A fuzzy vehicle routing assignment model with connection network based on priority-based genetic algorithm *

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Abstract. In this paper, we concentrate on developing a fuzzy chance-constrained model of vehicle routing assignment model according to fuzzy theory. In the model, we consider the total costs which include preparing costs of each type vehicle and the transportation costs as the objective function and the preparing costs and the commodity flow demand is regarded as fuzzy variables. So we try to minimize the total costs at the predetermined confidence level $\alpha$. Then we convert the fuzzy constraints into their crisp equivalents by using fuzzy theory. In addition, we propose a effective method of priority-based genetic algorithm to solve this kind of problem when there is no genetic algorithm which can give clearly procedure of solving it. The efficacy and efficiency of priority-based genetic algorithm are demonstrated by the numerical example in the article.

Keywords: fuzzy programming, possibility measure, vehicle routing assignment, priority-based genetical algorithm

1 Introduction

As the develop of urban economy, traffic congestion is becoming more and more serious. So every enterprise of vehicle transportation has to face a external fact that is economy lost is brought by traffic congestion. When they send commodities to customers, for the sake of induce the costs, they should consider how to arrange their vehicles and which routing to select. Hence, vehicle routing assignment problem has been highly considered by many scholars [20–22].

Usually the basic structure of a large assembly or transportation system is complicated connection network. In this network, many nodes such as depots, customers are linked by many physical and notional lines and the services are finished by many vehicles send commodities between all the nodes. Now many service suppliers and distributors recognized the importance of designing efficient distribution strategies to improve the level of customers service and reduce transportation costs. In a typical distribution system, vehicles provide delivery, customer pick-up, or repair and maintenance services to customers that are geographically dispersed in a given area. In many applications a common objective is to find a set of routes for the vehicles which satisfies a variety of constraint and so as to minimize the total fleet operating cost. The problem of minimizing total cost has traditionally been called vehicle routing assignment problem. In a word, vehicle routing assignment problem is one of the logistics network problems and is concern with determining the vehicle type to assignment to each routing leg in order to minimize the total costs while satisfying vehicle routing and availability constraints to serve a number of customers with demands for some commodity, and its basic routing assignment model is as follows Fig. 1 (5 centers for example).

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Because of its important strategic position and economic importance, vehicle routing assignment problem has gained much attention in recent years. This problem was first introduced by Dantzig and Ramser [1], and was developed by Clarke and Wright [2]. Then stochastic vehicle routing assignment problems arise when considering demands and travel times as stochastic variables. Tillman [3] was the first to propose an algorithm for the stochastic vehicle routing problem based on Clarke and Wright. After that, many researchers, such as Dror and Trudeau [4], Gendreau et al. [5], Laporte [6] studied various types of stochastic vehicle routing assignment problems.

Actually, because there are not enough data to analyze in some new systems, it is hard to describe the parameters of the problem as random variables. We found that some of the data actually showed some fuzzy characteristics [19]. For example, we often say “the demand of customer is about a ton”, so we can set these parameters as fuzzy variables which can better reflect the actual situation. In this article, we will consider the demand as fuzzy variable and formulate the fuzzy programming model, and also build a hybrid intelligent algorithm based on priority genetic algorithm to deal with the model.

The rest of the paper is organized as follows. In section 2, we present some basic knowledge of fuzzy theory and optimization theory. In the section 3, we study vehicle routing assignment problem model. First, we give a simple description of this problem, and then formulate the mathematic model with fuzzy coefficients. Later, the fuzzy model is changed into its crisp equivalent through using triangular fuzzy variables. In the following, in the section 4, we present priority-based genetic algorithm to solve it. In section 5, we provide a numerical example to show the application of the proposed models and algorithms. Finally the conclusion has been made in section 6.

2 Fuzzy vehicle routing assignment model

In this paper, the vehicle routing assignment is considered as a logistics network problem, which consists of many distribution centers. From each center to center, they have different commodity demand. The purpose of designing a vehicle routing assignment model is to minimize the total costs by arranging the vehicle type and selecting routing when satisfying the demand of customers as the Fig. 1 shows.

