

To study the parametric effects on the performance of brown stock washer in paper industry using MATLAB

Deepak Kumar*, Vivek Kumar, V. P. Singh

IIT Roorkee, Department of Paper Technology, Saharanpur Campus Saharanpur-247001, India

Abstract. In the present paper, the effects of various parameters on performance of brown stock washer in paper industry are studied for the species of sodium (Na^+) and lignin ions. The mechanism of the diffusion-dispersion washing of the bed of the pulp fibers is mathematically modeled by the basic material balance equation using the parameters namely, interstitial velocity, longitudinal dispersion coefficient, mass transfer coefficient and total porosity of the cake. Linear adsorption isotherm is used to describe the equilibrium between the concentration of the solute in the liquor and concentration of the solute on the fibers. For various values of parameters numerical results are obtained by the solution of the axial domain of the system (transport & adsorption isotherm equation) governed by partial differential equations using “pdepe” solver in MATLAB code. The analysis of the parametric effects is shown by the C vs. T graphical representations for the various values of parameters for both the species.

Keywords: MATLAB “pdepe” solver, pecelet number, porosity, mass transfer coefficient, adsorption isotherm, diffusion-dispersion

1 Introduction

The mathematical models describing the pulp washing process based on axial dispersion and particle diffusion are fairly well established in terms of transport equations using basic mass transfer relation like diffusion equation and material balance equations by some earlier researchers such as [2, 3, 7, 8, 11, 12, 15]. These transport equations together with the corresponding equations of adsorption isotherms to describe the equilibrium between the concentration of the solute in the liquor and concentration of the solute on the fibers, and various boundary conditions provide the mathematical model of pulp washing, which are extremely intricate in nature and solution of these models is highly complex. It is important to mention here that the model of pulp washing with its simplified versions (without adsorption isotherm) have been solved analytically using Laplace transform by [2] and numerically using Orthogonal Collocation by [3]. [1, 6] attempted to solve the washing model using different numerical techniques by taking various assumptions. All these methods are highly complex and time consuming. Application of such solution techniques in control systems is not possible due to the more processing time and involvement of high mathematical skills at operator level. Recently system of such equations has been successfully solved with easy approach, “pdepe” solver in MATLAB source code by [16] which is more convenient and consumes less time than the other techniques mentioned above.

All these pulp washing models characterize the performance of washer in terms of various input parameters such as interstitial velocity, longitudinal dispersion coefficient, mass transfer coefficient and total porosity of the cake. The input parameters further can be clubbed to obtain the dimensionless parameters. Peclet number ($Pe = u \times L/D_L$) is one of the dimensionless parameter which takes care of various local velocities, bed thickness and longitudinal dispersion coefficient. For a constant bed depth it can show the variational effects of both u and D_L . So that the effect of Peclet Number on exit solute concentration in pulp washing

* Corresponding author. Tel.: +919927090165; fax: +91-132-2714011.

E-mail address: dkr2009@gmail.com; vivekfpt@iitr.ernet.in; singhvp3@gmail.com.

gives the idea of the effect of both the parameters interstitial velocity and longitudinal dispersion coefficient. These parameters have been calculated experimentally by using the relations given by earlier investigators [5, 13, 14].

In the present paper parametric effects on the performance of pulp washing is studied by obtaining solution of mathematical model of pulp washing related to diffusion-dispersion during flow through multiparticle system. Numerical solutions are obtained by varying different values of input parameters using “pdepe” solver in MATLAB source code.

2 Description of mathematical models

2.1 Transport equation

Firstly using simple material balance for the mat of pulp fibers, assumed to be stationary packed bed of homogeneous symmetrical cylindrical fibers with constant area A_c and the total porosity ε_t , the mathematical equation is obtained. After that Using Fick's second law of diffusion and neglecting the molecular diffusion coefficient D_v ^[15], the transport equation can be obtained as:

$$D_L \left(\frac{\partial^2 c}{\partial z^2} \right) = u \left(\frac{\partial c}{\partial z} \right) + \left(\frac{\partial c}{\partial t} \right) + \frac{(1 - \varepsilon_t)}{\varepsilon_t} \left(\frac{\partial n}{\partial t} \right) \quad (1)$$

Where u is the velocity of the liquor in the mat, c is the concentration in the liquid phase and n is the concentration in fiber.

