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Kinematics and dynamics analysis of micro-robot for surgical applications

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Abstract. The applications of robotics in medical field have increased extensively in the last two decades. This paper aims at investigating the kinematics and dynamics of six DOF micro-robot intended for surgery applications. The kinematic equations of motion are derived using Denavit-Hartemberg representation. The workspace of the robot is investigated based on the kinematic equations as well as the physical limit of each joint. The dynamic equations of motion are derived using a Lagrangian-Euler technique. The required hub torque to move each joint based on prescribed trajectories are calculated for proper selection of the motor torques. This dynamic equation is also important to design the proper controller for the robot. A simulation results are carried out using MATLAB

Keywords: kinematics, dynamics, surgical, micro-robot, trajectory planning

1 Introduction

Robots are widely used in medical field for minimally invasive surgery efficiently and accurately. Minimally invasive surgery is an innovative approach that allows reducing patient trauma, postoperative pain and recovery time[2]. The kinematics and dynamics analysis of the robot are very important for any applications. The direct aim is to properly select the workspace and the actuator size. Silvia Frumento et al. designed a minimally invasive robot for heart surgery. They concentrated mainly in the kinematics analysis and the workspace of the robot. Tsai and Hsu[8] investigated a parallel surgical robot having six degree-of-freedom (DOF). They performed the kinematic analysis only to obtain the workspace of surgical robot and control it by Fuzzy Logic method. Miller et al.[6] focused on the dynamic of the Multi-rigid-body robot using Newton's method and applied the results to design the controller. Featherstone and Orin[4] investigated the robot dynamics equations using Newton-Euler algorithm to obtain the equations of motion for the robot arm.

The object of this paper is to present a full analysis of 6 DOF micro-robot for surgical applications. It is divided into three parts (kinematics analysis, trajectory planning and dynamic analysis). In all parts the simulation results are carried out using MATLAB .

2 Kinematics analysis

The surgical robot under consideration has 6 DOF and a redundant one for tool replacement. The end-effector has 3 rotations Roll, Pitch and Yaw as shown in Fig. 1. The kinematics analysis for the robot is performed using the Denavvit-Hartenberg representation. A schematic diagram assigning all the joint axes is represented in Fig. 1 as well.

The 6 DOF manipulator kinematic parameters are derived using Denavit Hartemberg formulation shown in Tab. 1.

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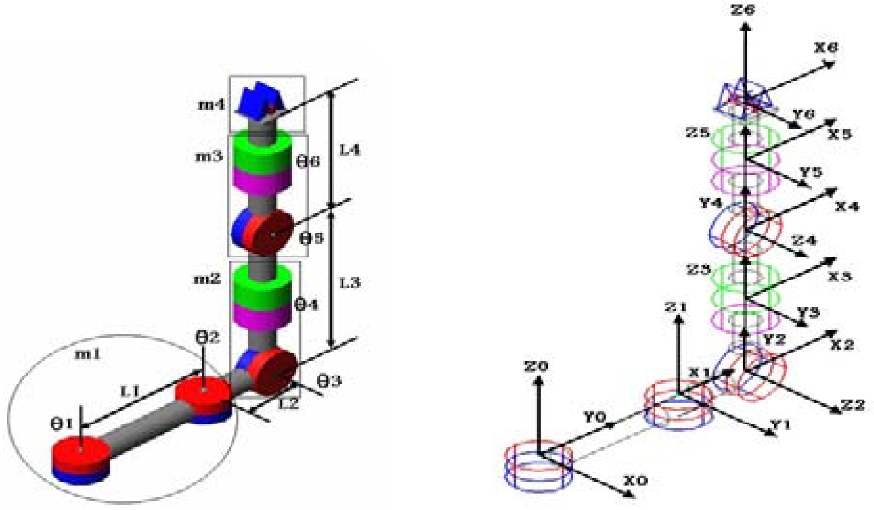

