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The Homotopy perturbation method for solving higher dimensional initial boundary value problems of variable coefficients

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Abstract. In this Letter, an application of homotopy perturbation method (HPM) is applied to solve the higher dimensional initial boundary value problems of variable coefficients. Comparison are made between the Adomians decomposition method and homotopy perturbation method. The results reveal that the homotopy perturbation method is very effective and simple and gives the exact solution.

Keywords: Homotopy Perturbation Method (HPM), higher dimensional initial boundary value problems, Adomian Decomposition Method (ADM)

1 Introduction

The numerical and analytical solutions of higher dimensional initial boundary value problems (IBVP) of variable coefficients, linear and nonlinear, are of considerable significance for applied sciences. Examples of linear models are Euler - Darboux equation [11], Larnbropoubs equation [13], and Tricomi equation [1] given by

$$(x-y)u_{xy} + (au_x - bu_y) = 0, \tag{1}$$

$$u_{xy} + axu_x - byu_y + cxyu + u_t = 0, \tag{2}$$

$$u_{yy} = yu_{xx}, \tag{3}$$

respectively. Examples of nonlinear models are introduced in KdV equation [9] of variable coefficients and Clairaut's equation [10] given by

$$u_t + \alpha t^n uu - x + \beta t^m uu_{xxx} = 0, \tag{4}$$

$$u = xu_x + yu_y + f(u_x, u_y), \tag{5}$$

respectively. For more details about these models, the reader is referred to [14].

The homotopy perturbation method [6], proposed first by He in 1998 and was further developed and improved by He [3, 5, 7, 8]. The method yields a very rapid convergence of the solution series in the most cases. Usually, one iteration leads to high accuracy of the solution. Although goal of He's homotopy perturbation method was to find a technique to unify linear and nonlinear, ordinary or partial differential equations for solving initial and boundary value problems. In this paper, we apply He's homotopy perturbation method to higher dimensional initial boundary value problems of variable coefficients. The results reveal that the proposed method is very effective and simple.

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2 Basic idea of homotopy perturbation method

To illustrate the basic ideas of this method, we consider the following non-linear differential equation

$$A(u) - f(r) = 0, \qquad r \in \Omega, \tag{6}$$

with the following boundary conditions

$$B\left(u, \frac{\partial u}{\partial n}\right), \qquad r \in \Gamma, \tag{7}$$

where A is a general differential operator, B a boundary operator, $f(r)$ is a known analytical function and Γ is the boundary of the domain Ω. The operator A can be decomposed into two operators, $\mathcal{L}$ and $\mathcal{N}$, where $\mathcal{L}$ is a linear, and $\mathcal{N}$ a nonlinear operator. Eq. (6) can be, therefore, written as follows:

$$\mathcal{L}(u) + \mathcal{N}(u) - f(r) = 0. \tag{8}$$

Using the homotopy technique, we construct a homotopy$U(r; p) : \Omega \times [0, 1] \to \mathcal{R}$, which satisfies:

$$\mathcal{H}(U, p) = (1-p)[\mathcal{L}(U) - \mathcal{L}(u_0)] + p[A(U) - f(r)] = 0, \qquad p \in [0,1], r \in \Omega. \tag{9}$$

or

$$\mathcal{H}(U, p) = \mathcal{L}(U) - \mathcal{L}(u_0) + p\mathcal{L}(u_0) + p[\mathcal{N}(U) - f(r)] = 0, \tag{10}$$

where $p \in [0, 1]$ is an embedding parameter, u_0 is an initial approximation for the solution of Eq. (6), which satisfies the boundary conditions. Obviously, from Eqs. (9) and (10) we will have

$$\mathcal{H}(U, 0) = \mathcal{L}(U) - \mathcal{L}(u_0) = 0, \tag{11}$$

$$\mathcal{H}(U, 1) = A(U) - f(r) = 0. \tag{12}$$

The changing process of p form zero to unity is just that of $U(r, p)$ from $u_0(r)$ to $u(r)$. In topology, this is called homotopy. According to the (HPM), we can first use the embedding parameter p as a small parameter, and assume that the solution of Eqs. (9) and (10) can be written as a power series in p:

$$U = U_0 + pU_1 + p^2 U_2 + \cdots . \tag{13}$$

Setting $p = 1$, results in the approximate solution of Eq. (6)

$$u = \lim_{p \longrightarrow 1} U = U_0 + U_1 + U_2 + \cdots . \tag{14}$$

The combination of the perturbation method and the homotopy method is called the homotopy perturbation method (HPM), which has eliminated the limitations of the traditional perturbation methods. On the other hand, this technique can have full advantage of the traditional perturbation techniques. The series (13) is convergent for most cases. Some criteria is suggested for convergence of the series (13), in [4].