Writing $\frac{(1-\varepsilon_t)}{\varepsilon_t}$ as μ for convenience, the Eq. (1) may be written as

$$D_L \left(\frac{\partial^2 c}{\partial z^2} \right) = u \left(\frac{\partial c}{\partial z} \right) + \left(\frac{\partial c}{\partial t} \right) + \mu \left(\frac{\partial n}{\partial t} \right) \quad (2)$$

This is a non homogeneous, non linear, first degree, second order, parabolic partial differential equation. Here u , ε_t and D_L are functions of z while c and n are functions of z and t .

2.2 Adsorption isotherm:

For the present study, the adsorption isotherm has been proposed to describe the equilibrium between the concentration of the solute in the liquor and fibers. The adsorption desorption isotherm dynamics, unfortunately not clearly known for Na^+ and lignin. Various adsorption-desorption equilibrium equations are available in the literature. In the present study adsorption isotherm given by [8] is used by taking the rate of adsorption finite and initial adsorbate concentration zero. For ready reference the equations are given as

$$\frac{\partial n}{\partial t} = k_1 c - k_2 n \quad (3)$$

2.3 Initial and boundary conditions

Initial and boundary conditions are used in the present investigation are given by [2, 12]. According to [2] Initial condition is $c(z, t) = n(z, t) = c_i$ for $0 < t < L/u$, where L/u corresponds to displacement time and boundary condition at the inlet of the bed is

$$uc - D_L \frac{\partial L}{\partial z} = uc_s \text{ at } z = 0 \text{ and } t > 0 \quad (4)$$

[12] gives the boundary condition at the inlet of the bed

$$c = c_s \text{ at } z = 0 \text{ and } t > 0 \quad (5)$$

Boundary condition at bed exit is same for both above cases i.e.

$$(\partial c / \partial z) = 0, \text{ at } z = L \text{ and } t > 0 \quad (6)$$

Thus in the present investigation two cases arise for two different boundary conditions at inlet of the bed given by Eq. (4) and Eq. (5) together with bed exit Eq. (6). Thus the set of two models is obtained for the solution with two different boundary conditions as depicted in Tab. 1.

2.4 Conversion into dimensionless form

Before obtaining the solution, pulp washing model with adsorption isotherm is converted into dimensionless form by using certain dimensionless parameters like Peclet number (or Bodenstein number), dimensionless time, dimensionless cake thickness and dimensionless concentration given below:

Peclet Number, $Pe = uL/D_L$.

Dimensionless cake thickness, $Z = z/L$.

Dimensionless concentrations of solute in liquor, $C = (c - c_s)/(c_i - c_s)$.

Dimensionless time, $T = ut/L$.

Dimensionless concentrations of solute in fiber $N = (n - c_s)/(c_i - c_s)$.

And some constant parameters are as follows, $K = k_1/k_2$, $G = k_2L/u$, and $H = (K - 1)c_s/(c_i - c_s)$.

Table 1. Existing mathematical models for washing zone used in present investigation (dimensionless form)

s.No.	Transport Equation	Adsorption Isotherm (dimensionless form)	Boundary Conditions (dimensionless form)
1	$\frac{\partial^2 C}{\partial Z^2}$ $= Pe(\partial C/\partial Z + \partial C/\partial T + \mu \partial N/\partial T)$	$\partial N/\partial T = G(H + KC - N)$	$C(0, T) = 0$ for $T > 0$ & $\partial C/\partial Z = 0$ ($Z = 1$)
2	$\frac{\partial^2 C}{\partial Z^2}$ $= Pe(\partial C/\partial Z + \partial C/\partial T + \mu \partial N/\partial T)$	$\partial N/\partial T = G(H + KC - N)$	$\partial C/\partial Z = Pe C$ for ($Z = 0, T > 0$) & $\partial C/\partial Z = 0$ ($Z = 1$)

3 Result and discussion

The effects of various parameters like Peclet number, porosity and mass transfer coefficient on the models given in Tab. 1 are obtained by solving the models with different values of the above parameters. In this paper the range of basic input parameters are used as given by [3]. Based on the value of dispersion coefficient given by [3], the minimum values of Peclet numbers for Na^+ is 27 and 20 for lignin. So for the effect of porosity and mass transfer coefficient is studied at the Peclet number 27 and 25 for Na^+ and lignin respectively.