Fig. 1. Geometric model of surgical manipulator

Table 1. The robot parameters

Link#	θ_i	d_i	a_i	α_i	T
1	θ_1	0	L_1	0	0_{T_1}
2	θ_2	0	L_2	0	1_{T_2}
3	θ_3	0	0	-90	2_{T_3}
4	θ_4	L_3	0	90	3_{T_4}
5	θ_5	0	0	-90	4_{T_5}
6	θ_6	L_4	0	90	5_{T_6}

The link transformations matrix can be given as:

$$
{}^0T_1 = \begin{bmatrix} C_1 & -S_1 & 0 & L_1C_1 \\ S_1 & C_1 & 0 & L_1S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad
{}^1T_2 = \begin{bmatrix} C_2 & 0 & -S_2 & L_2C_2 \\ S_2 & 0 & C_2 & L_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad
{}^2T_3 = \begin{bmatrix} C_3 & 0 & -S_3 & 0 \\ S_3 & 0 & C_3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^3T_4 = \begin{bmatrix} C_4 & 0 & -S_4 & 0 \\ S_4 & 0 & C_4 & 0 \\ 0 & -1 & 0 & L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad
{}^4T_5 = \begin{bmatrix} C_5 & 0 & -S_5 & 0 \\ S_5 & 0 & C_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\quad
{}^5T_6 = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & L_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

The kinematics equations of the end effectors are calculated by MATLAB symbolic Toolbox as follows:

$$
\begin{bmatrix} P_X \\ \\ P_Y \\ \\ P_Z \\ \\ Z \end{bmatrix} = \begin{bmatrix} ((-(C_1C_2 - S_1S_2)C_3 \times C_4 - (-C_1S_2 - S_1C_2)S_4)S_5 + \\ (C_1C_2 - S_1S_2)S_3C_5)L_4 - (C_1C_2 - S_1S_2)S_3L_3 + \\ C_1L_2C_2 - S_1L_2S_2 + L_1C_1 \\ \\ ((-(C_1S_2 + S_1C_2)C_3C_4 - (C_1C_2 - S_1S_2)S_4)S_5 + \\ (C_1S_2 + S_1C_2)S_3C_5)L_4 - (C_1S_2 + S_1C_2)S_3L_3 + \\ S_1L_2C_2 + C_1L_2S_2 + L_1S_1 \\ \\ (-S_3C_4S_5 - C_3C_5)L_4 + C_3L_3 \\ \\ 1 \end{bmatrix}
$$

Where: $C_i = \cos\theta_i$ and $S_i = \sin\theta_i$.

2.1 Robot workspace

The workspace of the surgical manipulator can be representing by solving the kinematic equations and taking into consideration all the physical limits of the joints. Tab. 2 represents the physical limits of the six joints[2] while Fig. 2 represents the three dimensional workspace of the end-effector.

Table 2. Joint physical limits[2]

Link #	1	2	3	4	5	6
$\theta_i(Degrees)$	$-180°$	$-90°$	0	$-180°$	$-90°$	$-180°$
$\theta_f(Degrees)$	180°	90°	180°	180°	90°	180°
$L(mm)$	0	50	15	36	0	39.5

