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Marine reserves and its consequences as a fisheries management tool *

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Abstract. This paper describes a prey-predator type fishery model with prey dispersal in a two-patch environment, one of which is a free fishing zone and other is protected zone. Different consequences of reserve are described by the simulation processes. Our simulation clearly indicates that prey-predator interactions do matter when the implementation of a reserve is considered. It is also observed that reserves will be most effective when coupled with fishing effort controls in adjacent fisheries. Optimal harvesting policy is discussed.

Keywords: marine protected areas, prey-predator, exploitation, ecotourism, value function

1 Introduction

Marine protected areas (MPAs) are an emerging tool for managing marine resources. Many nations recognized the economic potential of their marine resources though they do not perceive the exploitation of marine resources to be a threat to the sustainability of these marine areas. Therefore a heavy fishing pressure is resulted on the world's ecosystem due to the fact that the nations begin to look more towards the sea for economic growth and new food sources. Thus over exploitation of marine resources is a serious and immediate global problem that current management policies struggle to solve. There is an increasing call for the expansion of marine protected areas (MPAs) with no-take zones in order to protect fish stocks and make fisheries sectors more sustainable.

Even though scientists and researchers consider the increasing scope of closed areas for the conservation of marine biodiversity[1, 17]. The use of MPAs is directed towards ecosystem functioning where ecosystems are easily disrupted by fishing efforts, reserves may be a more appropriate option. MPAs also have the potential to provide a margin of safety and perhaps even enhance the productivity of some fisheries. These ecological benefits are used to enhance the long run sustainability of the fishery and to improvements in habitat[2, 4, 5, 10, 11, 14, 15].

Marine reserves, areas closed to exploitation are seen as an additional management tool that could control fishing mortality. Thus establishment of marine reserves or no fishing zones represent an important tool for future fisheries management and sustainable development of ecosystem. The major advantage of those establishments is that they are directly addressed to the problems related to the harvesting and exploitation of the resources and its related impacts. Recent studies show that using MPAs can improve yield as well as protect stocks and sustain fishery viability. The benefits associated with MPAs have been widely investigated and the field is an interesting area of research in theoretical ecology. Hartmann et al.[7] investigated the economic optimality of implementing an MPA to get more informative data about fish population, thereby allowing a better management strategy.

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Pitchford et al.[13] set up a deliberately simple deterministic model of a fishery with MPA where a deterministic evaluation of the model leads to the conclusion that a MPA is not required. The author then introduced stochasticity into simulations of the simple fishery model to illustrate that the deterministic evaluation is incorrect and that the fishery is only sustainable when there is a buffer to the uncertainty in the system. The dynamics of a fishery resource system in an adequate environment which consists of two zones such as a free fishing zone and a reserve zone where fishing is strictly prohibited was studied by Kar[8] using a non-linear mathematical model. Rodwell et al.[16] examined the contribution of fully protected tropical marine reserves to fishery enhancement by modeling marine reserve-fishery linkages. The consequences of reserve establishment on the long run equilibrium fish biomass and fishery catch levels are evaluated. They also concluded that marine reserves are an important component of sustainable tropical fisheries management and reserves will be most effective when coupled with fishing effort controls in adjacent fisheries.

Sumalia[19] indicated that MPAs can protect the discounted economic rent from the fishery if the habitat is likely to face a shock and fishers have a high discount rate. The total standing biomass increases with increasing MPA size but only up to a point. Sumalia et al.[18] also provided an overview of marine protected areas and the role of economic analysis and modeling in designing, implementing and evaluating such MPAs. MPAs are said to provide multiple benefits including: “Protection of habitats, conservation of biodiversity, protection or enhancement of ecosystem services, recovery of depleted stocks, export of individuals to fished areas, insurance against environmental or management uncertainty and sites for scientific investigation, baseline information, education, recreation and inspiration”(see Lubehenco et al [9]). Hallwood[6] investigated the effect of positive marginal monitoring and enforcement costs, ‘policing cost’, on the optimal exploitation of a fishery under the management of a MPA, and shown that with positive marginal policing cost, the objective of maximum economic yield is no longer optimal, and that some dissipation of economic rent is socially optimal. Tuck et al.[20] shown that in a two local population meta-population with unidirectional larval transfer, the optimal exploitation of the harvested population should be conducted as if it were independent of the reserved population.

The objective of this paper is to examine the effects of marine reserves on the equilibrium levels of fish biomass, catch, predation and rent in a fishery under the assumption that this region is divided into two patches. One of these patches is considered to be a marine reserve and the remaining adjacent patch is used for harvesting the resource. This assumption is familiar to the literature dealing with marine reserves or MPAs. The difference lies in the realistic dynamic model constructed to represent the direct and indirect impacts of resource across the patches. We first outline the basic theoretical model describing the biological dynamics of two homogeneous stocks and its importance in the reserves and fishing zones and then add exploitation to the system. We compute some numerical simulations to determine the conditions under which the biological steady state can be attained and to draw some important conclusions regarding reserve designs. We also consider the optimal area of the reserve and exploitation rate in the fishery.

2 Model description and assumptions

In this section we describe a deterministic model of the interaction between the fish stock in a fishing ground and that of an adjacent marine reserve. The model is used to evaluate the impact of marine protection on the equilibrium fish biomass, predation and catch by representing the with and without reserve biomass and catch levels. The basic nature of a fishery associated with the reserve and fishing ground is described by the net transfer of fish from the reserve to the fishing grounds. Let us assume that the pre-reserve growth of a population follow logistic law described by $\mathrm{d}N/\mathrm{d}t = rN(1-N)$ where r is the intrinsic growth rate, N is the size of the stock, measured as a fraction of its carrying capacity. Now to model the possible interaction between the fish stocks in the reserve area and the fishing grounds, let us assume x & y are respectively the stock into patch 1, reserve area and patch 2, the fishing grounds. The marine reserve is fully protected from fishing and if the fish are not in the protected region they are exploitable. Again we consider, the catch of fish by fishers which is the only form of exploitation from the region open to fishing. In most of the works of marine reserve authors assume independent logistic growth for species in each of the patches, but such a coupled differential system does not represent the dynamics of population in a patchy environment as it does not satisfy the material balance in the system.

Our study is based on a prey-predator interaction in the aforesaid two patches among which one is protected for the prey (patch 1) and another being accessible to both prey and predators (patch 2). Each patch is supposed to be homogeneous. Let us assume z is the biomass density of predator species at time t. If we consider the maximum relative increase in predation to the reserved and unreserved area are respectively m & n then the predation terms can be taken as mxz & nyz.

