ISSN 1 746-7233, England, UK
World Journal of Modelling and Simulation
Vol. 4 (2008) No. 3, pp. 188-195

A method for calculating pipe deformation in gas wells*

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(*Received February 1 2008, Accepted May 21 2008*)

Abstract. Buckling behavior of pipes in curved gas wells is a main problem in gas industry.Calculating axial deformation of pipes is the key to successful operating of exploiting gas.In this paper, aiming for calculating axial deformation of pipes in curved gas wells, we developed a series of basic tubing forces models, which are valuable in calculating axial deformation of pipes.Based on these models, a algorithm for calculating axial deformation is designed. Then the algorithm is used in solving a practical problem in exploiting gas.

Keywords: buckling behavior, axial deformation, tubing forces model, simulation

1 Introduction

Pipe buckling may cause problems such as deviation control while drilling, ineffective axial load transfer to the bit, and even pipe failure. Considering the importance of the subject matter, buckling behavior of pipes has long been investigated by many researchers[2, 6, 8]. Several models[5, 10, 15] have been proposed for prediction of forces causing helical shape (the so-called helical buckling) of pipes in vertical and curved wells. However, there is no agreement on which models should be used for better predictions.

When the pipes buckle/bend, an additional contact force between the pipe and well wall is induced. Contact forces together with the friction coefficient determine the severity of the drag forces, which are of major concern[7]. Johanscik et al.presented an equation to calculate contact force of drill pipe without considering buckling possibility due to excessive compressive loading. Mitchell presented an equation to predict contact forces due to helical buckling of drill pipes in vertical wells[15]. Wu and Juvkam-Wold developed a model to determine the contact force for drill pipe in deviated wells[6].

A selection of articles on buckling theory and coiled tubing behavior include the following. Lubinski et al. derived the well-known pitch-force relation for vertically and helically buckled tubing[2, 9, 10]. Dawson and Pasley discussed primarily sinusoidal buckling and gave a formula for critical compression force in an inclined but straight well[3]. Cheatham used the energy principle to derive a formula for the critical force for helical buckling. However, the results suffered from weaknesses. Mitchell discussed in particular the effect of friction in the pos-buckled state, but his analysis was based on the assumption of a weightless tubing, with no external normal forces acting on it[15]. A recent paper by Kyllingstad focuses on the effect of curvature on buckling resistance and generalizes previous results to realistic well trajectories[5].

In this paper, aiming for calculating axial deformation of pipes in curved gas wells, we developed a series of basic tubing forces models, which are valuable in calculating axial deformation of pipes.This paper is organized as follows. In section 2, the basic tubing forces model for pipes in curved wells will be set up. Next, in section 3, we analyze the axial deformation of the pipes in curved wells.Then, the presentation of

* This research was supported by the National Science Foundation for Distinguished Young Scholars, P. R. China (Grant No. 70425005) and the Teaching and Research Award Program for Outstanding Young Teachers in Higher Education Institutions of MOE of P. R. China (Grant No. 20023834-3).

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algorithm for the axial deformation and numerical results is in section 4. Some concluding remarks are finally given in section 5.

2 Basic tubing forces model(tfm)

The basic TFM calculation is performed by calculating the forces along the length of a pipe at a specific depth in a well, as the pipe is being either run into the hole (RIH) or being pulled out of the hole(POOH).

2.1 Basic forces on the pipe

The basic calculation is explained by using a simple example in which a tubing segment is located in a straight, inclined section of a well without fluids or pressures, shown in Fig. 1.

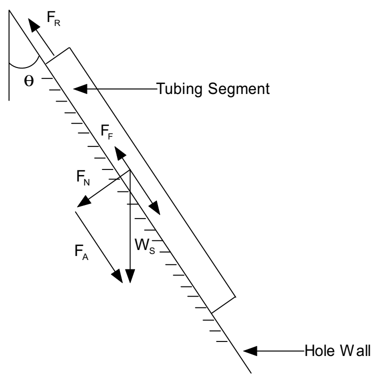

Fig. 1. Tubing segment in a straight , inclined section of a well

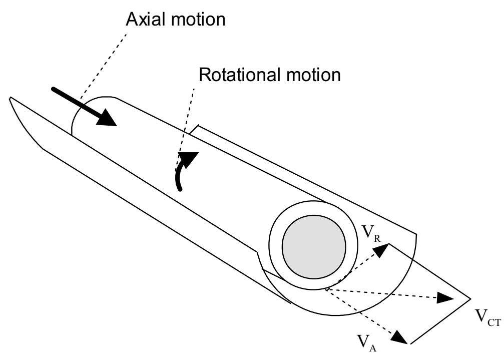

Fig. 2. Tubing segment moving axially and totationally

The vector triangle in Fig. 1 shows how a weight W_S can be broken into two component forces. F_A is the force component in the axial direction. F_N is the force component in the normal direction. The equations for each of these components are:

$$F_A = W_S \cos\theta \tag{1}$$

$$F_N = W_S \sin\theta \tag{2}$$

The friction force is calculated by multiplying the normal weight component by the friction coefficient μ.

$$F_F = \mu F_N \tag{3}$$

The real axial force is found by summing the weight component in the axial direction, F_A, with the friction caused by the normal component of the weight, F_F.

$$F_R = F_A \pm F_F \tag{4}$$

The basic TFM can be adapted to account for rotation of tubing in the well during either tripping or drilling[2]. T_F, the torque associated with the opposing friction, would be the frictional force multiplied by the outer radius of the tubing.

$$T_F = rF_{RF} \tag{5}$$

Since the friction force acts in the direction opposite of motion[12], if the axial velocity of the tubing is V_A and the equivalent linear velocity of rotation is V_R, then the resultant velocity V_{CT} is the vector sum of V_A and V_R, as shown in Fig. 2.

The actual friction force will be in the direction opposite of V_{CT}. Based on the velocity vector angle between V_{CT} and V_A, denoted by β, the axial and rotational components of friction resistance can be calculated.

$$F_{RF} = \mu F_N \sin\beta \qquad F_F = \mu F_N \cos\beta \tag{6}$$

2.2 Real force and effective force

In this section the more difficult concepts are discussed. Imagine a closed ended pipe suspended in a well as shown in Fig. 3 :

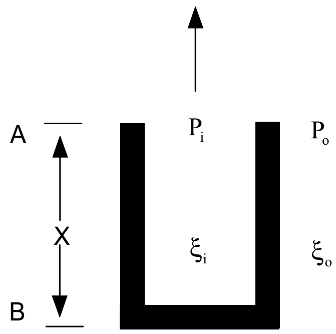

Fig. 3. Closed ended pipe suspended in a well

Let us consider only the lower section of this pipe from some point "A" downward.
The axial force components acting below point A are:
(1) weight of the pipe acting downward is W_SX
(2) upward force on the end of the pipe due to the external pressure is $P_{oB}A_o$
(3) downward force on the end of the pipe due to the internal pressure is $P_{iB}A_i$

2.3 Model equations

As was described previously, the model begins a calculation for the tubing pipe at one position in the well: the bottom end of the pipe. The effective force and torque calculations are performed for each successive "segment" of the tubing up to the surface.

2.3.1 Basic equation

According to equation (5), we can get the basic differential equation pair, which is integrated over the segment is[1]:

$$\frac{dF_E}{ds} = W_B \cos\theta \frac{d\gamma}{ds} + \mu \frac{dF_N}{ds} \cos\beta \qquad \frac{dT_F}{ds} = r\mu \frac{dF_N}{ds}\delta \tag{7}$$

where

$$\frac{dF_N}{ds} = \sqrt{\left(F_E \sin\theta \frac{d\gamma}{ds}\right)^2 + \left(F_E \frac{d\theta}{ds} + W_B \sin\theta \frac{d\gamma}{ds}\right)^2} \tag{8}$$

and

$$\delta = \begin{cases} -1, & \text{nonzoro torque is applied to non-rotaing CT} \\ \sin\beta, & \text{otherwise} \end{cases} \tag{9}$$

2.3.2 Effect of fluid flow

The flow of fluid in the tubing and in the annulus around the tubing produces two types of forces which must be accounted for in the equation of axial equilibrium[4].As a result, it has been shown that the following term, which accounts for both of the fluid flow effects mentioned here, must be added to equation (7):

$$\frac{dF_{Fl}}{ds} = \frac{2\pi r_o r_c}{r_c^2 - r_o^2}\left(r_o\tau_c - r_c\tau_o\right) \tag{10}$$