2.1 Problem description

As mentioned above, the vehicle routing assignment with connection network is considering that a set of commodity flow demand and a set of distribution centers. In order to satisfy the demand of customer, we must arrange vehicles to send commodity to them.
Questions is: How to arrange them to minimize the total transportation costs and preparing costs of each routing leg and center?

In order to formulate the mathematic model of the vehicle routing assignment problem reasonably, first we should make several assumptions as follows:

(1) From each center to center that required the commodity flow demand with a certain amount.
(2) Each vehicle has a container with a physical capacity limitation and the total loading of each vehicle cannot exceed its capacity.
(3) A vehicle will be assigned for only one route on which may be more than one center.
(4) The same number of vehicle of each type remain at each center every night.
(5) We have enough vehicle to satisfy the demand.
(6) Each vehicle must pay for the preparing cost when left.

2.2 Modelling

Let \( G = (N, A) \) be a network, consisting of a finite set of nodes \( N = \{0, 1, 2, \ldots, n\} \), and a set of arcs \( A = \{(i, j), (k, l), \ldots, (s, t)\} \) joining \( n \) pairs of nodes in \( N \). Arc \((i, j)\) is said to be incident with nodes \( i \) and \( j \), and its directed for node \( i \) to node \( j \). Suppose that each arc \((i, j)\) has assigned to it nonnegative number \( c_{ij} \).

The mathematical notation and formulation are as follows:

Indices:

- \( i, j \): the index of center, \((i, j) = 1, 2, \ldots, N\), where index 0 is a dummy node;
- \( F \): the index of vehicle type, \((f = 1, 2, \ldots, F\);

Parameters:

- \( c_{fij} \): unit transportation cost of the vehicle type \( f \) from node \( i \) to \( j \);
- \( d_{ij} \): commodity demand from node \( i \) to \( j \);
- \( e_{fi} \): the preparing cost of the vehicle type \( f \) in origin node \( i \);
- \( F \): set of all vehicles;
- \( N \): set of all nodes;

Decision variables:

- \( x_{fij} = \begin{cases} 1 & \text{vehicle type } f \text{ run from node } i \text{ to } j, \\ 0 & \text{otherwise}. \end{cases} \)
- \( y_{fi} = \begin{cases} 1 & \text{if vehicle type } f \text{ departure at origin node } i, \\ 0 & \text{otherwise}. \end{cases} \)

The total cost of the vehicle routing assignment problem mainly consists of two parts: total transportation cost and preparing cost. So the whole total cost can be described as follow:

\[
C(x, y) = \sum_{f=1}^{F} \sum_{i=0}^{N} \sum_{j=0}^{N} c_{fij} x_{fij} + \sum_{f=1}^{F} \sum_{i=0}^{N} e_{fi} y_{fi}
\]

A precondition of the model is the transport flow must satisfy the commodity flow demand for each vehicle type, so the constraint is

\[
\sum_{f=1}^{F} x_{fijk} \geq d_{ij}
\]

Then we formulate the problem by using the following mixed integer programming model:

\[
\min C(x, y) = \sum_{f=1}^{F} \sum_{i=0}^{N} \sum_{j=0}^{N} c_{fij} x_{fij} + \sum_{f=1}^{F} \sum_{i=0}^{N} e_{fi} y_{fi}
\]

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In the above model, the objective function minimizes the total cost made of: the transportation cost and preparing cost for each vehicle, constraint (2) ensures all the transport flow must satisfy the commodity flow demand for each vehicle, constraint (3) ensures the flow balance at each leg in the network for each vehicle type. constraint (4) ensures that the same number of vehicle of each type remain every day, constraint (5) and (6) define the nature of the decision variable.

In the past, when designing routing assignment problem, we often formulate the model by using accurate parameters. In practise, the critical parameters for the problem, such as commodity demands, are usually uncertain because of the absence of abundant information. For example, we often say “the demand is between 20 and 30 tons”, “about 100 ton”. It is more appropriate to describe them with uncertain parameters. In this paper, we consider the commodity demand and preparing cost as fuzzy variables. That is to say \( \tilde{d}_{ij} \) and \( \tilde{e}_{fi} \) and fuzzy variables.