3 Applications

In this section, four distinct problems will be tested by using the proposed method presented above. The first two problems are linear, where as the last two examples are nonlinear.

Example 1

We first consider the two-dimensional IBVP:

$$u_{tt} = \frac{1}{2} y^2 u_{xx} + \frac{1}{2} x^2 u_{yy}, \qquad 0 < x, y < 1, \qquad t > 0, \tag{15}$$

subject to the boundary conditions,

$$u(0,y,t) = y^2 e^{-t}, \qquad u(1,y,t) = (1+y^2)e^{-t}, \tag{16}$$
$$u(x,0,t) = x^2 e^{-t}, \qquad u(x,1,t) = (1+x^2)e^{-t}, \tag{17}$$

and the initial conditions

$$u(x,y,0) = x^2 + y^2, \qquad u_t(x,y,0) = -(x^2+y^2), \tag{18}$$

To solve Eq. (15)$\sim$ Eq. (18) by homotopy perturbation method, we construct the following homotopy:

$$(1-p)\left(\frac{\partial^2 v}{\partial t^2} - \frac{\partial^2 u_0}{\partial t^2}\right) + p\left(\frac{\partial^2 v}{\partial t^2} - \frac{1}{2}y^2\frac{\partial^2 v}{\partial x^2} - \frac{1}{2}x^2\frac{\partial^2 v}{\partial y^2}\right) = 0, \tag{19}$$

or

$$\frac{\partial^2 v}{\partial t^2} - \frac{\partial^2 u_0}{\partial t^2} = p\left(\frac{1}{2}y^2\frac{\partial^2 v}{\partial x^2} + \frac{1}{2}x^2\frac{\partial^2 v}{\partial y^2} - \frac{\partial^2 u_0}{\partial t^2}\right). \tag{20}$$

Suppose the solution of Eq. (20) to be in the following form

$$v = v_0 + pv_1 + p^2 v_2 + \cdots . \tag{21}$$

Substituting (21) into (20), and equating the coefficients of the terms with the identical powers of p,

$$\begin{aligned}
&p^0 : \frac{\partial^2 v_0}{\partial t^2} = \frac{\partial^2 u_0}{\partial t^2}, \\
&p^1 : \frac{\partial^2 v_1}{\partial t^2} = \frac{1}{2}y^2\frac{\partial^2 v_0}{\partial x^2} + \frac{1}{2}x^2\frac{\partial^2 v_0}{\partial y^2} - \frac{\partial^2 u_0}{\partial t^2}, \qquad v_1(x,y,0) = 0, \\
&p^2 : \frac{\partial^2 v_2}{\partial t^2} = \frac{1}{2}y^2\frac{\partial^2 v_1}{\partial x^2} + \frac{1}{2}x^2\frac{\partial^2 v_1}{\partial y^2}, \qquad v_2(x,y,0) = 0, \\
&p^3 : \frac{\partial^2 v_3}{\partial t^2} = \frac{1}{2}y^2\frac{\partial^2 v_2}{\partial x^2} + \frac{1}{2}x^2\frac{\partial^2 v_2}{\partial y^2}, \qquad v_3(x,y,0) = 0, \\
&\quad\vdots \\
&p^j : \frac{\partial^2 v_j}{\partial t^2} = \frac{1}{2}y^2\frac{\partial^2 v_{j-1}}{\partial x^2} + \frac{1}{2}x^2\frac{\partial^2 v_{j-1}}{\partial y^2}, \qquad v_j(x,y,0) = 0, \\
&\quad\vdots
\end{aligned} \tag{22}$$

For simplicity we take

$$\begin{aligned}
&v_0(x,y,t) = u_0(x,y,t), \\
&v_1(x,y,t) = \int_0^t \int_0^t \left\{\frac{1}{2}y^2\frac{\partial^2 v_0}{\partial x^2} + \frac{1}{2}x^2\frac{\partial^2 v_0}{\partial y^2} - \frac{\partial^2 u_0}{\partial t^2}\right\} dtdt.
\end{aligned} \tag{23}$$