2.3.3 Helical buckling load

The primary equation for the helical buckling load, ignoring the effect of friction on the helical buckling load[11], is

$$F_{HB} = -2\sqrt{\frac{2EI}{r_c}}\sqrt[4]{\left(F_{HB}\sin\theta\frac{d_\gamma}{ds}\right)^2 + \left(F_{HB}\frac{d\theta}{ds} + W_B\sin\theta\right)^2} \tag{11}$$

2.3.4 Wall contact force

If the tubing is helically buckled, an additional wall contact force must be added to equation (8) to account for the additional wall contact force caused by the helix. This additional wall contact force due to the helix is given by the following equation[13, 14]:

$$\frac{dF_{NHB}}{ds} = \frac{r_c F_E^2}{4EI} \tag{12}$$

3 Axial deformation of pipes in curved gas wells

The deformation of pipes in curved gas wells contains two components:axial and horizontal deformation.Because the horizontal scale of the tubing is much smaller than the axial scale, the horizontal deformation is so small that we often ignore it.

3.1 Effects of changing temperature

Assume the temperature at point S is $T(s)$, it's initial temperature is $T_0(s)$, the thermal expansion coefficient of tubing is α, then the equation can be obtained as follows[16–18]:

$$\frac{du_T(s)}{ds} = \alpha\left[T(s) - T_0(s)\right] \tag{13}$$

$$u_T(s) = u_T(s_0) + \alpha\int_0^s \left[T(s) - T_0(s)\right] ds \tag{14}$$

3.2 Effects of internal and external pressure

Under the internal and external pressure of fluid, the tubing will have axial defromation[16–18].We can obtain the deformation ε_p by using Hooke's law:

$$\frac{du_p}{ds} = \varepsilon_p(s) = \frac{2\mu\left(p_o r_o^2 - p_i r_i^2\right)}{E\left(r_o^2 - r_i^2\right)} \tag{15}$$

$$u_p(s) = u_p(s_0) + \frac{2\mu}{E\left(r_o^2 - r_i^2\right)}\left[r_o^2\int_0^s p_o(s)\,ds - r_i^2\int_0^s p_i(s)\,ds\right] \tag{16}$$

3.3 Effects of axial force

According to Hooke's law, the deformation $\varepsilon_F(s)$ caused by axial force $\sigma_F(s)$ is[16–18]:

$$\frac{du_F}{ds} = \varepsilon_F(s) = -\frac{F_\tau(s)}{E(A_o - A_i)} \tag{17}$$

$$u_F(s) = u_F(s_0) - \frac{1}{E(A_o - A_i)}\int_0^s F_\tau(s)\,ds \tag{18}$$

4 Algorithm for axial deformation and numerical simulation

To simplify the calculation, we divide the tubing in several short segments of the same length.The length of a segment varies depending on variations in wall thickness, hole diameter, fluid density inside and outside the CT and well geometry. Here we set the length of 1 ft.

4.1 Algorithm

Based on the afore-mentioned discussion, a algorithm is designed[19, 20]:

Step 1. Obtain each point's inclination and azimuth

$$\varphi_j = \varphi_{j-1} + \frac{\varphi_k - \varphi_{k-1}}{\Delta s_k}\Delta s_j \tag{19}$$

$$\psi_j = \psi_{j-1} + \frac{\psi_k - \psi_{k-1}}{\Delta s_k}\Delta s_j \tag{20}$$

Step 2. Calculating the distribution of internal and external fluid pressure

$$p_{i,j} = p_{i,j-1} - \rho_{i,j} g \cos\varphi_j \Delta s_j \tag{21}$$

$$p_{o,j} = p_{o,j-1} - \rho_{o,j} g \cos\varphi_j \Delta s_j \tag{22}$$

Step 3. Calculate effective pipe weight

$$q_{ej} = q_j + (\rho_{ij} g A_{ij} - \rho_{oj} g A_{oj}) h_j \tag{23}$$

Step 4. Assume the effective axial force at dot j is already known as $F_{\tau ej}$, then f_{nj} and β_i can be obtained

$$f_{nj} = \left\{ \left[F_{\tau ej} \left(\frac{\varphi_j - \varphi_{j-1}}{s_j - s_{j-1}} \right)_j + q_{ej} \sin\varphi_j \right]^2 + \left[F_{\tau ej} \sin\varphi_j \left(\frac{\psi_j - \psi_{j-1}}{s_j - s_{j-1}} \right)_j \right]^2 \right\}^{\frac{1}{2}} \tag{24}$$

$$\beta_j = \frac{F_{\tau ej}}{2} \left(\frac{r_j}{E_j I_j f_{nj}} \right)^{\frac{1}{2}} \tag{25}$$