**Definition 1.**
Let \( \Theta \) be a nonempty set, \( \mathcal{P}(\Theta) \) the power set of \( \Theta \), for each \( A \in \mathcal{P}(\Theta) \), there is a nonnegative number \( \text{Pos}(A) \), called its possibility, such that:
(i) \( \text{Pos}(\emptyset) = 0, \text{Pos}(\Theta) = 1 \),
(ii) \( \text{Pos}(\bigcup_k A_k) = \text{Sup}_k \text{Pos}(A_k) \), for any arbitrary collection \( A_k \) in \( \mathcal{P}(\Theta) \).

Then the triplet \( (\Theta, \mathcal{P}(\Theta), \text{Pos}) \) is called a possibility space, and the function \( \text{Pos} \) is referred to as a possibility measure.

**Definition 2.**
Given a domain \( X \). If \( \tilde{A} \) is a fuzzy subset of \( X \), for any \( x \in X \)

\[
\mu_{\tilde{A}}: X \rightarrow [0, 1], \quad x \rightarrow \mu_{\tilde{A}}(x),
\]

\( \mu_{\tilde{A}} \) is called a membership function of \( x \) with respect to \( \tilde{A} \), \( \mu_{\tilde{A}}(x) \) denotes the grade to each point in \( X \) with a real number in the interval \([0, 1]\) that represents the grade of membership of \( x \) in \( A \). \( \tilde{A} \) is called a fuzzy set and described as follows

\[
\tilde{A} = \{(x, \mu_{\tilde{A}}(x))| x \in X \},
\]

If \( \alpha \) is possibility level and \( 0 \leq \alpha \leq 1 \), \( A_\alpha \) consist of all elements whose degrees of membership in \( \tilde{A} \) are greater than or equal to \( \alpha \),

\[
A_\alpha = \{x \in X|\mu_{\tilde{A}}(x) \geq \alpha\},
\]

then \( A_\alpha \) is called the \( \alpha \)-level set of fuzzy set \( \tilde{A} \).

Because of the existence of fuzzy parameters, the objective function and the constraints (2) have no exact sense. Based on the definition above, we can change the model into fuzzy chance constrained model (FCCP):

\[
\min \bar{f}
\]

subject to:

\[
\text{Pos}\{\sum_{f=1}^{F} \sum_{i=0}^{N} \sum_{j=0}^{N} c_{fij}x_{fij} + \sum_{f=1}^{F} \sum_{i=0}^{N} e_{fi}y_{fi} \leq \bar{f}\} \geq \alpha
\]

\[
\text{Pos}\{\sum_{f=1}^{F} x_{fijk} \geq d_{ij}\} \geq \beta, \quad \forall i, j
\]
Let triangular fuzzy variable $\alpha$ be determined confidence level $\beta$ given confidence function is:

$$\begin{align*}
\sum_{j=0}^{N} x_{fij} - \sum_{k=0}^{N} x_{fki} &= 0, \quad \forall i, f \\
x_{f0j} &= x_{fj0}, \quad \forall f, j \\
x_{fij} &\geq 0, \quad \forall f, i, j \\
y_{fi} &\in [0, 1], \quad \forall i, f, j
\end{align*}$$

(10) (11) (12) (13)

Objective chance constraint (8) means the objective value $\hat{f}$ should be the minimum value at the predetermined confidence level $\alpha$, constraint (9) means the possibility of the satisfying commodity flow demand is $\beta$ at least.

### 2.3 Crisp equivalents of fuzzy constraints

In order to solve the fuzzy chance constrained model (FCCP) above, we can change the fuzzy constraints into their crisp equivalents. In this article, we use triangular fuzzy variable to express the fuzzy parameters $d_{ij}$ and $e_{fi}$: $d_{ij} = (d_{ij1}, d_{ij2}, d_{ij3})$, $e_{fi} = (e_{f1}, e_{f2}, e_{f3})$.

**Definition 3.** [8] $\text{Pos}\{\hat{r} \leq m\} = \sup \{\mu_{r}(x)|x \leq m, x \in R\}$, $m$ is the function of decision variable.

