	has not any vertex critical	C_6 is not vertex critical
C_7	3	1, 2, 3, 4, 5, 6, 7 are vertices critical	C_7 is vertex critical
		⋮	
C_{10}	3	1, 2, 3, 4, 5, 6, 7, 8, 9, 10 are vertices critical	C_{10} is vertex critical
		⋮	

A fact in the C_n position of vertices this is that, if one a vertex be critical then all vertices are critical. In particular for $n = 3k + 1$, we can see C_n is vertex critical.
With apply this Algorithm for $H_{2m+1,2n+1}$ we have:

With apply this Algorithm for $H_{2m,n}$ we can see:

Table 3.

$H_{2m+1,2n+1}$	γ	position of criticality	
$H_{3,5}$	1	has not any vertex critical	$H_{3,5}$ is not vertex critical
$H_{3,7}$	2	has not any vertex critical	$H_{3,7}$ is not vertex critical
$H_{3,9}$	3	2, 4, 5, 6, 8 are vertices critical	$H_{3,9}$ is not vertex critical
$H_{3,11}$	3	has not any vertex critical	$H_{3,11}$ is not vertex critical
$H_{3,13}$	3	has not any vertex critical	$H_{3,13}$ is not vertex critical
$H_{3,15}$	4	has not any vertex critical	$H_{3,15}$ is vertex critical
$H_{3,17}$	5	2, 6, 8, 9, 10, 12, 16 are vertices critical	$H_{3,17}$ is not vertex critical
$H_{3,19}$	5	has not any vertex critical	$H_{3,19}$ is not vertex critical
		⋮	
$H_{3,25}$	7	2, 6, 10, 12, 13, 14, 16, 20, 24 are vertices critical	$H_{3,25}$ is not vertex critical
		⋮	

Table 4.

$H_{2m,n}$	$\gamma(H_{2m,n})$	position of criticality	
$H_{4,5}$	1	has not any vertex critical	$H_{4,5}$ is not vertex critical
$H_{4,6}$	2	any vertex is critical	$H_{4,6}$ is vertex critical
$H_{4,7}$	2	has not any vertex critical	$H_{4,7}$ is not vertex critical
		⋮	
$H_{4,11}$	3	any vertex is critical	$H_{4,11}$ is vertex critical
$H_{4,12}$	3	has not any vertex critical	$H_{4,12}$is not vertex critical
		⋮	
$H_{4,16}$	4	any vertex is critical	$H_{4,16}$ is vertex critical
		⋮	

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