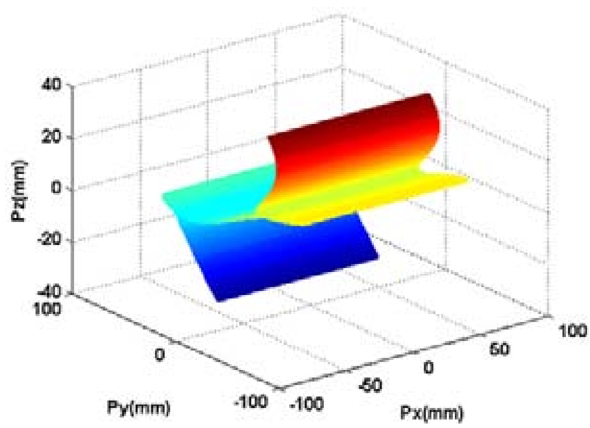

Fig. 2. The workspace of the robot

3 Trajectory planning

The most common techniques for trajectory planning for industrial robots are polynomial of different orders, Cubic and B-splines, linear segments with parabolic blends and the soft motion trajectory[1]. The Linear Segments with Parabolic Blends trajectories are faster and more suitable for Industrial applications. On the other hand, the higher order polynomials trajectory as well as the Soft Motion trajectory[5] are easy to design and control especially for the jerk. They are accurate and precise and they are suitable for medical applications. In this paper, the trajectory planning for each joint is designed using fifth order polynomial as a rest-to-rest maneuvering where the link starts from rest, increases gradually and stops at the end of the trajectory. Unlike the third order polynomial trajectory which controls the initial and final angular velocities, the fifth order takes into account starting and ending accelerations. In this case, the total number of boundary conditions is six, allowing a fifth polynomial to control the jerk at start and goal positions. The trajectory is given by:

$$\theta(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3 + C_4 t^4 + C_5 t^5 \tag{1}$$

In which C_0, C_1, C_2, C_3, C_4 and C_5 are coefficients to be determined from the initial conditions as follows:

$$
\begin{aligned}
C_0 &= \theta_i \\
C_1 &= \dot{\theta}_i \\
C_2 &= \frac{\ddot{\theta}_i}{2} \\
C_3 &= \frac{20\theta_f - 20\theta_i - (8\dot{\theta}_f + 12\dot{\theta}_i)t_f - (3\ddot{\theta}_i - \ddot{\theta}_f)t_f^2}{2t_f^3} \\
C_4 &= \frac{30\theta_f - 30\theta_i + (14\dot{\theta}_f + 16\dot{\theta}_i)t_f + (3\ddot{\theta}_i - 2\ddot{\theta}_f)t_f^2}{2t_f^4} \\
C_5 &= \frac{12\theta_f - 12\theta_i - (6\dot{\theta}_f + 6\dot{\theta}_i)t_f + (\ddot{\theta}_i - \ddot{\theta}_f)t_f^2}{2t_f^5}
\end{aligned}
$$

Where θ_i and θ_f are the initial and final values of each joint and t_f is the duration time. The parameters of six joints are obtained using inverse kinematics analysis as shown in Tab. 3.

Table 3. Robot parameters for trajectory planning[2]

Link#	1	2	3	4	5	6
$\theta_i(Degrees)$	$-180°$	$-90°$	0	$-180°$	$-90°$	$-180°$
$\theta_f(Degrees)$	180°	90°	180°	180°	90°	180°
L(mm)	0	50	15	36	0	39.5
$\ddot{\theta}_i(Degrees/s^2)$	5	5	5	5	5	5
$\ddot{\theta}_f(Degrees/s^2)$-5	-5	-5	-5	-5	-5	-5
$t_f(sec)$	0	0	0	0	0	0
$t_f(sec)$	5	5	5	5	5	5

The flowchart representing the sequence of generating the fifth-order polynomial trajectory is shown in the Fig. 3. As the joints limits for some of the joints are similar, a sample for the trajectories for joints 1 and 2

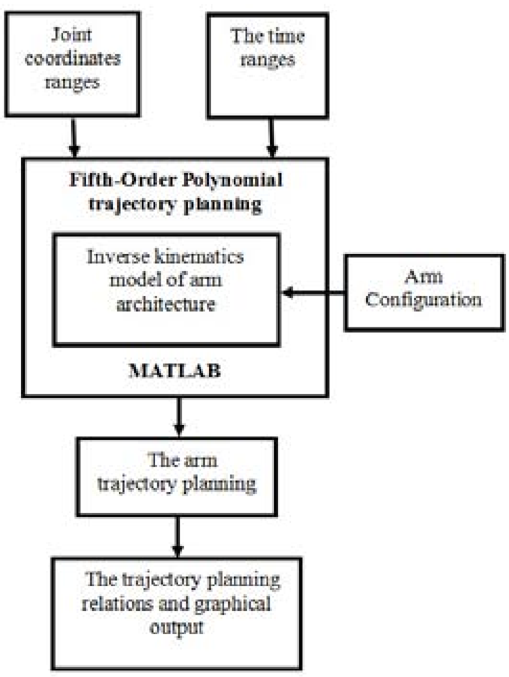

Fig. 3. The flowchart of the trajectory planning

are presented in Fig. 4 and Fig. 5.