**Definition 4.** [8] Let $\hat{r} = (r_1, r_2, r_3)$ is a triangular fuzzy variable, and $r_1 \leq r_2 \leq r_3$, then the membership function is:

$$\mu_{x} = \begin{cases} 
\frac{x - r_1}{r_2 - r_1}, & \text{if } r_1 \leq x \leq r_2 \\
\frac{x - r_2}{r_3 - r_2}, & \text{if } r_2 \leq x \leq r_3 \\
0, & \text{otherwise}
\end{cases}$$

(14)

For the fuzzy chance constrained (8), because of $e_{fi}$ is triangular fuzzy variable and $y_{fi} \geq 0$, based on the algorithm of addition and subtraction, we know that $\sum_{f=1}^{F} \sum_{i=0}^{N} \sum_{j=0}^{N} c_{fij} x_{fij} + \sum_{f=1}^{F} \sum_{i=0}^{N} e_{fi} y_{fi}$ is also triangular fuzzy variable, and expressed as follows:

$$\begin{align*}
&\sum_{f=1}^{F} \sum_{i=0}^{N} \sum_{j=0}^{N} c_{fij} x_{fij} + \sum_{f=1}^{F} \sum_{i=0}^{N} e_{fi} y_{fi}, \\
&\sum_{f=1}^{F} \sum_{i=0}^{N} \sum_{j=0}^{N} c_{fij} x_{fij} + \sum_{f=1}^{F} \sum_{i=0}^{N} e_{fi} y_{fi}.
\end{align*}$$

Theorem 1. Let triangular fuzzy variable $\hat{r} = (r_1, r_2, r_3)$, and its membership function is $\mu_{r}(x)$, then for any given confidence $\alpha (0 \leq \alpha \leq 1)$, we have $\text{Pos}\{\hat{r} \leq m\} \geq \alpha$ if and only if $m \geq (1 - \alpha) r_1 + \alpha r_2$.

**Proof.** See Appendix.

By Theorem (1), we know that objective chance constraint (8) can be changed into its crisp equivalent,

$$\text{Pos}\{\sum_{f=1}^{F} \sum_{i=0}^{N} \sum_{j=0}^{N} c_{fij} x_{fij} + \sum_{f=1}^{F} \sum_{i=0}^{N} e_{fi} y_{fi} \leq \hat{f}\} \geq \alpha$$

$$\iff \sum_{f=1}^{F} \sum_{i=0}^{N} \sum_{j=0}^{N} c_{fij} x_{fij} + \sum_{f=1}^{F} \sum_{i=0}^{N} y_{fi} [(1 - \alpha) e_{f1} + \alpha e_{f2}] \leq \hat{f}$$

In the same way, we can change the constraint (9) in to its crisp equivalent,

$$\text{Pos}\{\sum_{f=1}^{F} x_{fijk} \geq d_{ijk}\} \geq \beta \iff (1 - \beta) d_{ij1} + \beta d_{ij2} \leq \sum_{f=1}^{F} x_{fijk}$$
From the discussion above, we know that the fuzzy chance constraints of FCCP model all can be changed into their crisp equivalents when considering the demand and preparing cost as triangular fuzzy variable. Hence the uncertain model is transformed into certain model which can be solved easily.

\[
\begin{align*}
\min & \quad \bar{f} \\
\text{s.t.} & \quad \sum_{f=1}^{F} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} c_{fij} x_{fij} + \sum_{f=1}^{F} \sum_{i=0}^{N-1} y_{fi} \left[ (1 - \alpha)e_{fi1} + \alpha e_{fi2} \right] \leq \bar{f} \\
& \quad (1 - \beta)d_{ij1} + \beta d_{ij2} \leq \sum_{f=1}^{F} x_{fijk} \\
& \quad \sum_{j=0}^{N-1} x_{fij} - \sum_{k=0}^{N-1} x_{fki} = 0 \\
& \quad x_{f0j} = x_{fj0}, \quad \forall f,j \\
& \quad x_{fij} \geq 0, \quad \forall f,i,j \\
& \quad y_{fi} \in [0,1], \quad \forall f,i \\
\end{align*}
\]  

(15)

Because some of the variables are fuzzy variables, we can calculate the objective function by using the expected value of them.