Having this assumption we get the following iterative equation

$$v_j = \int_0^t \int_0^t \left\{\frac{1}{2}y^2\frac{\partial^2 v_{j-1}}{\partial x^2} + \frac{1}{2}x^2\frac{\partial^2 v_{j-1}}{\partial y^2}\right\} dtdt, \qquad j = 2,3,4,\cdots . \tag{24}$$

Starting with$v_0 = u_0 = (x^2+y^2) - t(x^2+y^2)$, by using Eq. (23) and (24), we obtain

$$\begin{aligned}
v_1(x,y,t) &= (\frac{t^2}{2} - \frac{t^3}{6})(x^2+y^2),\\
v_2(x,y,t) &= (\frac{t^4}{24} - \frac{t^5}{120})(x^2+y^2),\\
v_3(x,y,t) &= (\frac{t^6}{720} - \frac{t^7}{5040})(x^2+y^2),\\
&\vdots
\end{aligned}$$

The approximate solution of (15) can be obtained by setting $p = 1$,

$$u = \lim_{p\to 1} v = v_0 + v_1 + v_2 + \cdots = (1 - t + \frac{t^2}{2!} - \frac{t^3}{3!} + \frac{t^4}{4!} - \frac{t^5}{5!} + \frac{t^6}{6!} - \frac{t^7}{7!} + ...)(x^2+y^2).$$

Thus the exact solution will be as

$$u(x,y,t) = (x^2+y^2)e^{-t}.$$

which is exactly the same as obtained by Adomain decomposition method [12].
Example 2

We next consider the three-dimensional IBVP:

$$u_{tt} = \frac{1}{45}x^2u_{xx} + \frac{1}{45}y^2u_{yy} + \frac{1}{45}z^2u_{zz} - u. \qquad 0 < x,y,z < 1, \quad t > 0, \tag{25}$$

subject to the Neumann boundary conditions

$$u_x(0,y,z,t) = 0, \quad u_x(1,y,z,t) = 6y^6z^6 \tag{26}$$
$$u_y(x,0,z,t) = 0, \quad u_y(x,1,z,t) = 6x^6z^6 \tag{27}$$
$$u_z(x,y,0,t) = 0, \quad u_z(x,y,1,t) = 6x^6y^6 \tag{28}$$

and initial conditions

$$u(x,y,z,0) = 0, \qquad u_t(x,y,z,0) = x^6y^6z^6, \tag{29}$$

To solve Eq. (25) $\sim$ (29) by homotopy perturbation method, we construct the following homotopy:

$$(1-p)\left(\frac{\partial^2 v}{\partial t^2} - \frac{\partial^2 u_0}{\partial t^2}\right) + p\left(\frac{\partial^2 v}{\partial t^2} - \frac{1}{45}x^2v_{xx} - \frac{1}{45}y^2v_{yy} - \frac{1}{45}z^2v_{zz} + v\right) = 0, \tag{30}$$

or

$$\frac{\partial^2 v}{\partial t^2} - \frac{\partial^2 u_0}{\partial t^2} = p\left(\frac{1}{45}x^2v_{xx} + \frac{1}{45}y^2v_{yy} + \frac{1}{45}z^2v_{zz} - v - \frac{\partial^2 u_0}{\partial t^2}\right). \tag{31}$$

Suppose the solution of Eq. (31) to be in the following form

$$v = v_0 + pv_1 + p^2v_2 + \cdots. \tag{32}$$

Substituting (32) into (31), and equating the coefficients of the terms with the identical powers of p,

$$
\begin{aligned}
&p^0: \frac{\partial^2 v_0}{\partial t^2} = \frac{\partial^2 u_0}{\partial t^2}, \\
&p^1: \frac{\partial^2 v_1}{\partial t^2} = \frac{1}{45}x^2 v_{0,xx} + \frac{1}{45}y^2 v_{0,yy} + \frac{1}{45}z^2 v_{0,zz} - v_0 - \frac{\partial^2 u_0}{\partial t^2}, \qquad v_1(x,y,z,0)=0, \\
&p^2: \frac{\partial^2 v_2}{\partial t^2} = \frac{1}{45}x^2 v_{1,xx} + \frac{1}{45}y^2 v_{1,yy} + \frac{1}{45}z^2 v_{1,zz} - v_1, \qquad v_2(x,y,z,0)=0, \\
&p^3: \frac{\partial^2 v_3}{\partial t^2} = \frac{1}{45}x^2 v_{2,xx} + \frac{1}{45}y^2 v_{2,yy} + \frac{1}{45}z^2 v_{2,zz} - v_2, \qquad v_3(x,y,z,0)=0, \\
&\quad \vdots \\
&p^j: \frac{\partial^2 v_j}{\partial t^2} = \frac{1}{45}x^2 v_{j-1,xx} + \frac{1}{45}y^2 v_{j-1,yy} + \frac{1}{45}z^2 v_{j-1,zz} - v_{j-1}, \qquad v_j(x,y,z,0)=0, \\
&\quad \vdots
\end{aligned} \tag{33}
$$