Hence the basic dynamics of the marine reserve and fishing grounds are described by the following model:

$$\begin{aligned}\frac{\mathrm{d}x}{\mathrm{d}t}&=rx(1-x-y)-M-mxz,\\ \frac{\mathrm{d}y}{\mathrm{d}t}&=ry(1-x-y)+M-nyz-h\\ \frac{\mathrm{d}z}{\mathrm{d}t}&=sz\left(1-\frac{\gamma z}{x+y}\right)\end{aligned}\tag{1}$$

where s is the intrinsic growth rate of predator species, γ is the equilibrium ratio between prey biomass and predator biomass.

In order to simulate the model let us first specify appropriate functional forms of catch and transfer functions.

We generally use the phrase catch-per-unit-effort hypothesis to describe an assumption that catch per unit effort is proportional to the stock level or,

$$\frac{h}{E}\propto y\,\text{implies}\,h=qEy\tag{2}$$

where E is the effort applied for harvesting the fish population in the unreserved area and q is a constant, called the catchability coefficient.

The transfer is mainly depend on the size of the reserve, the mobility coefficient and the comparative densities of the stocks in the reserve zone and fishing grounds. Assuming total region under consideration is unit and $\alpha(0<\alpha<1)$ is the reserved area, consequently $(1-\alpha)$ is the unreserved area. As we have already considered both the patches are homogeneous therefore the area of the patches is proportional to their corresponding carrying capacity. Therefore with a mobility coefficient σ the net transfer rate can be written as,

$$M=\sigma\left(\frac{x}{\alpha}-\frac{y}{1-\alpha}\right)\tag{3}$$

After using (2) and (3), the system (1) is reduced to

$$\begin{aligned}\frac{\mathrm{d}x}{\mathrm{d}t}&=rx(1-x-y)-\sigma\left(\frac{x}{\alpha}-\frac{y}{1-\alpha}\right)-mxz,\\ \frac{\mathrm{d}y}{\mathrm{d}t}&=ry(1-x-y)+\sigma\left(\frac{x}{\alpha}-\frac{y}{1-\alpha}\right)-nyz-qEy,\\ \frac{\mathrm{d}z}{\mathrm{d}t}&=sz\left(1-\frac{\gamma z}{x+y}\right).\end{aligned}\tag{4}$$

3 Numerical simulation

In this section we assign some values to the parameters of the system (4) and compute some simulations using those data. For the purpose of simulation experiments we mainly use the software MATLAB.

The numerical values of the parameters are as follows: $r=0.3$, $\sigma=0.2$, $m=1$, $n=1$, $q=0.0025$, $s=0.1$, $\gamma=3$.

All the simulation results are obtained in equilibrium. The results can be classified into two categories. First category represents the results where the predation term is considered to be zero i.e., the results are represented without predation but in the second category the results are represented with predation. Thus the following results of the simulations given in the next section represent the qualitative properties and the dynamics of population in a protected area.

3.1 Simulation without prey-predator interactions

In this section the simulation results are considered in absence of predators, i.e. the only form of exploitation is catch. The effect of fishing effort on fish biomass is represented in Fig. 1. From Fig. 1 it is evident that the fishing effort has great influence on fish biomass. When 30% reserve is considered on fish biomass, the fraction of stock in fishing zone as well as in reserve zone tends to their extinct limit with increasing fishing effort. The fraction of stock inside the reserve zone tends to its extinct limit due to the transfer of biomass from reserve area to open area which ultimately control the harvesting efficiency and enhance the fishing stock to reach its extinct limit. It is observed that the presence of marine reserve do not eliminate the extinction of the total stock but in the presence of marine reserve fishing effort increases compared to the fishing effort used for the extinction of the total stock in case of no reserve.

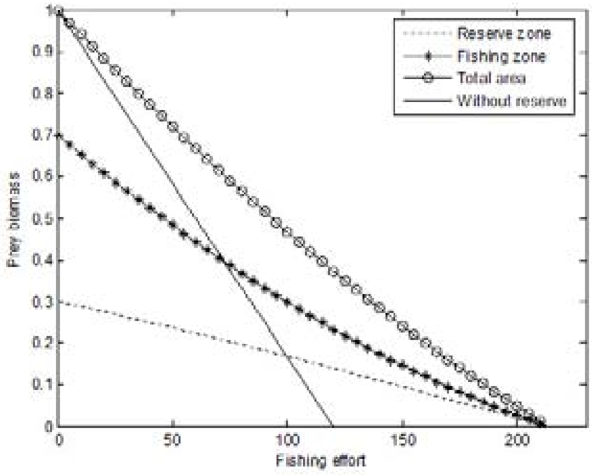

Fig. 1. Variation of transfer and catch with the increasing fishing effort when 30% reserve is taken into consideration.

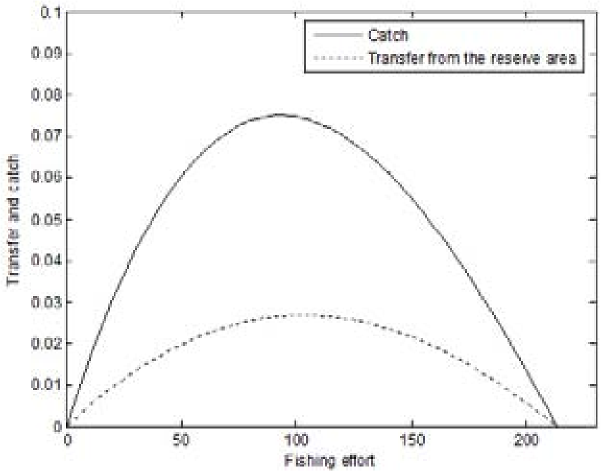

Fig. 2. Variation of prey biomass with the increasing fishing effort when 30% reserve is taken into consideration

Fig. 2 represents the relation between fishing effort, transfer of stock from reserve zone to fishing zone and the catch from fishing zone. It is observed from the figure that both transfer and catch increase up to a certain limit as fishing effort increases, but after this limit both are decreased and tend to zero with the increasing fishing effort. This is due to the fact that at the preliminary stage when the biomass in the open zone is heavily fished the density of stock in both the zones differ and the flow of transfer from reserve zone to fishing zone increases, consequently catch reaches to its maximum level as the flow of transfer reaches to its maximum level but after this level i.e. in the later stage when the rate transfer decreases, catch is also decrease and tend to zero as fishing effort increases.