For control purpose the transient behavior of the solute concentration in the black liquor is of more interest, rather than solute concentration in fiber, therefore the value of $\partial N/\partial T$ from adsorption isotherm is substituted in the equation of flow (Transport equation) for both the models depicted in Tab. 1 and then solution obtained accordingly.

3.1 Effect of peclet number

The range of Peclet number from 20 to 110 is found to be valid for pulp washing as suggested by earlier investigators such as [3, 6]. The effect of Peclet number is studied for the same range of Peclet numbers as mentioned above. For Peclet number below 20 (say 10 to 19), deviation in the profile is observed and hence the higher Peclet number shows the minimum deviation in the shape of breakthrough curves. The effects of Peclet Number on the exit solute concentration are shown in the Fig. 1 and Fig. 2 for sodium ions and Fig. 3 and Fig. 4 for lignin ions respectively.

Stream of the curves start from $c = 1.0$ descending differently but however converge at $T = 1.1$ and again diverge from each other in topsy-turvy manner. It means that C-T profiles for $Pe = 20$ to 110 which are at lower side of the curve, go upward, this fact is found to be in agreement with [6]. Thus the Peclet number has significant influence on C-T profiles.

3.2 Effect of porosity

Porosity is defined as the ratio of the volume available for flow to the total volume. Porosity of the mat is an important factor as the hydrodynamics of the filtration is highly influenced by the porous path through which the fluid will move. In the present case, porosity is based on the average consistency of the mat during

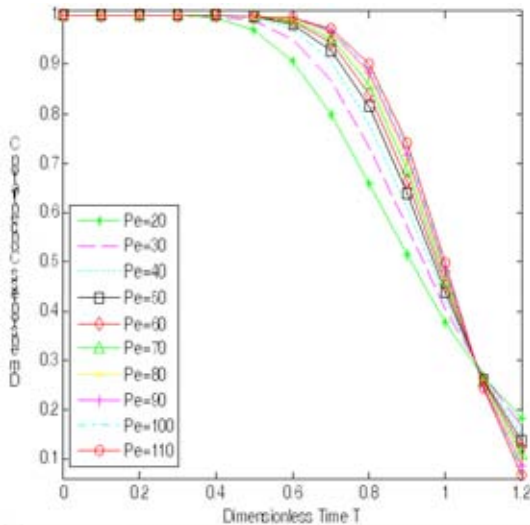


Fig. 1. C vs T profiles for Na^+ for different values of Pe at Bed exit for model-1

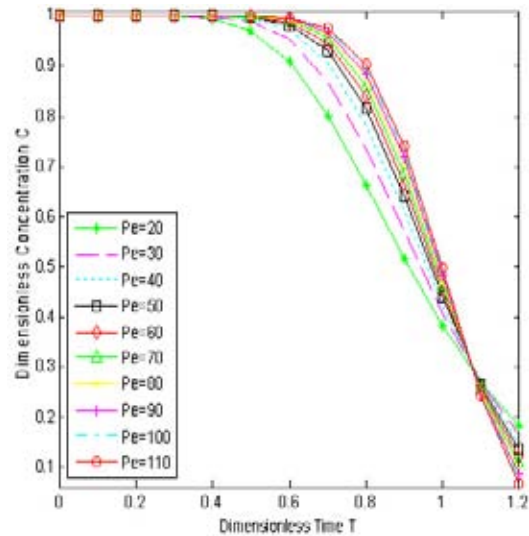


Fig. 2. C vs T profiles for Na^+ for different values of Pe at Bed exit for model-2

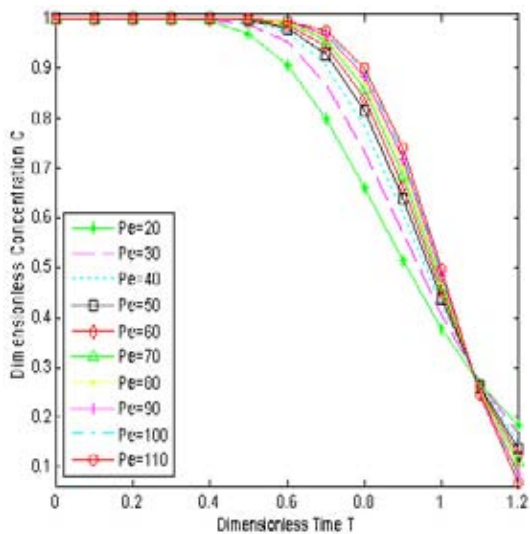


Fig. 3. C vs T profiles for Lignin for different values of Pe at Bed exit for model-1