For simplicity we take

$$
\begin{aligned}
&v_0(x,y,z,t) = u_0(x,y,z,t), \\
&v_1(x,y,z,t) = \int_0^t \int_0^t \left\{ \frac{1}{45}x^2 v_{0,xx} + \frac{1}{45}y^2 v_{0,yy} + \frac{1}{45}z^2 v_{0,zz} - v_0 - \frac{\partial^2 u_0}{\partial t^2} \right\} dtdt.
\end{aligned} \tag{34}
$$

Having this assumption we get the following iterative equations

$$
v_j = \int_0^t \int_0^t \left\{ \frac{1}{45}x^2 v_{j-1,xx} + \frac{1}{45}y^2 v_{j-1,yy} + \frac{1}{45}z^2 v_{j-1,zz} - v_{j-1} \right\} dtdt, \quad j = 2,3,4,\cdots \tag{35}
$$

Starting with $v_0 = u_0 = tx^6y^6z^6$, by using (34) and (35), we obtain

$$
\begin{aligned}
v_1(x,y,z,t) &= \frac{t^3}{6}x^6y^6z^6, \\
v_2(x,y,z,t) &= \frac{t^5}{120}x^6y^6z^6, \\
v_3(x,y,z,t) &= \frac{t^7}{5040}x^6y^6z^6, \\
&\vdots
\end{aligned}
$$

he approximate solution of Eq. (25) can be obtained by setting $p = 1$,

$$
u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \cdots = (t + \frac{t^3}{3!} + \frac{t^5}{5!} + \frac{t^7}{7!} + ...)x^6y^6z^6.
$$

Thus the exact solution will be as

$$
u(x,y,z,t) = x^6y^6z^6 \sinh t.
$$

which is exactly the same as obtained by Adomain decomposition method [12].

Example 3

Consider the two-dimensional nonlinear inhomogeneous IBVP:

$$
u_{tt} = 2x^2 + 2y^2 + \frac{15}{2}(xu_{xx}^2 + yu_{yy}^2), \tag{36}
$$

$$
u(0,y,t) = y^2t^2 + yt^6, \qquad u(1,y,t) = (1+y^2)t^2 + (1+y)t^6, \tag{37}
$$

$$
u(x,0,t) = x^2t^2 + xt^6, \qquad u(x,1,t) = (1+x^2)t^2 + (1+x)t^6, \tag{38}
$$

and initial conditions

$$u(x, y, 0) = 0, \qquad u_t(x, y, 0) = 0, \tag{39}$$

To solve Eq. (36) $\sim$ (39) by homotopy perturbation method, we construct the following homotopy:

$$(1-p)\left(\frac{\partial^2 v}{\partial t^2} - \frac{\partial^2 u_0}{\partial t^2}\right) + p\left(\frac{\partial^2 v}{\partial t^2} - (2x^2 + 2y^2) - \frac{15}{2}(xv_{xx}^2 + yv_{yy}^2)\right) = 0, \tag{40}$$

or

$$\frac{\partial^2 v}{\partial t^2} - \frac{\partial^2 u_0}{\partial t^2} = p\left(2x^2 + 2y^2 + \frac{15}{2}(xv_{xx}^2 + yv_{yy}^2) - \frac{\partial^2 u_0}{\partial t^2}\right). \tag{41}$$

Suppose the solution of Eq. (41) to be in the following form

$$v = v_0 + pv_1 + p^2 v_2 + \cdots. \tag{42}$$

Substituting (42) into (41), and equating the coefficients of the terms with the identical powers of p,