Step 5. Calculate axial pressure N_j and dimensionless axial pressure n_j

$$n_j = \begin{cases} 1, & \beta_j \le 1 \\ 1 + \frac{4}{11}(\beta_i - 1) + \frac{84}{121}(\beta_j - 1)^2, & 1 < \beta_j \le 1.37 \\ \beta_j^2, & \beta_j > 1.377 \end{cases} \tag{26}$$

$$N_j = n_j f_{nj} \tag{27}$$

Step 6. Calculate the effective axial force at dot $j+1$

$$F_{\tau e,j+1} = F_{\tau e,j} + (q_{ej} \cos\varphi_j + f_{vej} - f_j N_j)\Delta s_j \tag{28}$$

Step 7. Calculate the axial force $F_{\tau j}$

$$F_{\tau j} = F_{\tau ej} - p_{ij} A_{ij} + p_{oj} A_{oj} \tag{29}$$

Step 8. Assume deformation of tubing at dot j(u_{Fj}, u_{Pj}, u_{Tj}) are already known, then we can obtain the deformation at dot $j+1$

$$u_{p,j+1} = u_{p,j} + \frac{2\mu_j}{E_j (A_{oj} - A_{ij})} (A_{oj} p_{oj} - A_{ij} p_{ij}) \Delta s_j \tag{30}$$

$$u_{T,j+1} = u_{T,j} + \alpha_j (T_j - T_o) \Delta s_j \tag{31}$$

$$u_{F,j+1} = u_{F,j} - \frac{F_{\tau j} \Delta s_j}{E_j (A_{oj} - A_{ij})} \tag{32}$$

4.2 Numerical simulation

As was described previously, the algorithm begins a calculation for the tubing pipe at the bottom end of the pipe.The deformation calculations are performed for each successive “segment” of the tube to the surface.

4.2.1 Parameters

In this simulation, we study the axial deformation of pipe in X well, which is at Dayi country.All the needed parameters are given as following: (1) Internal fluid density is $1000\,kg/m^3$. (2) External fluid density is $1000\,kg/m^3$. (3) Depth of the well is $5160\,m$. (4) Friction coefficient is 1.2. (5) Ground temperature is $16\,^oC$. (6) Ground temperature gradient is $2.18\,^oC/100m$. (7) Length of one segment is $1\,m$. (8) Parameters of pipes, inclined well, inclination, azimuth and vertical depth are given in appendix.

4.2.2 Results

Through the TFM and simulation, we obtain a series of results.They are shown in the following:

Table 1. Results of pipe deformation

Temperature deformation	Pressure deformation	Axial force deformation	Total deformation
3.105	0.365	2.627	6.097