**Definition 5.** Let \( \xi \) be a fuzzy variable, then the expected value of \( \xi \) is defined by

\[
E[\xi] = \int_{0}^{+\infty} Cr\{\xi \geq r\} \, dr - \int_{-\infty}^{0} Cr\{\xi \leq r\} \, dr,
\]

provided that at least one of the two integrals id finite.

Especially, the triangular fuzzy variable \( \xi = (r_1, r_2, r_3) \) has an expected value

\[
E[\xi] = \frac{1}{4}(r_1 + 2r_2 + r_3).
\]

3 Priority-based genetic algorithm

The usual form of genetic algorithm (GA) is described by Globerg\(^{[11]}\). Based on the mechanics of natural selection and computer science, genetic algorithms are regarded as a very important approach to solve uncertain programming problems. Generally, genetic algorithms for solving optimization problems are a sequence of computational steps which asymptotically converge to optimal solution. GA has received considerable attentions regarding their potential as a novel optimization technique because of theirs lots of advantages. For example, they do not need many mathematical requirements about the optimization problems; they provide us a great flexibility to hybridize to make an efficient implementation for a special problem; especially, they are often combined with many other techniques to solve problems. Now GAs have been well discussed and summarized by several authors, e.g., Holland\(^{[12]}\), Koza\(^{[13]}\), Fonseca and Fleming\(^{[14]}\), Orosh\(^{[15]}\), Gen and Cheng\(^{[16]}\) and Liu\(^{[17]}\).

Recently, Lin and Gen proposed a priority-based genetic algorithm encoding method\(^{[18]}\). As it is known, a gene in a chromosome is characterized by two factors: lotus, i.e. the position of gene located within the structure of chromosome, and allele, i.e. the value the gene takes. In this encoding method, the position of a gene is used to represent node ID and its value is used to represent the priority of the node for constructing a path among candidates. A path can be uniquely determined from this encoding.

In the following, we attempts to present a priority-based genetic algorithm to obtain a compromise solution of the model referred above, provided that the representation, initialization, genetic operations are revised as follows.
3.1 Representation and feasibility of the chromosome

For this problem, we design each chromosome consisting of two parts. The first part is locus to represent the task ID: the route between each center. The second part is allele to represent the priority of the task: the sequence of each route to satisfying the commodity require. To develop a priority-based genetic representation of the vehicle routing assignment problem with connection network, there are mainly 3 phases:

**Phase 1**: Creating a vehicle route.
- step 1.1: Generate a random network structure with the connection relationship.
- step 1.2: Generate a random priority to each vehicle route using encoding procedure.

**Phase 2**: Assigning vehicles to each route.
- step 2.1: Assign vehicles to satisfied with commodity demand using decoding procedure.
- step 2.2: Complete all the connection routes.
- step 2.3: Obtain feasible vehicle routes.

**Phase 3**: Drawing a vehicle route.
- step 3.1: Achieve the vehicle connection relationship.
- step 3.2: Draw a vehicle route S.

Initialization: Firstly, we should make certain of the popSize (popSize chromosomes). This procedure is actually selecting popSize points randomly as initial solutions in the solution space. If the generated chromosome \( V = (x, y) \) is proven to be feasible, then it is accepted as a chromosome; otherwise we repeat the process until a feasible chromosome is obtained.

Parameters selection: Confirm the crossover probability \( P_c \) and mutation probability \( P_m \) in order to make sure the variety of solution each generation.

Encoding: The priority-based encoding method is easily verified that permutation of the encoding corresponds to the service sequences, so that the genetic operators can easily be applied to the encoding. As priority-based encoding method, depending on the following 2 steps, the initial chromosome are randomly generated first.

- step 1.1: Generate a random network structure with the connection relationship.
- step 1.2: Generate a random priority to each vehicle route using encoding procedure.