$$\begin{aligned}
&p^0 : \frac{\partial^2 v_0}{\partial t^2} = \frac{\partial^2 u_0}{\partial t^2}, \\
&p^1 : \frac{\partial^2 v_1}{\partial t^2} = 2x^2 + 2y^2 + \frac{15}{2}(xv_{0,xx}^2 + yv_{0,yy}^2) - \frac{\partial^2 u_0}{\partial t^2}, \qquad v_1(x, y, 0) = 0, \\
&p^2 : \frac{\partial^2 v_2}{\partial t^2} = \frac{15}{2}(xv_{0,xx}v_{1,xx} + yv_{0,yy}v_{1,yy}), \qquad v_2(x, y, 0) = 0, \\
&p^3 : \frac{\partial^2 v_3}{\partial t^2} = \frac{15}{2}x(v_{1,xx}^2 + 2v_{0,xx}v_{2,xx}) + \frac{15}{2}y(v_{1,yy}^2 + 2v_{0,yy}v_{2,yy}), \quad v_3(x, y, 0) = 0, \\
&\quad \vdots \\
&p^j : \frac{\partial^2 v_j}{\partial t^2} = \frac{15}{2}x(\sum_{i=0}^{j-1} v_{i,xx}v_{j-1-i,xx}) + \frac{15}{2}y(\sum_{i=0}^{j-1} v_{i,yy}v_{j-1-i,yy}), \quad v_j(x, y, 0) = 0, \\
&\quad \vdots
\end{aligned} \tag{43}$$

For simplicity we take

$$\begin{aligned}
&v_0(x, y, t) = u_0(x, y, t), \\
&v_1(x, y, t) = \int_0^t \int_0^t \left\{2x^2 + 2y^2 + \frac{15}{2}(xv_{0,xx}^2 + yv_{0,yy}^2) - \frac{\partial^2 u_0}{\partial t^2}\right\} dtdt.
\end{aligned} \tag{44}$$

Having this assumption we get the following iterative equation

$$v_j = \int_0^t \int_0^t \left\{\frac{15}{2}x(\sum_{i=0}^{j-1} v_{i,xx}v_{j-1-i,xx}) + \frac{15}{2}y(\sum_{i=0}^{j-1} v_{i,yy}v_{j-1-i,yy})\right\} dtdt, j = 2, 3, 4, \cdots. \tag{45}$$

Starting with$v_0 = u_0 = t^2(x^2 + y^2)$, by using Eq. (45) and Eq. (46), we obtain

$$\begin{aligned}
&v_1(x, y, t) = (x + y)t^6, \\
&v_2(x, y, t) = 0, \\
&v_3(x, y, t) = 0, \\
&\quad \vdots
\end{aligned}$$

The approximate solution of (36) can be obtained by setting $p = 1$,

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \cdots = (x^2 + y^2)t^2 + (x + y)t^6 + \cdots$$

Thus the exact solution will be as

$$u(x,y,t) = (x^2+y^2)t^2 + (x+y)t^6.$$

which is exactly the same as obtained by Adomain decomposition method [12].

Example 4

We finally consider the three-dimensional nonlinear nonhomogeneous IBVP:

$$u_{tt} = 2 - t^2 + u - (e^{-x}u_{xx}^2 + e^{-y}u_{yy}^2 + e^{-z}u_{zz}^2). \qquad 0 < x, y, z < 1, \quad t > 0, \tag{46}$$

subject to the Neumann boundary conditio

$$u_x(0,y,z,t) = 1, \quad u_x(1,y,z,t) = e \tag{47}$$

$$u_y(x,0,z,t) = 1, \quad u_y(x,1,z,t) = e \tag{48}$$

$$u_z(x,y,0,t) = 1, \quad u_z(x,y,1,t) = e \tag{49}$$

and initial conditions

$$u(x,y,z,0) = e^x + e^y + e^z, \qquad u_t(x,y,z,0) = 0, \tag{50}$$

To solve Eq. (47) $\sim$ (50) by homotopy perturbation method, we construct the following homotopy:

$$(1-p)\left(\frac{\partial^2 v}{\partial t^2} - \frac{\partial^2 u_0}{\partial t^2}\right) + p\left(\frac{\partial^2 v}{\partial t^2} - 2 + t^2 - v + e^{-x}v_{xx}^2 + e^{-y}v_{yy}^2 + e^{-z}v_{zz}^2\right) = 0,$$

or

$$\frac{\partial^2 v}{\partial t^2} - \frac{\partial^2 u_0}{\partial t^2} = p\left(2 - t^2 + v - e^{-x}v_{xx}^2 - e^{-y}v_{yy}^2 - e^{-z}v_{zz}^2 - \frac{\partial^2 u_0}{\partial t^2}\right). \tag{51}$$