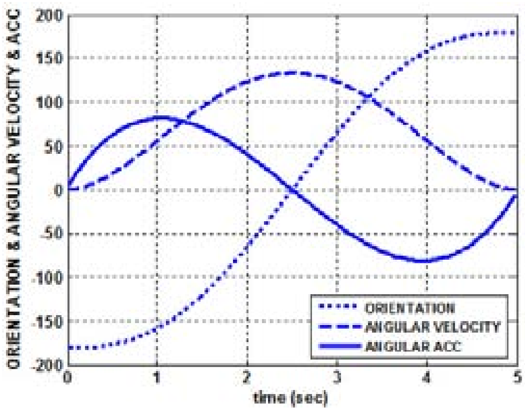

Fig. 4. The trajectory of the first joint

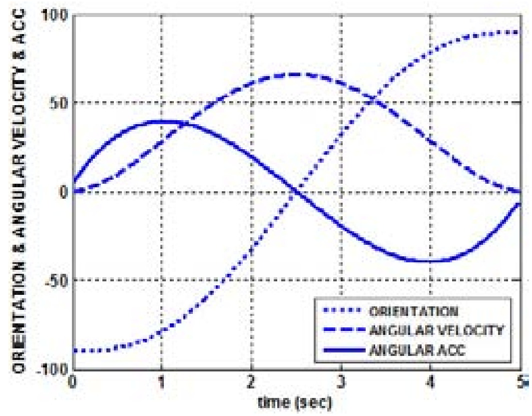

Fig. 5. The trajectory of the second joint

4 The dynamic model

Manipulator dynamics is concerned with the equations of motion, the way in which the manipulator moves in response to torques applied by the actuators, or external forces. The history and mathematics of the dynamics of serial-link manipulators are well covered in the literature. The equations of motion for an n-axis manipulator are given by[7]:

$$Q = M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) \tag{2}$$

Where:

q	is the vector of generalized join coordinates
$\dot{q}$	is the vector of joint velocities
$\ddot{q}$	is the vector of joint accelerations
M	is the inertia matrix
C	is the Coriolis and Centrifugal matrix
G	is the gravity matrix
Q	is the vector of generalized force associated.

The Lagrang- Euler is utilized here to calculate the kinetic energy, potential energy and derive the dynamic equations in symbolic form using the MATLAB symbolic Toolbox for the six-degree-of-freedom robot because the multiple-degree-of-freedom robot has equations very long and complicated. The equations of motion can be given in a compact form as:

$$T_i = \sum_{j=1}^{n} D_{ij}\ddot{q}_i + \sum_{j=1}^{n}\sum_{k=1}^{n} D_{ijk}\dot{q}_j\dot{q}_k + D_i \tag{3}$$

where

$$D_{ij} = \sum_{p=\max(i,j)}^{n} Trace(U_{pj} J_p U_{pi}^T) \tag{4}$$

$$D_{ijk} = \sum_{p=\max(i,j,k)}^{n} Trace(U_{pjk} J_p U_{pi}^T) \tag{5}$$

$$D_i = \sum_{p=1}^{n} -m_p g^T U_{pj} \bar{r}_p \tag{6}$$

$$J_i = \begin{bmatrix} \frac{(-I_{xx}+I_{yy}+I_{zz})}{2} & I_{xy} & I_{xz} & m_i\bar{x}_i \\ I_{xy} & \frac{(I_{xx}-I_{yy}+I_{zz})}{2} & I_{yz} & m_i\bar{y}_i \\ I_{xz} & I_{yz} & \frac{(I_{xx}+I_{yy}-I_{zz})}{2} & m_i\bar{z}_i \\ m_i\bar{x}_i & m_i\bar{y}_i & m_i\bar{z}_i & m_i \end{bmatrix} \tag{7}$$

In which:
J is the moment of inertia matrix for each link
n is the total number of links
j is the total number of joints
k is the coefficient (1, 2, 3 . . . n),
p is the coefficient (1, 2, 3 . . . n).