In the Fig. 3 the comparison of total fish biomass is plotted against the fishing effort depending on the relative size of the reserve. It is evident from the figure that the fraction of stock inside the reserve zone give some protection against the extinction of the total fish biomass with the increasing fishing effort but it is not possible to eliminate the extinction of the total stock in each reserve area as it is clear from the figure that the protection of stock against extinction is increased when reserve area is increased.

The relation between catch and fishing effort depending on the relative size of the reserve are shown in Fig. 4. It is clear from the figure that the relative size of the reserve has a great influence on catch against the fishing effort. It is observed from the figures that when there is no reserve area, catch of stock increases and reach to its maximum level with the increasing fishing effort and after this limit catch decreases and tend to zero due to the fact that the stock has already been collapsed. But as the reserve area increases the total stock get some protection from extinction against the increasing fishing effort, consequently the catch increases simultaneously with the increasing fishing effort as the flow of transfer from reserve zone to open zone increases due to the difference between the densities in both the zones. Though catch is decreased when the fraction of stock in reserve zone collapse and ultimately tends to zero with the increasing fishing effort.

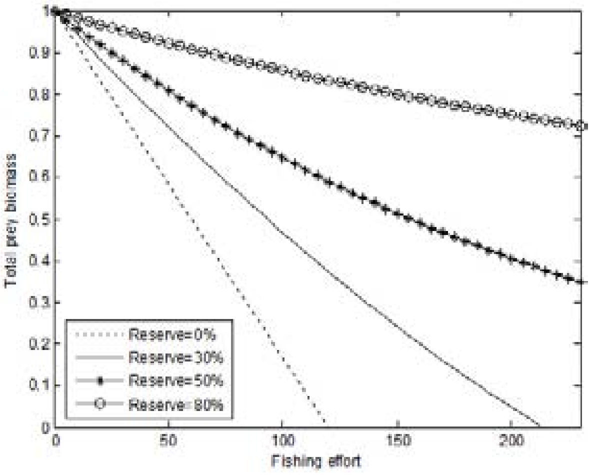

Fig. 3. Variation of prey biomass with the increasing fishing effort depending on the size of the reserve

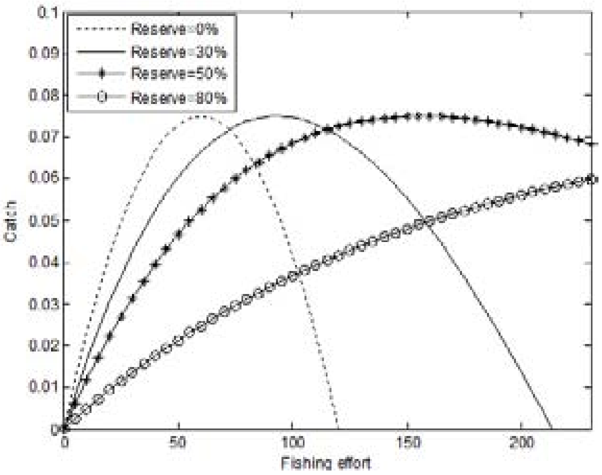

Fig. 4. Variation of catch with the increasing fishing effort depending on the size of the reserve

3.2 Simulation with prey-predator interaction

In this section, we consider the predation term mxz and nyz as exploited terms respectively for the reserved and unreserved area in addition to the catch by fishers. In this section, the simulation results are also compared with simulation results obtained in the previous section in absence of predation term. In order to represent the impacts of marine reserve in a fishery aforesaid comparisons are shown.

The effects of prey biomass and predator biomass with the increasing fishing effort are depicted in Fig. 5. It is observed from the figure that both the prey and predator biomass reach to their extinct limit with the increasing fishing effort. It is also observed that the prey biomass in presence of predators is always lower than the prey biomass in absence of predators at each level of increasing fishing effort though both are reached to their extinction for a certain value of fishing effort. This is an interesting effect of prey-predator interaction which may be considered as an important consequence for fishery management. Due to increasing fishing effort the stock of prey biomass reduces and tends to zero and consequently in absence of food storage, the stock of predator biomass decreases and reached to their extinct limit.

Various impacts of exploited terms i.e., catch and predation are represented in Fig. 6 for a given size of marine reserve, when fishing effort increases. It is clearly understandable from the Fig. 6 that how prey biomass is shared between predators and fishers. It is evident from the figure that the catch by fishers in presence of predators remains below compared to the catch in absence of predators. However, if fishing effort increases, it is noted that total exploitation i.e. predation along with catch by fishers decreases and ultimately tend to zero for a certain fishing effort. It is also depicted that the total predation term decreases and tend to zero with the increasing fishing effort since it is clear from Fig. 5 that the predator biomass is reached to its extinction due to competition for food.

The establishment of the reserve itself may influence the exploitation rate if fishing effort becomes more concentrated. In the forth coming figures the relationship between equilibrium level of biomass (prey & predator), catch, predation and rent are presented with respect to the relative size of the reserve area for a constant fishing effort.

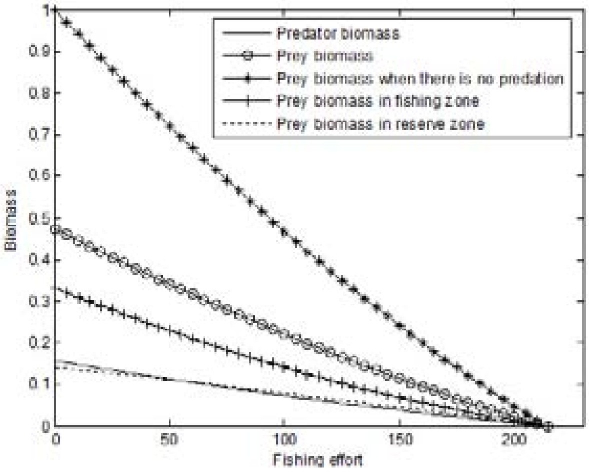

Fig. 5. Variation of biomass with the increasing fishing effort when 30% reserve is taken into consideration in presence of predators