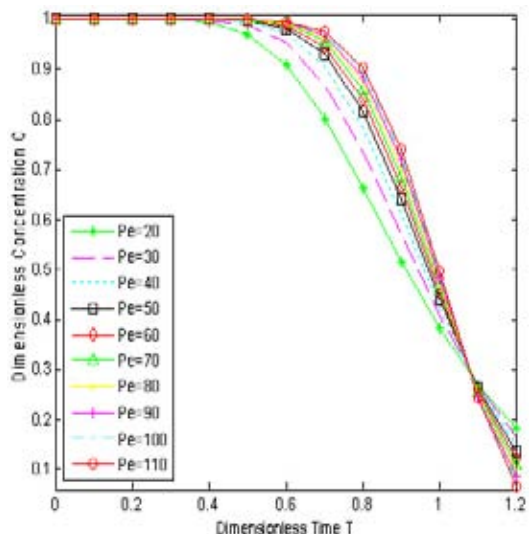


Fig. 4. C vs T profiles for Lignin for different values of Pe at Bed exit for model-2

the cake formation zone. The air effect on the porosity has been neglected. Porosity of the pulp suspension varies from 0.1 to 0.99, but is always less than 1. Longer range is considered to cover not only the entire range of suspension hebetated by the stock in the paper making process in practice but also to cover the porosity value of the flowing zone (0.7 to 0.85), stagnant zone (0.1 to 0.2) and total mean porosity values above 0.90. So for the study of the effect of porosity on exit solute concentration the vau of the porosity is taken from 0.1 to 0.95. The C vs T profiles for different porosity at the mid point of cake thickness i.e at $Z = 0.5$, are shown in the Fig. 5 and Fig. 6 for Na^+ and Fig. 7 and Fig. 8 for lignin ion. This shows no noticeable change for lignin ion with the ε ranging from 0.1 to 0.95 by both the models. But for Na^+ the models 1 and 2 shows a small change with the ε_t ranging from 0.1 to 0.4, this deviation may be ignored in engineering practice. The results are again in agreement with [6].

3.3 The effect of mass transfer coefficient k_1

The $C - T$ profiles for the six values of the mass transfer coefficient k_1 are shown in the Fig. 9 and Fig. 10 for Na^+ and Fig. 11 and Fig. 12 for lignin ion. The range $5 \times 10^{-3}/s$ to $500 \times 10^{-3}/s$ which is given by [3] and further modified by [6] for both the models as depicted in Tab. 1. Similarly as porosity, the profiles

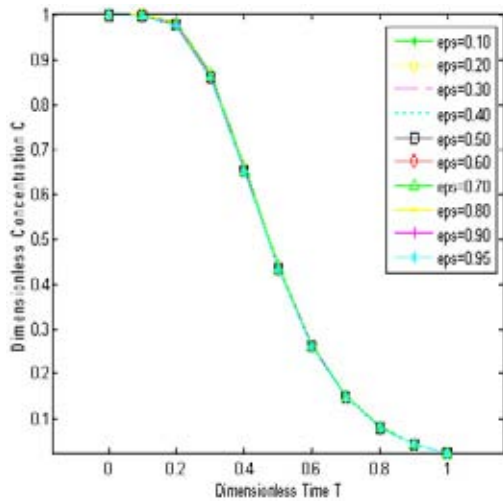


Fig. 5. C vs T profiles for Na^+ for different values of Porosity at Mid of Cake Thick ness for model-1