Now in order to explain the vehicle route effectively, a sample example is illustrated as follows. We consider 5 centers for connection with each other. We generate a chromosome for example, the ID 2 to 4 is showing from center 2 to another centers 1, 3, 4 and 5. Based on this rule with considering 5 centers, an initial chromosome is showed in Fig. 2. Then we can exchange the priority values each other for 10 times, and we get popSize chromosomes with different priority values.

**Fig. 2.** The chromosome by priority-based encoding for 5 centers

Decoding: As priority-based decoding method, depending on the following 3 steps, assign the vehicles to each vehicle route.

- step 2.1: Assign vehicles to satisfied with commodity demand using decoding procedure.
- step 2.2: Complete all the connection routes.
- step 2.3: Obtain feasible vehicle routes.

The priority-based decoding procedure can generate a vehicle route. And the route is development from the example of chromosome via priority-based decoding is showed as follow. For example, the highest priority value is 20 (see Fig. 2), it mean an vehicle from center 3 to 1, then the higher priority value in center 1 is 18, therefore, this vehicle come to 4, then 4 to 2, by the same way, we can generate the one vehicle route. We can use this priority-based decoding method to get all the routes. Then we calculate the total costs by this chromosome.
Routs for example:
(1) 3 → 1, 1 → 4, 4 → 2, 2 → 4, 4 → 3.
(2) 3 → 5, 5 → 2, 2 → 1, 1 → 2, 2 → 3.
(3) 3 → 4, 4 → 5, 5 → 1, 1 → 3.
(4) 3 → 2, 2 → 5, 5 → 4, 4 → 1, 1 → 5, 5 → 3.

3.2 Fitness evaluation

Fitness evaluation is to check the solution value of the objective function subjected to the problem constraints. The objective function provides the mechanism evaluating each individual. Evaluation function is to assign a probability of reproduction to each chromosome so that its likelihood of being selected is proportional to its fitness relative to the other chromosomes in the population. The objective functions in models can be calculated. Normalization is carried out for each objective as follows:

\[ v = \frac{f - f_{\text{min}}}{f_{\text{max}} - f_{\text{min}}} \quad i = 1, 2, \ldots, n \]

where \( f_{\text{min}} \) and \( f_{\text{max}} \) are the minimum and maximum value of \( i \)th objective on the current generation, respectively. Then we rearrange all chromosomes from large to small according to their objective function values and get a sequence \((v_1, v_2, \ldots, v_n)\).

3.3 Genetic operations

In this section, we discuss the procedure of selection, crossover, mutation and evaluation as follows:

3.3.1 Selection

Selection process: Selection process is based on spinning the roulette wheel \( \text{popSize} \) times. Each time we select a single chromosome for a new population. The roulette wheel is a fitness proportional selection. The operation is as follows:

Step 1: Calculate the probability \( P_i \) of a chromosome \( i \) which is copied to generate the offsprings (\( i = 1, 2, \ldots, \text{popSize} \)). Here \( P_1 = v_1 / \sum_{i=1}^{\text{popSize}} v_i \), \( P_2 = v_2 / \sum_{i=1}^{\text{popSize}} v_i, \ldots, P_i = v_i / \sum_{i=1}^{\text{popSize}} v_i \).

Step 2: Calculate the accumulative probability of each chromosome \( q_k \):

\[ q_k = \sum_{i=1}^{k} P_i, \quad k = 1, 2, \ldots, \text{popSize} \]

Step 3: Randomly generate a number \( \lambda \) with uniformity distribution in \([0, 1]\). If \( \lambda \leq q_1 \), we chose the chromosome 1 to copy; if \( q_{k-1} \leq \lambda \leq q_k \), we choose the chromosome \( \lambda \) to generate the offsprings. Repeat the process for \( \text{popSize} \) times, and we get the offsprings.

3.3.2 Crossover

As crossover operator, we adopt the arithmetical crossover. We denote \( v_i, i = 1, 2 \), and generate a random number \( \lambda_1, \lambda_2 \) from the open interval \((0, 1)\) and \( \lambda_1 + \lambda_2 = 1 \), then we can get the offspring:

\[ v'_1 = \lambda_1 v_1 + \lambda_2 v_2, \quad v'_2 = \lambda_2 v_1 + \lambda_1 v_2 \]

Here, the new \( v'_1, v'_2 \) will be checked, if they do not satisfy the chance constraints, we will randomly get new \( v'_1, v'_2 \) until they are feasible.