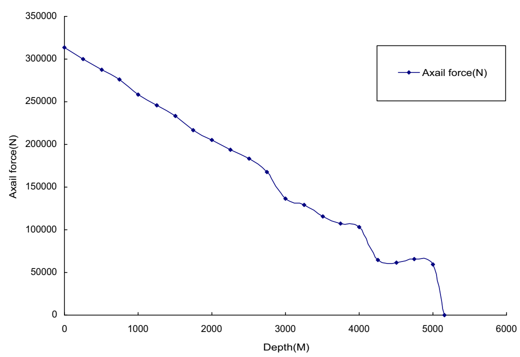

Fig. 4. Distribution of axial force

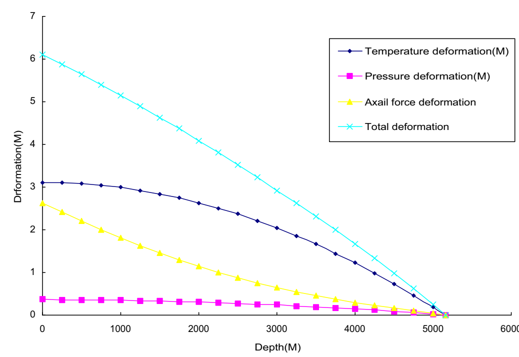

Fig. 5. Deformation of stirng

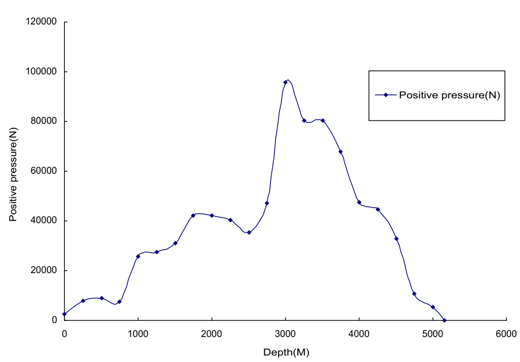

Fig. 6. Distribution of positive pressure

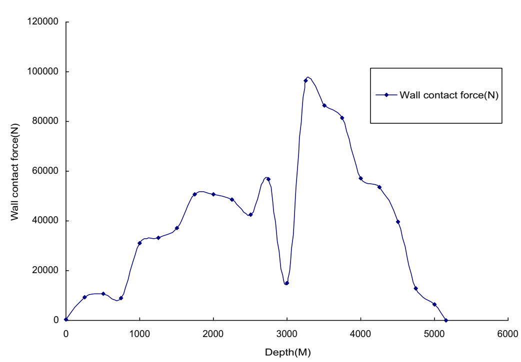

Fig. 7. Distribution of wall contact force

4.2.3 Discussion

From all the results above, we can obtain some information of the buckling behavior of pipes.

(1) The main factor causes the deformation of tubing is axial force and changing temperature as shown in Fig. 5. The axial force has less important effects on deformation compared with temperature in this case .Internal and external pressure has little effects on deformation, we can even ignore it.

(2) Friction coefficient, ground temperature gradient and material's thermal expansion coefficient have significant effects on axial force.To calculate the deformation, we must obtain the exact data of these parameters.

(3) Weight of per unit length and thickness of the tubing have huge impact on axial force.To prevent the pipe failure, we should use pipes of less weight and more thickness.

By using this method, we can obtain not only pipe deformation, but also distribution of axial force, positive pressure, and wall contact force. These results are helpful to predict when will the pipe sinusoidally, helically buckle and lock up.Then before the pipe run into the hole, we can predict it's behavior in the hole, this will help us to void some pipe failure.So we can see this method is valuable in calculating pipe deformation.

5 Conclusion

In this paper, we have established basic TFM for the pipes in curved gas wells and analyze the axial deformation of tubing.The basic TFM offers valuable insight into buckling phenomenon.Through the simulation and analysis, we can obtain the exact deformation of tubing in curved gas wells.It may help us to know exactly what happens in the wells and how should we to deal with it.Though the basic TFM can calculate the deformation of tubing in this paper, it is obvious that some assumption is different from practical processing and there are still some limitations and inconsistencies.So how to apply the basic TFM into general using is still our future work.

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Appendix

Table 2. Parameters of tubal pipes

Diameter	Thickness	Weight	Thermal expansion coefficient	Young's Modulus	Poisson's ratio	Using length
88.9	6.45	136	0.000011	210	0.3	4220
73.02	5.51	95	0.000011	210	0.3	940

Table 3. Parameters of inclined well

Measured depth	Internal diameter	External diameter
326.96	149.3	177.8
326.96	151.6	177.8
832.54	153.9	177.8
2594.15	153.9	177.8
2638.84	153.9	177.8
4371.5	149.3	177.8
4581.54	149.3	177.8
5160	151.6	177.8

Table 4. Parameters of Azimuth, inclination and vertical depth

Number	Measured depth	Azimuth	Inclination	Vertical depth
1	0	0	244.9	0
2	305.46	0.87	146.62	305.44
3	507.51	0.87	132.82	507.47
4	738.06	0.72	160.51	738
5	940.25	2.08	143.45	940.11
6	1200.23	3.33	148.24	1199.72
7	1517.94	3.85	147.67	1516.83
8	1835.25	5.68	154.45	1832.8
9	2268.3	6.1	177.08	2263.27
10	2643.59	7.76	183.77	2636.24
11	3043.42	21.21	155.02	3021.29
12	3476.18	25.21	159.64	3422.64
13	3908.85	24.67	147.18	3815.82
14	4342.02	30.51	159.3	4193.32
15	4761.6	16.1	210.6	4563
16	5160	13	125	4945.51