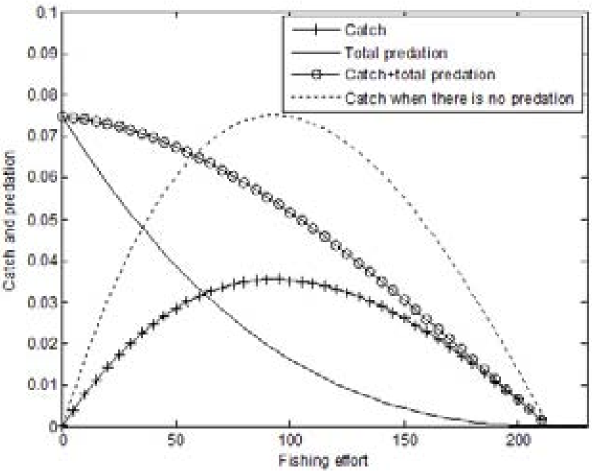

Fig. 6. Variation of catch and predation with the increasing fishing effort when 30% reserve is taken into consideration

Fig. 7 illustrates the impact of the relative size of the reserve area on equilibrium level of biomass, both on prey and predator. It is observed from the figure that both prey and predator biomass increase with the increasing size of the reserve. The prey biomass may get protection due to the fact that with the increasing size of the reserve the rate of harvesting decreases i.e. the harvesting rate is inversely proportional to the relative size of the reserve and the predator biomass get protection due to enough collection of food (prey biomass). It is also noted that in absence of predators the prey biomass reached to its peak level when the relative size of the reserve is unit as expected.

It is clear from Fig. 8 that for a given fishing effort the relative size of the reserve has great influence on exploited terms i.e. catch, predation etc. It is observed from the figure that with the increasing size of the reserve catch by fishers decreases not only in presence of predators but also in absence of predators and ultimately at the unit size of the reserve both are equal to zero. It is interesting to see that with the increasing size of the reserve total predation term also increases and this is due to the fact that the stock of prey biomass increases with the increasing size of the reserve. The figure also illustrates that the total exploited term i.e., the sum of catch by fishers and total predation, increases with the increasing size of the reserve and it cannot be equal to zero at unit reserve area, and this is due to the exponential nature of the total predation with the increasing size of the reserve.

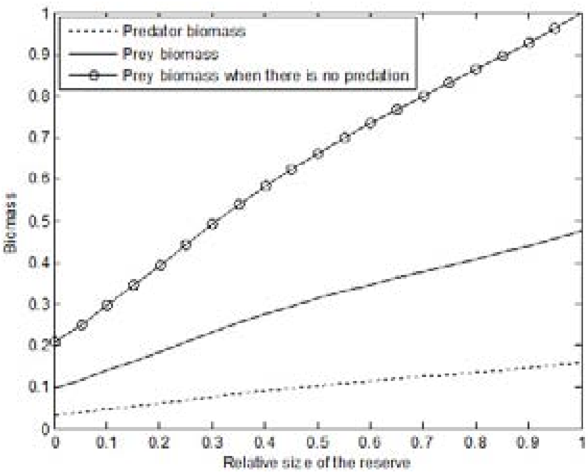

Fig. 7. Variation of biomass with the increasing size of the reserve at constant fishing effort ($E = 95$)

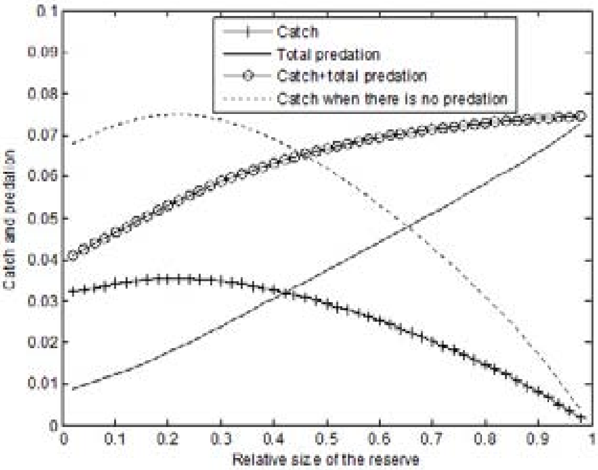

Fig. 8. Variation of catch and predation with the increasing size of the reserve at constant fishing effort ($E = 95$)

The effects of prey and predator stock in two different system namely open access and marine reserve with the increasing rate of migration are shown in Fig. 9. It is observed that the stock of prey and predator remain constant when open access is considered. This is due to the fact that the considered model is independent of migration rate. Again in presence of reserve when migration of fish from reserve area to open area is considered it is observed that both prey and predator stock get protection compared to the stock in open access though after a fixed value of migration rate the stock of prey and predator in marine reserve also remain constant but the prey and predator stock in marine reserve are always in higher level compared to the constant stock in case of open access. This is an important consequence of the presence of marine reserve.

The relation between catch and migration rate in two different system are represented in Fig. 10. It is evident from the figure that the catch in open access remains constant since it is independent of migration rate. It is also clear from the figure that the fish migration from reserve area to fishing area has a great impact on fishing i.e. on the term catch. In presence of reserve it is noted that catch is always in higher level compared to the catch in open access and this is due to the higher stock which supports the consequences of Fig. 9.

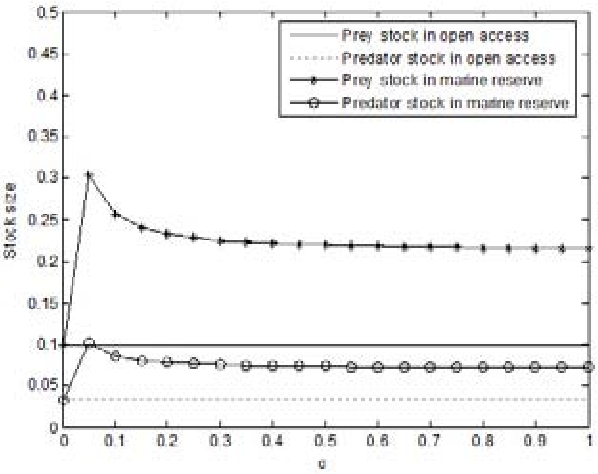

Fig. 9. Variation of stock size with the increasing rate of migration when 30% reserve is taken into consideration in presence of predators

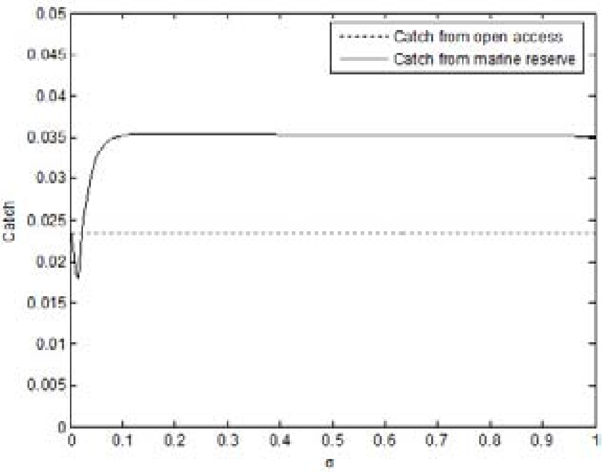

Fig. 10. Variation of catch with the increasing rate of migration when 30% reserve is taken into consideration in presence of predators

The change of exploitation rate is plotted against the rate of migration in Fig. 11. Exploitation rate is considered as the catch per unit stock. It is found from the figure that the exploitation rate in marine reserve is always in lower level compared to the exploitation rate in open access and due to the lower exploitation rate, the stock in presence of reserve has got protection which ultimately support the consequence explained in Fig. 9.