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3.3.3 Mutation

Similar to crossover, mutation is used to prevent the premature convergence and explore new solution space. However, unlike crossover, mutation is usually done by modifying gene within a chromosome. We adopt the swap mutation which selects two elements at random and then swaps the elements on these position as Fig. 3.

3.4 Overall procedure

The steps of our algorithm for solving the vehicle routing assignment problem as follows.

**Step 1.** Set the initial values and the parameters of genetic algorithms: population size $\text{pop}_{-}\text{size}$, crossover rate $p_c$, and maximum generation $\text{max}_{-}\text{gen}$.

**Step 2.** Generate the initial population by priority-based encoding routine.

**Step 3.** Evaluate $P_t$ by priority and lotus mapping-based decoding routine.

**Step 4.** Generic operators: arithmetical crossover and swap mutation.

**Step 5.** Check the feasibility of the offspring and repair the infeasible offspring.

**Step 6.** Evaluation and select the chromosomes.

**Step 7.** Repeat the second to sixth steps for the given number of $\text{max}_{-}\text{gen}$.

**Step 8.** Report the best chromosome as the optimal solution.

4 Numerical example

For the numerical example, we consider the vehicle routing assignment problem with 2 type vehicles and 5 centers for commodity send. The commodity flow demand is showed as Tab. 1. The transportation cost by type 1 and type 2 is showed as Tab. 2, and Tab. 3. The preparing cost for each center by vehicle types is showed in Tab. 4. The loading capacity of each vehicle type is 2 and 3.5 tons.

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Based on the initial random chromosome showed in Fig. 2 and the data in Fig. 1, we can get the result of the vehicle routing assignment problem. The routes, max commodity flow demand, vehicle type and route costs are shown in Tab. 5. The final objective value of minimum total cost is 246200.

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Table 2. Unit transportation cost by vehicle type 1. (unit:hundred)

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<td>8.0</td>
<td>0</td>
<td>4.0</td>
</tr>
<tr>
<td>5</td>
<td>3.0</td>
<td>6.0</td>
<td>5.0</td>
<td>8.0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3. Unit transportation cost by vehicle type 2. (unit:hundred)

<table>
<thead>
<tr>
<th>i/j</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>6.0</td>
<td>8.0</td>
<td>5.0</td>
<td>3.0</td>
</tr>
<tr>
<td>2</td>
<td>5.0</td>
<td>0</td>
<td>4.0</td>
<td>7.0</td>
<td>6.0</td>
</tr>
<tr>
<td>3</td>
<td>3.0</td>
<td>9.0</td>
<td>0</td>
<td>2.0</td>
<td>7.0</td>
</tr>
<tr>
<td>4</td>
<td>7.0</td>
<td>4.0</td>
<td>2.0</td>
<td>0</td>
<td>5.0</td>
</tr>
<tr>
<td>5</td>
<td>6.0</td>
<td>8.0</td>
<td>2.0</td>
<td>4.0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4. Preparing cost for each type vehicle. (unit:thousand)

<table>
<thead>
<tr>
<th>i/j</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(0.5,0.6,0.7)</td>
<td>(0.3,0.4,0.5)</td>
<td>(0.4,0.5,0.6)</td>
<td>(0.5,0.6,0.7)</td>
<td>(0.3,0.4,0.5)</td>
</tr>
<tr>
<td>2</td>
<td>(0.7,0.8,0.9)</td>
<td>(0.5,0.6,0.7)</td>
<td>(0.7,0.8,0.9)</td>
<td>(0.7,0.8,0.9)</td>
<td>(0.6,0.7,0.8)</td>
</tr>
</tbody>
</table>

Table 5. The result of initial chromosome for 5 centers.