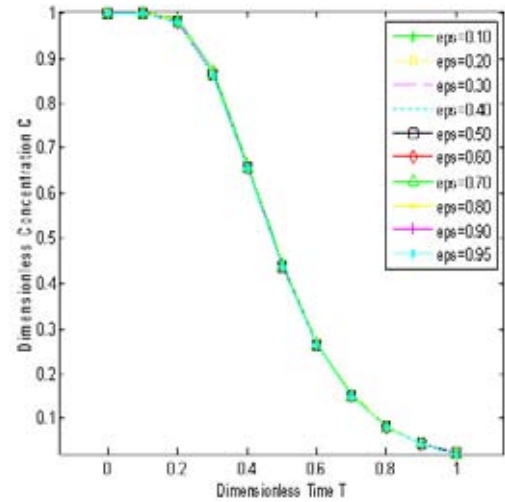


Fig. 6. C vs T profiles for Na^+ for different values of Porosity at Mid of Cake Thick ness for model-2

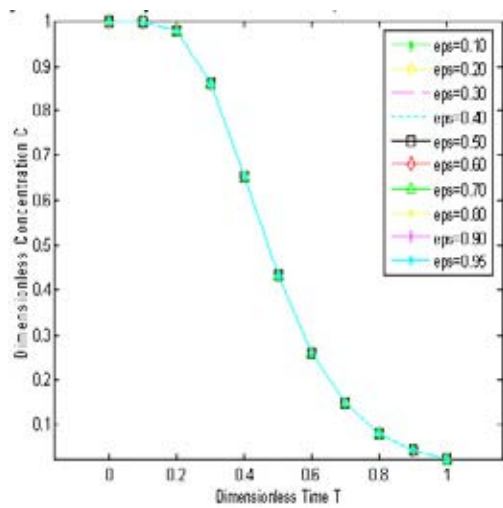


Fig. 7. C vs T profiles for Lignin for different values of Porosity at Mid of Cake Thick ness for model-1

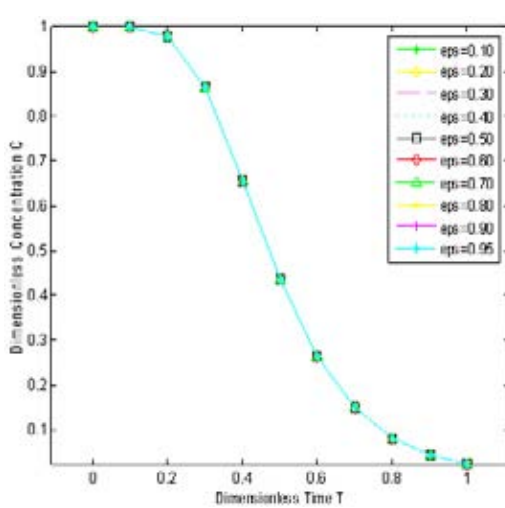


Fig. 8. C vs T profiles for Lignin for different values of Porosity at Mid of Cake Thick ness for model-2

show that there is no appreciable effect of varying value of k_1 for lignin ion for both models. But for Na^+ species, the models 1 and 2 shows small deviations from time $T = 0.1$ to 0.7 , which are also negligible. Again our results tend to agree with that of [6], who has mentioned that weak influence of this parameter.

3.4 The effect of mass transfer coefficient k_2

The $C - T$ profiles for the six values of the mass transfer coefficient k_2 , are shown in the Fig. 13 and Fig. 14 for Na^+ and Fig. 15 and Fig. 16 for lignin ions. The range of mass transfer coefficient is different for both the species i.e. Na^+ and lignin ion which is given by [3] and further modified by [6] for both the models given in Tab. 1. The range of k_2 is $2.0 \times 10^{-7}/s$ to $2.0 \times 10^{-3}/s$ for Na^+ and for lignin ion the range is of very low order i.e. $2.0 \times 10^{-100}/s$ to $6.0 \times 10^{-100}/s$. The C-T profiles show that there is no effect of varying value of k_2 for Na^+ for any of the model of the considered. Similarly the profiles show no effect of k_2 for the lignin ion also for any of the model as depicted in Tab. 1. This is the fact that the value of k_2 is of very low order for Na^+ and of much lower (of order 10^{-100}) for lignin ion. The results of the [6] are of the same nature.