3.3 Economic rents in the fishery

For a fishery manager, the management objective is to maximize the rents of the fishery using certain management tools. And it should be mentioned that under certain condition marine reserve may serve to increase the flow of rents. We can develop ecotourism to overcome the dilemma between the need for long-term resource conservation and the immediate necessity to provide jobs and income to the local population. Here we consider fishing and ecotourism rents and these are defined as follows.

$$\text{Fishery Rent}: T_1 = p_1 h_1 - c_1 E_1$$
$$\text{Ecotourism Rent}: T_2 = p_2 h_2 - c_2 E_2$$

where p_1 & c_1 are respectively the unit price of the prey and unit cost of effort devoted and p_2 & c_2 are respectively the unit price of the predator and unit cost of effort devoted. We assume a Cob-Douglas type production

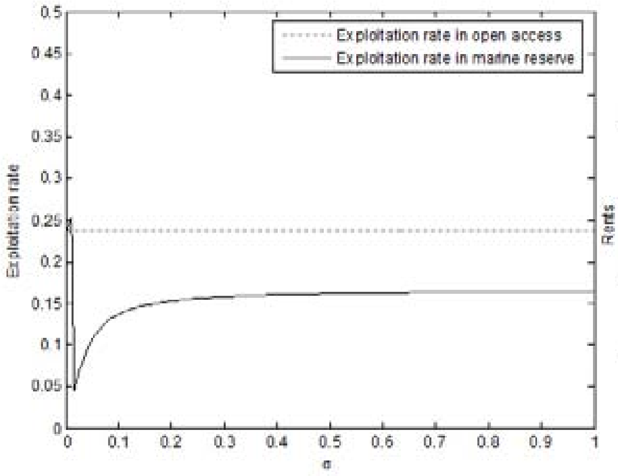

Fig. 11. Variation of exploitation rate with the increasing rate of migration when 30% reserve is taken into consideration in presence of predators

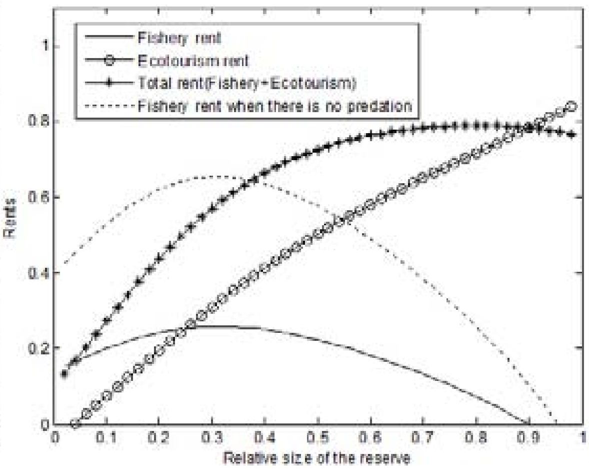

Fig. 12. Variation of rents with the increasing size of the reserve

function for ecotourism rent as follows:

$$h_2 = az^b E_2^c$$

where h_2 is the flow of ecotourism visits of the area, E_2 is the effort devoted to the ecotourism industry, a is a positive dimension parameter, b is the elasticity of visits with regard to the abundance of parameters and c is the elasticity of visits with regard to the effort devoted to promoting ecotourism.

The numerical values of the parameters are as follows: $p_1 = 10$,$p_2 = 5.5$, $c_1 = 0.001$, $c_2 = 40$, $E_1 = 0.01$, $a = 1$, $b = 0.8$, $c = 0.2$, and remaining parameters are same as used earlier.

The effects of rents of a fishery are illustrated in Fig. 9 with the increasing size of the marine reserve. From the figure it is clear that for a certain value of fishing effort fishery rent increases up to a certain value of the reserve and after that fishery rent decreases with the increasing size of the reserve and ultimately tend to zero. The fishery rent is directly proportional to the catch by fishers, thus at the preliminary stage fishery rent increases as catch increases up to a certain level of reserve and then catch decreases consequently fishery rent decreases and tend to zero. It is also observed that the fishery rent in absence of predators is always greater compared to the fishery rent in presence of predator, thus prey-predator interaction reduces the profit to the fishers. As it has already been seen that the predator biomass increases with the increasing size of the reserve (see Fig. 7) thus ecotourism rent increases with the increasing size of the reserve. Consequently total economic rent (fishery + ecotourism) is always in upper level compared to the fishery rent.

4 Optimal harvesting policy

Let c be the constant fishing cost per unit effort and p the constant price per unit biomass of landed fish in the unreserved area. The economic rent (revenue at any time) is given by

$$\pi(x, y, z, E) = (qpy - c)E$$

In this section our objective is to select a harvesting that maximizes the present value of a continuous time-stream of revenues. Here, δ is the instantaneous annual discount rate.

$$J = \int_0^\infty e^{-\delta t}\pi(x, y, z, E)\mathrm{d}t \tag{5}$$

The problem (5), subject to population equations (4) and control constraints $0 \leq E \leq E_{\max}$, can be solved by applying Pontryagin's maximum principle. The Hamiltonian is given by

$$\begin{aligned} H = e^{-\delta t}(pqy-c)E + \lambda_1\left[rx(1-x-y) - \sigma\left(\frac{x}{\alpha} - \frac{y}{1-\alpha}\right) - mxz\right] \\ + \lambda_2\left[ry(1-x-y) + \sigma\left(\frac{x}{\alpha} - \frac{y}{1-\alpha}\right) - nyz - qEy\right] + \lambda_3\left[sz\left(1-\frac{\gamma z}{x+y}\right)\right] \end{aligned}$$

where λ_1, λ_2, λ_3 are the adjoint variables and $\mu(t) = e^{-\delta t}(pqy-c) - \lambda_2 qy$ is called the switching function[3].