For the MATLAB simulation, the parameters of the robotic arm as given by[2] are represented in Tab. 4. The flowchart for the algorithm employed to calculate the torque history for each actuator based on the derived equations of motion is shown in the Fig. 6. The simulated results for the actuators torques are presented in Fig. 7 $\sim$ Fig. 12. It is found that the torque history over the selected period of time (5 seconds) has a lot of fluctuations as it seen clearly from Fig. 7 for the first actuator.

Table 4. Full mobility robotic parameters

Link#	1	2	3	4	5	6
$I_{act}(N.mm)$	10000	10000	10000	10000	10000	10000
m(kg)	6.055	6.055	6.055	6.055	6.055	6.055
L(mm)	0	50	15	36	0	39.5
$g(m/s^2)$	9.81	9.81	9.81	9.81	9.81	9.81
R(mm)	75	75	75	75	75	75
I_{xx}	$(1/12) \times m_i \times (3 \times (R^2) + (L_i/4)^2)$					
I_{yy}	$(1/12) \times m_i \times (3 \times (R^2) + (L_i/4)^2)$					
I_{zz}	$m_i \times R^2$					
I_{xz}	0					
I_{xy}	0					
I_{yz}	0					
X	$L_i/2$					
Y	0					
Z	0					

5 Discussion and conclusions

A kinematics and dynamics analysis for a six-degree-of-freedom surgical robot is presented in this paper. The kinematics model is presented based on Denavit-Hartenberg representation and the workspace of the end-effector is defined by solving the inverse kinematics problem. Fifth order polynomial trajectory for the six joints have been designed and employed to calculate the torque history for the six actuators. The dynamic equations of motion in symbolic form are derived using a Lagrange-Euler technique and the torque history is obtained using MATLAB for each joint. The proposed algorithm is flexible and can be extended to any robot