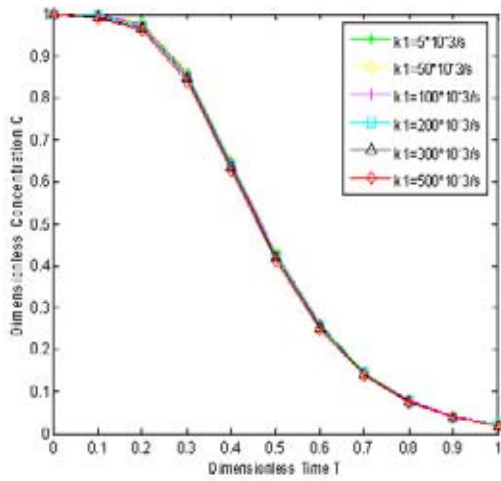


Fig. 9. C vs T profiles for Na^+ for various values of k_1 at Mid of Cake Thick ness for model-1

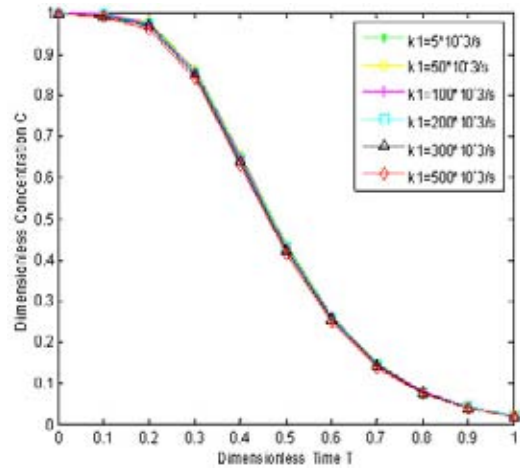


Fig. 10. C vs T profiles for Na^+ for various values of k_1 at Mid of Cake Thick ness for model-2

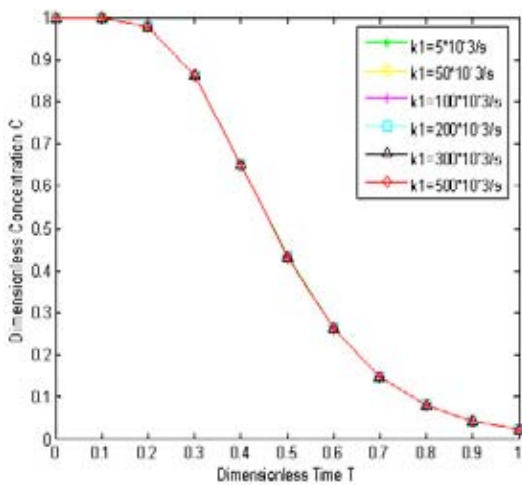


Fig. 11. C vs T profiles for Lignin for various values of k_1 at Mid of Cake Thick ness for model-1

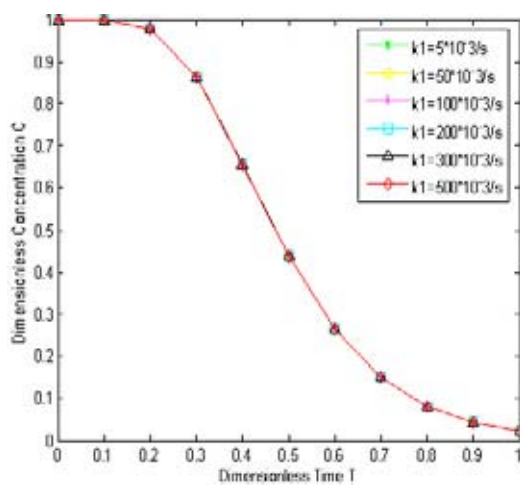


Fig. 12. C vs T profiles for Lignin for various values of k_1 at Mid of Cake Thick ness for model-2

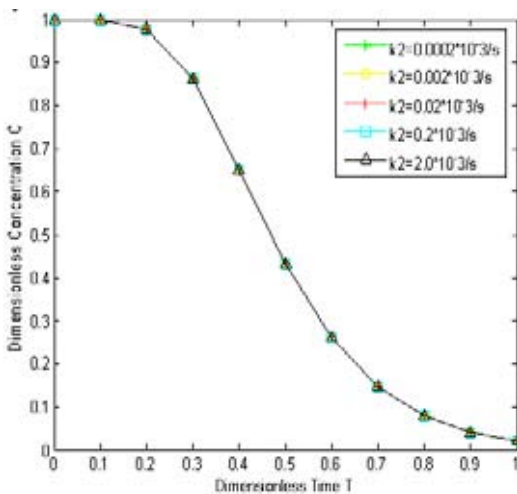


Fig. 13. C vs T profiles for Na^+ for various values of k_2 at Mid of Cake Thick ness for model-1