Since Hamiltonian H is linear in the control variable, the optimal control will be a combination of extreme controls and the singular control. The optimal control $E(t)$ that maximizes H must satisfy the following conditions:

$$\begin{aligned} &E = E_{\max}, \text{when } \mu(t) > 0, i.e. \quad \lambda_2(t)e^{\delta t} < p - c/qy \\ &E = 0, \text{ when } \mu(t) < 0, i.e. \quad \lambda_2(t)e^{\delta t} > p - c/qy \end{aligned}$$

$\lambda_2(t)e^{\delta t}$ is the usual shadow price[3] and $p - c/qy$ is the net economic revenue on a unit harvest. This shows that $E = E_{\max}$ or zero, according to the shadow price, is less than or greater than the net economic revenue on a unit harvest. Economically, the first condition implies that if the profit after paying all the expense is positive, then it is beneficial to harvest up to the limit of available effort. The second condition implies that, when the shadow price exceeds the fishermen's net economic revenue on a unit harvest, then the fishermen will not exert any effort.

When $\mu(t) = 0$, i.e., when the shadow price equals the net economic revenue on a unit harvest, then the Hamiltonian H becomes independent of the control variable $E(t)$, i.e., $\partial H/\partial E = 0$. This is the necessary condition for the singular control $E^*(t)$ to be optimal over the control set $0 < E^* < E_{\max}$. Thus the optimal harvesting policy is

$$E(t) = \begin{cases} E_{\max} & \text{when } \mu(t) > 0 \\ 0, & \text{when } \mu(t) < 0 \\ E^* & \text{when } \mu(t) = 0 \end{cases}$$

When $\mu(t) = 0$, it follows that

$$\lambda_2 qy = e^{-\delta t}(pqy - c) = e^{-\delta t}\frac{\partial \pi}{\partial E} \tag{6}$$

This implies that the user's cost of harvest per unit of effort equals the discounted value of the future marginal profit of the effort at the steady state level. Now, the adjoint equations are

$$\frac{\mathrm{d}\lambda_1}{\mathrm{d}t} = -\frac{\partial H}{\partial x} = -\left[\lambda_1\left(r - 2xr - ry - \frac{\sigma}{\alpha} - mz\right) + \lambda_2\left(-ry + \frac{\sigma}{\alpha}\right) + \lambda_3\left(-\frac{s\gamma z^2}{(x+y)^2}\right)\right] \tag{7}$$

$$\begin{aligned} \frac{\lambda_2}{\mathrm{d}t} = -\frac{\partial H}{\partial y} = -\Bigg[e^{\delta t}pqE + \lambda_1\left(-rx + \frac{\sigma}{1-\alpha}\right) + \lambda_2\left(r - rx - 2ry - \frac{\sigma}{1-\alpha} - nz - qE\right) \\ +\lambda_3\left(-\frac{s\gamma z^2}{(x+y)^2}\right)\Bigg] \end{aligned} \tag{8}$$

$$\frac{\mathrm{d}\lambda_3}{\mathrm{d}t} = -\frac{\partial H}{\partial z} = -\left[-\lambda_1 mx - \lambda_2 ny + \lambda_3\left(s - \frac{2s\gamma z}{x+y}\right)\right] \tag{9}$$

Now using (7), (8) and (9) in (6) we have the following optimal equation for E;

$$\frac{A_6}{A_5+\delta} = \frac{1}{qy}(c - qpy)$$

$$\text{where } A_1 = s\left(1 + \frac{z^* m x^*(1-\alpha)}{(x^*+y^*)(\sigma - rx^* + rx^*\alpha)}\right)$$

$$A_2 = \frac{\delta m x^*(pqy^* - c)(1-\alpha)}{qy^*(\sigma - rx^* + rx^*\alpha)} - \frac{pqEmx^*(1-\alpha)}{(\sigma - rx^* + rx^*\alpha)} + \frac{mx^*(pqy^* - c)(1-\alpha)}{qy^*(\sigma - rx^* + rx^*\alpha)}\left(\frac{\sigma x^*}{\alpha y^*} + ry^*\right) + \frac{n(pqy^* - c)}{q}$$

$$A_3 = rx^* + \frac{\sigma y^*}{x^*(1-\alpha)}$$

$$A_4 = \frac{(pqy^* - c)}{qy^*}\left(ry^* - \frac{\sigma}{\alpha}\right) - \frac{A_2 s z^*}{(A_1+\delta)(x^*+y^*)}$$

$$A_5 = \frac{\sigma x^*}{\alpha y^*} + ry^*$$

$$A_6 = \frac{A_4}{A_3+\delta}\left(\frac{\sigma}{1-\alpha} - rx^*\right) - \frac{A_2 s z^*}{(A_1+\delta)(x^*+y^*)} - pqE$$

4.1 Numerical examples

In order to ensure the existence of the optimal equilibrium we use the following values:

$$r = 0.3, \sigma = 0.2, m = 1, n = 1, q = 0.8, s = 0.1, \gamma = 3, \delta = 0.01, p = 15, c = 0.05$$

Using these parameter values when 40% reserve is considered from optimal harvesting policy we get the optimal value of $E = 0.360847$ and the corresponding optimal equilibrium is $(0.116597, 0.123048, 0.07988)$.

Again using these parameter values when 70% reserve is considered from optimal harvesting policy we get the optimal value of $E = 0.952923$ and the corresponding optimal equilibrium is $(0.209233, 0.0463112, 0.0851813)$.

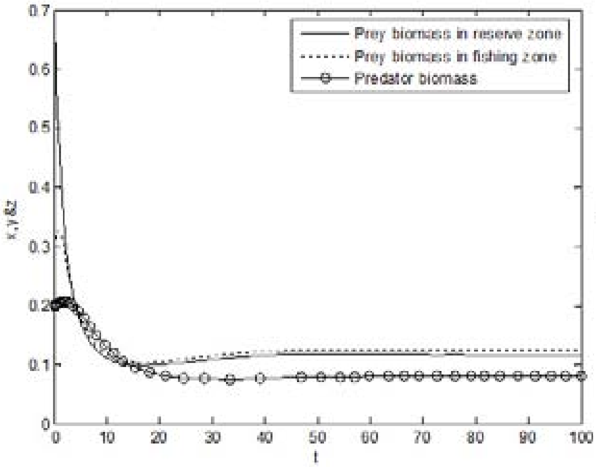

Fig. 13. Variation of optimal biomass with the increasing time when 40% reserve is considered