<table>
<thead>
<tr>
<th>Rout</th>
<th>Max demand flow</th>
<th>vehicle type</th>
<th>route costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(3,1),(1,4),(4,2), (2,4),(3)</td>
<td>9</td>
<td>T1:1, T2:2</td>
</tr>
<tr>
<td>2</td>
<td>(3,5),(5,2),(2,1), (1,2),(2,3)</td>
<td>8</td>
<td>T1:1, T2:2</td>
</tr>
<tr>
<td>3</td>
<td>(3,4),(4,5),(5,1), (1,3)</td>
<td>8</td>
<td>T1:1, T2:2</td>
</tr>
<tr>
<td>4</td>
<td>(3,2),(2,5),(5,4), ((4,1),(1,5),(5,3)</td>
<td>8</td>
<td>T1:1, T2:2</td>
</tr>
</tbody>
</table>

We set the GA parameters: $\alpha = 0.9$, popSize = 20, maxGen = 1000, $p_C = 0.5$, $p_M = 0.5$. Base on the priority-based genetic algorithm routine to calculate the 5 centers model, we can get the best chromosome as Fig. 4 showing and Tab. 6 is the best result for the 5 vehicle routing assignment. The final objective value of minimum total cost is 221200, which is better than the previous random generated case show in Tab. 5.

Table 6. The best solution for 5 centers.

<table>
<thead>
<tr>
<th>Rout</th>
<th>Max demand flow</th>
<th>vehicle type</th>
<th>route costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(3,1),(1,4),(3,3),(4,2),(2,3),(3,5)</td>
<td>8</td>
<td>T1:1, T2:2</td>
</tr>
<tr>
<td>2</td>
<td>(5,2),(2,4),(4,5)</td>
<td>9</td>
<td>T1:1, T2:2</td>
</tr>
<tr>
<td>3</td>
<td>(5,4),(4,1),(1,5)</td>
<td>8</td>
<td>T1:1, T2:2</td>
</tr>
<tr>
<td>4</td>
<td>(3,3),(3,2),(2,5)</td>
<td>4</td>
<td>T1:2</td>
</tr>
<tr>
<td>5</td>
<td>(1,3),(3,1)</td>
<td>8</td>
<td>T1:1, T2:2</td>
</tr>
<tr>
<td>6</td>
<td>(1,2),(2,1)</td>
<td>6</td>
<td>T2:2</td>
</tr>
</tbody>
</table>
5 Conclusion

In this paper, based on fuzzy environments, we have considered a fuzzy chance constrained model of vehicle routing assignment problem and proposed priority-based genetic algorithm to solve it. In the model, we mainly consider total costs comprised of preparing costs and transportation costs for the objective and we try to minimize it at the predetermined confidence level $\alpha$. Then we change the fuzzy constraints in to its crisp equivalents. Though the route selection problem based on genetic algorithm has proposed already, there is no clearly procedure of solving this kind of problem. This article propose an effective priority-based genetic algorithm to solve this problem and at the same time, consider it in fuzzy environments.

Although the priority-based genetic algorithm mentioned in this paper is only used to solve the vehicle routing assignment problem with fuzzy chance constrained, we think it useful to be applied to some other problems. In the future, we will consider other fields using this method.

References

Appendix

From the definition (3): \( \text{Pos}\{\tilde{r} \leq m\} = \sup \{\mu_{r}(x)|x \leq m, x \in R\} \), we know that,

\[
\text{Pos}\{\tilde{r} \leq m\} \geq \alpha \iff \sup\{\mu_{r}(x)|x \leq m, x \in R\} \geq \alpha \\
\iff m \geq H_{\alpha}, H_{\alpha} = \inf\{H|H = \mu_{\tilde{r}}^{-1}(\alpha)\}
\]

For triangular fuzzy variable, see Fig. 5, from the membership function of definition 4, we know that,

\[
(H_{\alpha} - r_1)/(r_2 - r_1) = \alpha
\]

So we get that,

\[
H_{\alpha} = (1 - \alpha)r_1 + \alpha r_2
\]

That is to say,

\[
m \geq (1 - \alpha)r_1 + \alpha r_2
\]

Hence,

\[
\text{Pos}\{\tilde{r} \leq m\} \geq \alpha \iff m \geq (1 - \alpha)r_1 + \alpha r_2.
\]

Fig. 5. The membership function of \( r \)