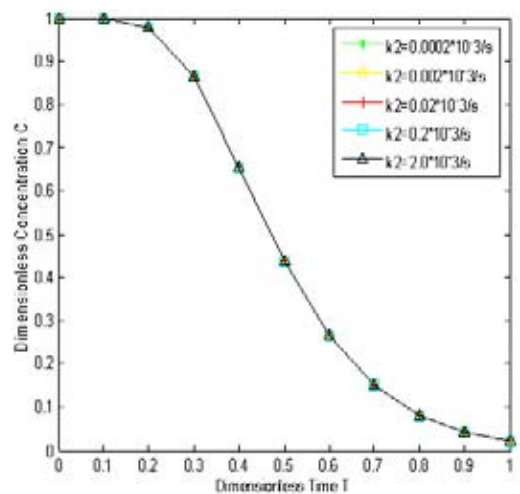


Fig. 14. C vs T profiles for Na^+ for various values of k_2 at Mid of Cake Thick ness for model-2

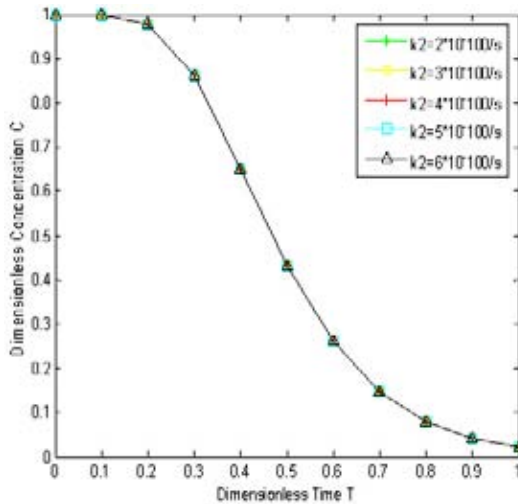


Fig. 15. C vs T profiles for Lignin for various values of k_2 at Mid of Cake Thickness for model-1

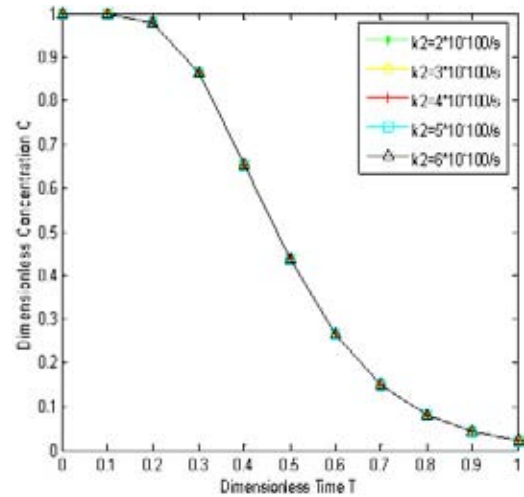


Fig. 16. C vs T profiles for Lignin for various values of k_2 at Mid of Cake Thickness for model-2

4 Conclusion

It may be concluded that the effect of all the parameters in terms of C vs. T (Dimensionless concentration in liquid phase vs. Dimensionless Time) for both the models that are given in Tab. 1 with various boundary conditions through 'pdepe solver' in MATLAB source code are interpreted in terms of various graphs. The main conclusion in respect of the various parameters mentioned above may be summarized as below.

- (1) For Peclet number below 20, deviation in the profile is observed and hence the higher Peclet number shows the minimum deviation in the shape of breakthrough curves.
- (2) There is no noticeable change of the C-T profiles with for lignin ion with the ε_t ranging from 0.1 to 0.95 for both the models. But for Na^+ the models 1 and 2 show a small change with the ε_t ranging from 0.1 to 0.4.
- (3) The $C - T$ profiles show that there is no appreciable effect of varying value of k_1 for lignin ion for both the models. But for Na^+ models 1 and 2 show small deviations from time $T = 0.1$ to 0.7, which are also negligible.
- (4) The $C - T$ profiles show no effect of k_2 for the Na^+ & lignin ion for any of the model which are given in Tab. 1. This is the fact that the value of k_2 is of very low order for Na^+ and of much lower (of order 10^{-100}) for lignin ion.
- (5) The pattern of the graphs of models 1 & 2 tend to depict similar profiles for Na^+ as well as for lignin ions. Therefore we may conclude that boundary conditions do not have much influence on the shapes of profiles.