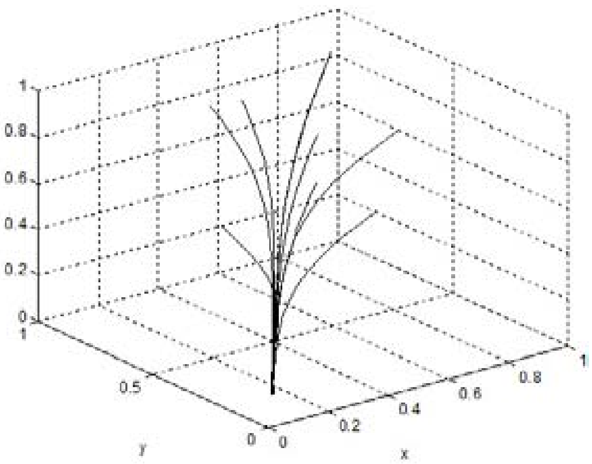

Fig. 14. Phase space trajectories corresponding to the optimal effort $E = 0.360847$ beginning with different initial levels when 40% reserve is considered

In Fig. 20 the optimal stock of prey and predator are plotted against the fishing effort when 40% reserve is considered. It is found from the figure that at $E = 0.995$ stocks are reached to their extinction and therefore $E = 0.995$ is considered as maximum effort for bang bang theory with 40% reserve.

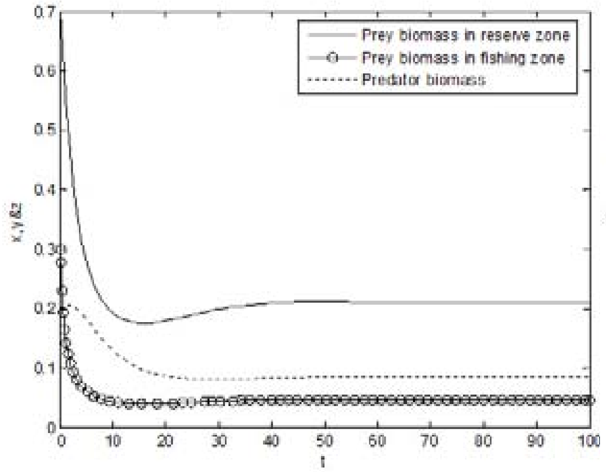

Fig. 15. Variation of optimal biomass with the increasing time when 70% reserve is considered

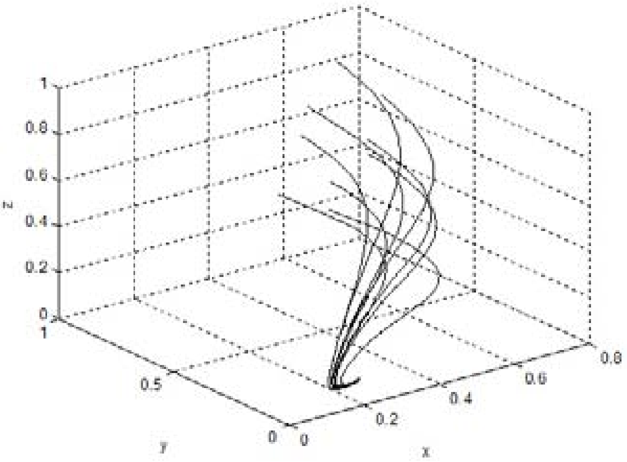

Fig. 16. Phase space trajectories corresponding to the optimal effort $E = 0.952923$ beginning with different initial levels when 70% reserve is considered