Suppose the solution of Eq. (51) to be in the following form

$$v = v_0 + pv_1 + p^2 v_2 + \cdots. \tag{52}$$

Substituting (52) into (51), and equating the coefficients of the terms with respect to p,

$$\begin{aligned}
p^0 &: \frac{\partial^2 v_0}{\partial t^2} = \frac{\partial^2 u_0}{\partial t^2},\\
p^1 &: \frac{\partial^2 v_1}{\partial t^2} = 2 - t^2 + v_0 - e^{-x}v_{0,xx}^2 - e^{-y}v_{0,yy}^2 - e^{-z}v_{0,zz}^2 - \frac{\partial^2 u_0}{\partial t^2}, \qquad v_1(x,y,z,0) = 0,\\
p^2 &: \frac{\partial^2 v_2}{\partial t^2} = v_1 - 2e^{-x}v_{0,xx}v_{1,xx} - 2e^{-y}v_{0,yy}v_{1,yy} - 2e^{-z}v_{0,zz}v_{1,zz}, \qquad v_2(x,y,z,0) = 0,\\
p^3 &: \frac{\partial^2 v_3}{\partial t^2} = v_2 - e^{-x}v_{0,xx}v_{2,xx} - e^{-y}v_{0,yy}v_{2,yy} - e^{-z}v_{0,zz}v_{2,zz}\\
&\qquad - 2e^{-x}v_{1,xx}^2 - 2e^{-y}v_{1,yy}^2 - 2e^{-z}v_{1,zz}^2, v_3(x,y,z,0) = 0,\\
&\vdots\\
p^j &: \frac{\partial^2 v_j}{\partial t^2} = v_{j-1} - e^{-x}\sum_{i=0}^{j-1} v_{i,xx}v_{j-1-i,xx} - e^{-y}\sum_{i=0}^{j-1} v_{i,yy}v_{j-1-i,yy}\\
&\qquad - e^{-z}\sum_{i=0}^{j-1} v_{i,zz}v_{j-1-i,zz}, \qquad v_j(x,y,z,0) = 0,\\
&\vdots
\end{aligned}$$

For simplicity we take

$$v_0(x,y,z,t) = u_0(x,y,z,t),$$
$$v_1(x,y,z,t) = \int_0^t \int_0^t \left\{ 2 - t^2 + v_0 - e^{-x} v_{0,xx}^2 - e^{-y} v_{0,yy}^2 - e^{-z} v_{0,zz}^2 - \frac{\partial^2 u_0}{\partial t^2} dtdt \right\}.$$

Having this assumption we get the following iterative equation

$$v_j = \int_0^t \int_0^t (v_{j-1} - e^{-x} \sum_{i=0}^{j-1} v_{i,xx} v_{j-1-i,xx} - e^{-y} \sum_{i=0}^{j-1} v_{i,yy} v_{j-1-i,yy}$$
$$- e^{-z} \sum_{i=0}^{j-1} v_{i,zz} v_{j-1-i,zz}) dtdt, j = 2,3,4,\cdots. \tag{53}$$

Starting with $v_0 = u_0 = tx^6 y^6 z^6$, by using (53), we obtain

$$v_1(x,t) = \frac{t^4}{12} - \frac{t^6}{360},$$
$$v_2(x,t) = \frac{t^6}{360} - \frac{t^8}{20160},$$
$$v_3(x,t) = \frac{t^8}{20160} - \frac{t^{10}}{1814400},$$
$$\vdots$$

The approximate solution of (47) can be obtained by setting $p = 1$,

$$u = \lim_{p\to 1} v = v_0 + v_1 + v_2 + \cdots = (e^x + e^y + e^z) + t^2 - \frac{t^4}{12} + \frac{t^4}{12} - \frac{t^6}{360} + \frac{t^6}{360} - \frac{t^8}{20160} + \frac{t^8}{20160} + \cdots.$$

Thus the exact solution will be as

$$u(x,y,z,t) = (e^x + e^y + e^z) + t^2.$$

which is exactly the same as obtained by Adomain decomposition method[12].

4 Conclusions

The main goal of this paper has been to drive an analytical solution for the linear higher dimensional initial boundary value problems of variable coefficients. We have achieved this goal by applying He's homotopy perturbation method. Results are compared with those in open literature [12], revealing that the obtained solutions are exactly same with those obtained by Adomian decomposition method [12]. The analytical approximation to the solution are reliable and confirms the power and ability of the He's homotopy perturbation method as an easy device for computing the solution of a linear and nonlinear partial differential equations. The computations associated with the examples in this paper were performed using Matlab 7.

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