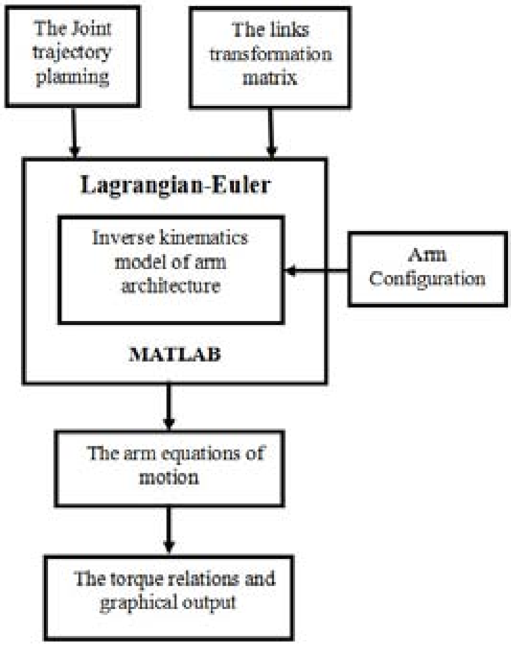

Fig. 6. The flowchart of the dynamic model

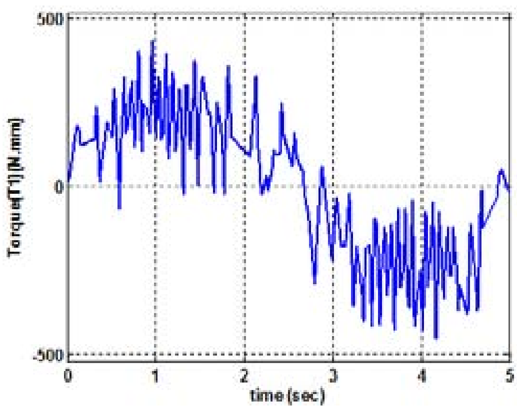

Fig. 7. Torque history for joint 1

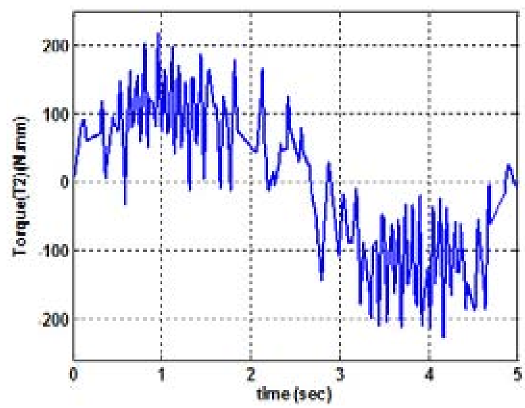

Fig. 8. Torque history for joint 2

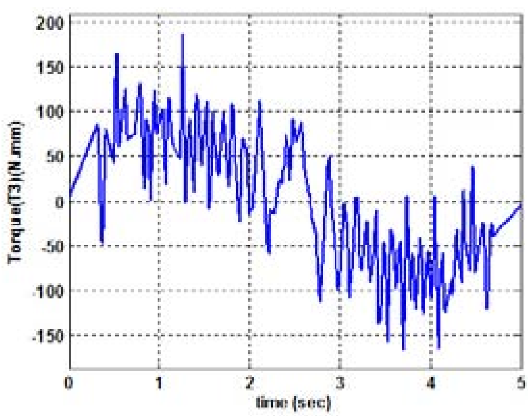

Fig. 9. Torque history for joint 3

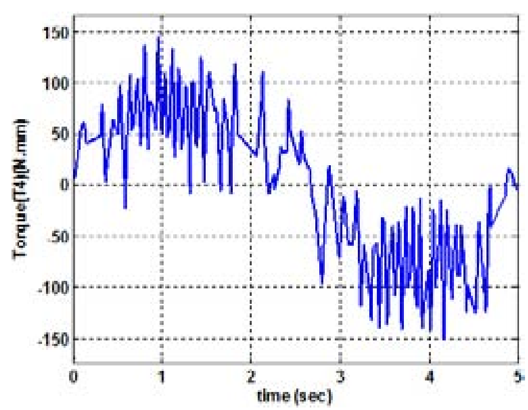

Fig. 10. Torque history for joint 4

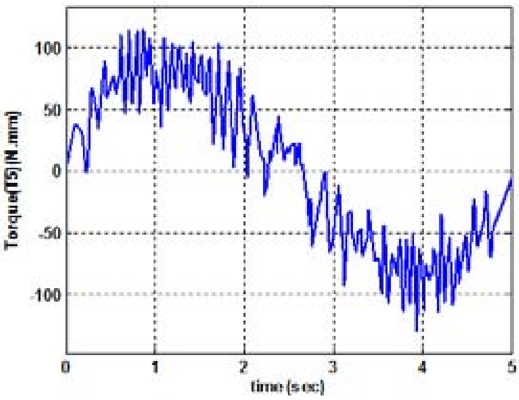

Fig. 11. Torque history for joint 5

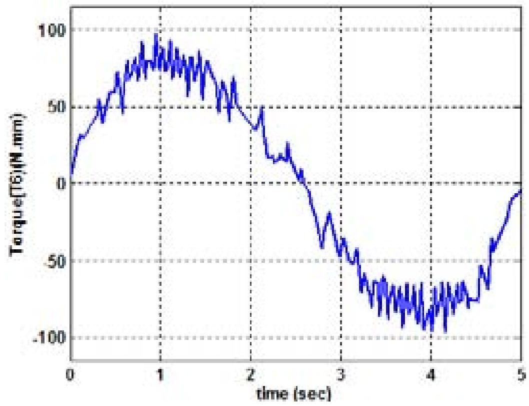

Fig. 12. Torque history for joint 6

configuration provided that the Denavit-Hartenberg presentation is available and the physical limits of joints are defined. The original torque history has many fluctuations. It is clear that the higest hub torque is for joint 1 while actuator torque of joint 6 is the lowest. It should be also noted that changing the final time for the joint trajectory changes the torque history considerably. For $t_f = 5$ seconds, the dominant part in the torque history is the inertia matrix. Increasing the final time to 10 and 20 seconds, shifts the dominant term from inertia matrix to Centrifugal and Coriolis matrics since the effect of angular velocity will be obviously high. This is due to the vanishing of the acceleration at most of the joint trajectory. Another consequence is of increasing the final time is the dramatic change in the peak value of the joint torque which requires big actuator size for the same task.

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