5 Nomenclature

c	: Concentration of the solute in the liquor, kg/m^3
c_i	: Concentration of solute inside the vat, kg/m^3
c_s	: Concentration of solute in the wash liquor, kg/m^3
D_L	: Longitudinal dispersion coefficient, m^2/s
D_v	: Molecular diffusion coefficient, m^2/s
k_1, k_2	: Mass transfer coefficients, $1/s$
L	: Cake thickness, m
n	: Concentration of solute on fibers, kg/m^3
t	: Time, s
C	: Dimensionless Concentration of solute in the liquor
N	: Dimensionless Concentration of solute in the fiber

Z	: Dimensionless Distance
T	: Dimensionless Time
u	: Liquor speed in cake pores, m/s
z	: Variable cake thickness, m
Δz	: Small increment in cake thickness, m
ε_t	: Porosity of the bed

References

- [1] S. Arora, S. Dhaliwal, V. Kukreja. Modeling of the displacement washing of the pulp fiber bed. *Indian Journal of Chemical Technology*, 2006, **13**(9): 1–6.
- [2] H. Brenner. The diffusion model of longitudinal mixing in beds of finite length, numerical values. *Chemical Engineering Science*, 1962, **17**: 229–243.
- [3] L. Grahs. *Washing of cellulose fibers, analysis of displacement washing operation*. Phd. thesis., Chalmers University of Technology, Goteborg, Sweden, 1974.
- [4] C. Han. Washing theory of the porous structure of aggregated materials. *Chemical engineering science*, 1967, **22**: 837–846.
- [5] V. Kukreja. *Modeling of washing of brown stock on rotary vacuum washer*. Phd. thesis., University of Roorkee, Roorkee, India, 1996.
- [6] M. Kumar. *Mathematical modeling of pulp washing systems and solutions*. Phd. thesis., Indian Institute of Technology Roorkee, Roorkee, India, 2002.
- [7] M. Kuo, E. Barret. Continuous filter cake washing performance. *AICHE J.*, 1970, **16**(4): 633–638.
- [8] L. Lapidus, N. Amundson. Mathematics of adsorption in beds, part-vi: The effect of longitudinal diffusion in ion exchange and chromatographic columns. *Journal of Physical Chemistry*, 1952, **56**: 984–988.
- [9] I. Neretnieks. Mathematical model for continuous counter current adsorption. *Svensk Papperstidning*, 1974, **77**(11): 407–411.
- [10] M. Pekkanen, H. Norden. Review of pulp washing models. *Paperija Puu*, 1985, **67**(11): 689–696.
- [11] G. Pellett. Longitudinal dispersion, intra particle diffusion and liquid-phase mass transfer during flow through multi particle systems. *Tappi J.*, 1966, **49**(2): 75–82.
- [12] M. Perron, B. Lebeau. A mathematical model of pulp washing on rotary drums. *Pulp and paper Canada*, 1977, **78**(3): TR1–TR5.
- [13] F. Potucek. Chemical engineering view of pulp washes. *Papir a celuloza*, 2005, **60**(4): 114–117.
- [14] F. Potucek, M. Pulcer. Displacement of black liquor from pulp bed. **in**: *31st International Conference of the Slovak Society of Chemical Engineering*, vol. 58, Tatranske Matliare, 2004, 337–381. 6.
- [15] W. Sherman. The movement of a soluble material during the washing of a bed of packed solids". *AICHE J.*, 1964, **10**(6): 855–860.
- [16] V. Singh, V. Kumar, D. Kumar. Numerical solution of diffusion model of brown stock washing beds of finite length using matlab. **in**: *IEEE Proceeding of '2nd UKSim European Symposium on Computer Modeling and Simulation*, 2008, 295–300.