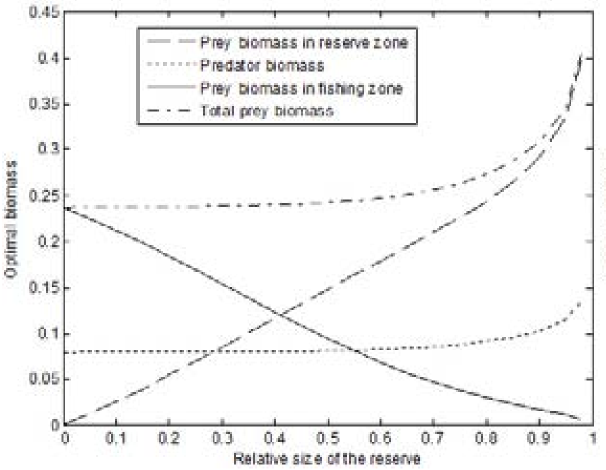

Fig. 17. Variation of optimal biomass with the increasing size of the reserve

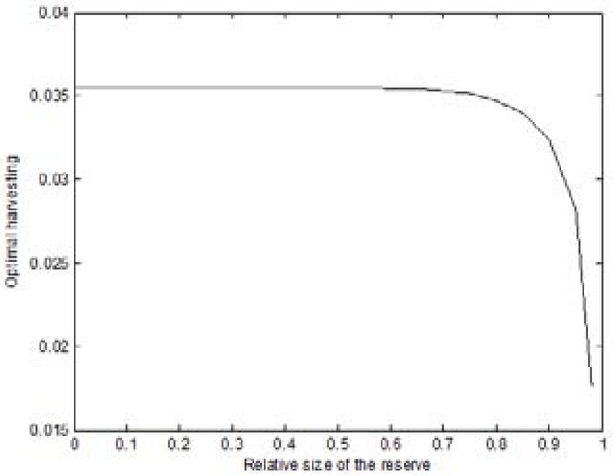

Fig. 18. Variation of optimal harvesting with the increasing size of the reserve

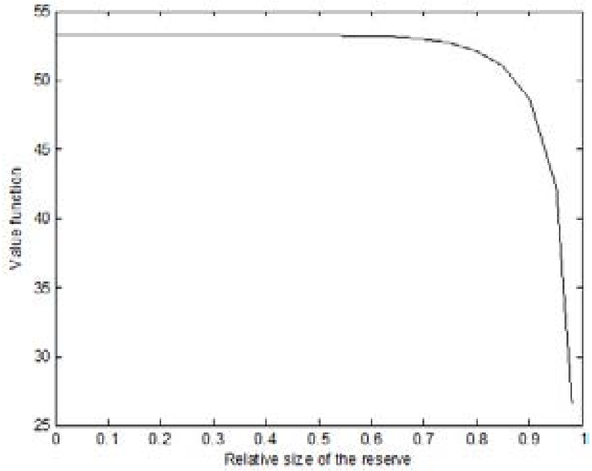

Fig. 19. Variations of value function with the increasing size of the reserve

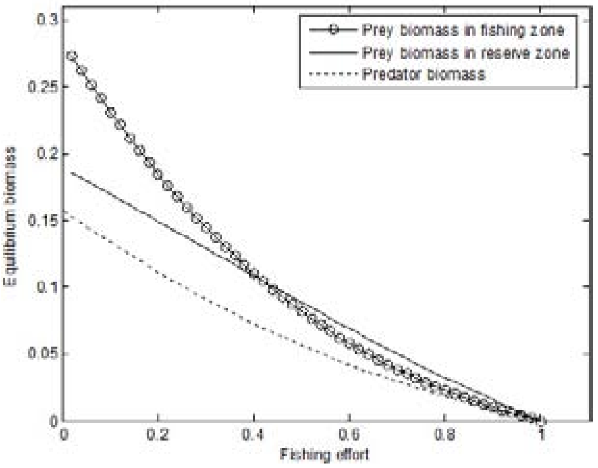

Fig. 20. Variation of optimal biomass with the increasing fishing effort when 40% reserve is considered

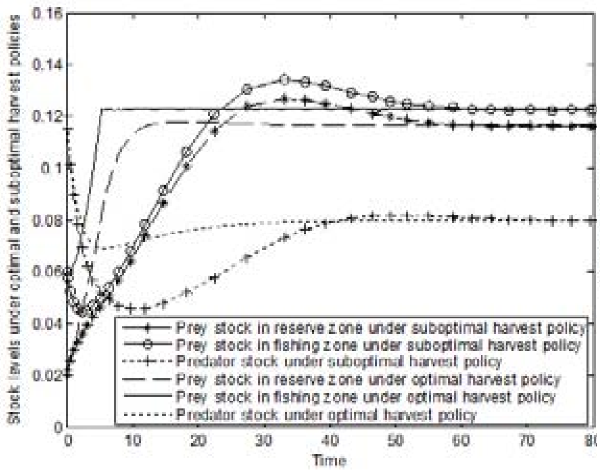

Fig. 21. Variation of stock levels under optimal and suboptimal harvest policies when 40% reserve is considered. Both optimal and suboptimal paths approach their respective singular optimal solutions

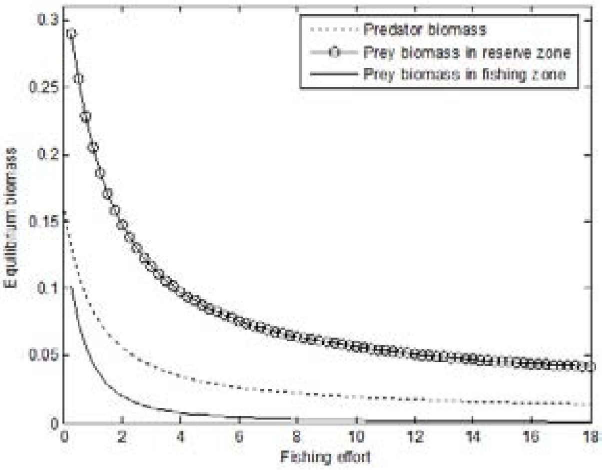

Fig. 22. Variation of optimal biomass with the increasing fishing effort when 70% reserve is considered

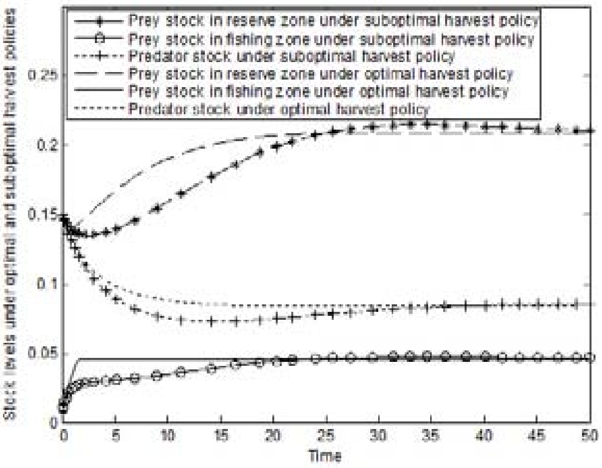

Fig. 23. Variation of stock levels under optimal and suboptimal harvest policies when 70% reserve is considered. Both optimal and suboptimal paths approach their respective singular optimal solutions

Table 1. Time taken by different approach paths

Figure number	Coordinates		Path type	Time taken to reach the end point
	Initial point	End point		
Fig. 21	$(0.02, 0.06, 0.12)$	$(0.117, 0.123, 0.08)$	Optimal	53.60
	$(0.02, 0.06, 0.12)$	$(0.117, 0.123, 0.08)$	Suboptimal	79.63
Fig. 23	$(0.15, 0.01, 0.15)$	$(0.209, 0.046, 0.086)$	Optimal	39.19
	$(0.15, 0.01, 0.15)$	$(0.2092, 0.0463, 0.0855)$	Suboptimal	60

In Fig. 22 the optimal stock of prey and predator are plotted against the fishing effort when 70% reserve is considered. It is found from the figure that at $E = 15.57$ predator stock is reached to the extinction and therefore $E = 15.57$ is considered as maximum effort for bang bang theory with 70% reserve.

Details of the figures are given in Tab. 1. It is observed that suboptimal paths always take more time than optimal paths to reach the optimal singular solution.

5 Conclusion

Over-exploitation and extinction of fisheries is a serious global problem in recent times, that current management policies struggle to solve. Traditionally, the management of fisheries are based on adjusted quotas or effort control. Unfortunately, such management strategies are expensive to implement due to its enormous amount of data collection needed for exact stock assessments.

This paper describes a prey-predator fishery model with prey dispersal in a two patch environment, one is assumed to be a free fishing zone and the other is a reserved zone where fishing is prohibited but it can be used for ecotourism purposes. Our simulations indicate that MPAs can substantially reduce the risk of fisheries collapse. Other possible benefits ranging from preservation of biodiversity and ecosystem integrity to increased tourism revenue are considered here and it likely to strengthen the case of MPAs. Our simulation clearly indicates that prey-predator interactions do matter when the implementation of a reserve is considered. It is also observed that reserves will be most effective when coupled with fishing effort controls in adjacent fisheries.

We have not considered the cost and benefits of marine reserves versus conventional management methods. More sophisticated integrated models are needed to facilitate comparison between reserves and conventional fishery management. Analysis of costs and benefits of reserves thus requires more research.

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