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# Efficiency optimization of electric motors: a comparative study of stochastic algorithms

Millie Pant*, Radha Thangaraj, V. P. Singh

Department of Paper Technology, Indian Institute of Technology Roorkee, Saharanpur - 247001, India.

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**Abstract.** This paper presents a comparative study of three popular, population based stochastic algorithms viz. Genetic Algorithms, Particle Swarm Optimization and Differential Evolution for maximizing the efficiency of electric motors. The simulation results for a hypothetical textile mill load diagram show that although all the three algorithms gave more or less similar results in comparison to each other, their performance is better than the traditional techniques.

**Keywords:** genetic algorithms, particle swarm optimization, differential evolution, induction motor, efficiency, textile mill

## 1 Introduction

The Evolutionary computation community has shown a significant interest in optimization for many years[7, 15, 18, 20]. In particular, there has been a focus on global optimization of numerical, real- valued problems for which exact and analytical methods do not apply. Since many general-purpose optimization algorithms have been proposed for finding optimal solutions; notably: Evolution strategies[22], evolutionary programming[5], Genetic algorithms (GA)[11], Particle Swarm Optimization (PSO)[14] and Differential Evolution (DE)[24]. Many efforts have been devoted to compare these algorithms to each other. Typically, such comparisons have been based on artificial numerical benchmark problems. In this study we investigate the performance of PSO, DE and GA on the efficiency optimization of electric motor (induction motor).

Electric motors consume more than 60% of the electric energy used by the industry sector. Therefore, motor energy saving solutions by increasing its efficiency has received considerable attention during the last three decades due to the increase in energy cost[27]. The induction motor (IM), especially the squirrel cage type is widely used in electric drives and is responsible for most of the energy consumed by electric motors[6]. Partial load applied to IM results in lower efficiency and power factor[3]. One example of practical partial load occurrence is spinning drive motor in the textile industry, where shaft load of the motor determines by the quantity of yarn in the spindle.

The efficiency and power factor can be improved by making the motor excitation a monotone increasing function of the load[16]. Many researchers have been reported several strategies using different variables to minimize/maximize losses/efficiency in the IM. Based on the control strategy[10] and the motor parameters[17] available in the literature, this paper compares four stochastic algorithms (SAs) in terms of efficiency optimization of three-phase induction motor for textile mill load diagram.

The remaining of the paper is organized as follows: In Section 2 a brief overview of SAs is presented; Section 3 gives the model of efficiency optimization of IM and the results. Finally the paper concludes with Section 6. Numerical results of some selected unconstrained benchmark problems using the above mentioned algorithms are given in Appendix A.

* Corresponding author. Tel.: +91-9759561464;fax: +91-132-2726456.
*E-mail address*: millifpt@iitr.ernet.in.

## 2 Stochastic search methods for global optimization

The process of finding the maximum or minimum value of an objective subjected to various constraints is known as optimization. The most general non linear optimization problem may be defined as,

$$\text{Minimize } f(X);\ f:\ R_n \rightarrow R \text{ subject to } X \in S \subseteq Rn \tag{1}$$

If $X \in S$ and $\exists$ an $\varepsilon -$ neighbourhood $N\varepsilon(X)$ around $X$ such that $f(X) \geq f(\overline{X})$ for each $X \in S \bigcap N_\varepsilon$ then $(\overline{X})$ is called a local optimal solution. In certain situations of practical interest it is often necessary to obtain the global optimal solution rather than the local optimal solution. There are various possible algorithms for solving nonlinear optimization problems. A comprehensive coverage of these algorithms can be found in Pinter[19], and Torn and Zilinskas[26]. Broadly the global Optimization techniques may be divided into stochastic and deterministic search techniques. Whereas, deterministic techniques depend on the mathematical nature (like differentiability, continuity etc.) of the problem, stochastic techniques are considered to be more user friendly because they do not depend on the mathematical properties of a given function and are hence more appropriate for finding the global optimal solutions for any type of objective function.

For the present study we have chosen three popular population based search techniques viz. Genetic Algorithms (GA), Particle Swarm Optimization (PSO) and Differential Evolution (DE) for maximizing the efficiency of an electric motor. A brief description of these algorithms is given below.

### 2.1 Real coded genetic algorithms

Genetic algorithms are perhaps the most commonly used search techniques for obtaining the global optimal solution of optimization problems. These are based on the principles of natural genetics and natural selection as introduced by Holland[11] and further described by Goldberg[8]. Like RST2, GA maintains a set of candidate solutions $\Omega$. In each generation, a new $\Omega$ is evolved from the old $\Omega$, and as the generation proceeds, the set of solutions in $\Omega$ converges to global minimum. New solution points are generated with the help of selection, crossover and mutation operators. The arithmetic crossover operator was used in the present work[1]. It linearly combines two parent chromosome vectors to produce two new offspring as follows:

$$Offspring\ 1 = a \times Parent\ 1\ +\ (1-a) \times Parent\ 2; \tag{2}$$
$$Offspring\ 2 = (1-a) \times Parent\ 1\ +\ a \times Parent\ 2; \tag{3}$$

where ‘a’ is a random weighting factor which can take a value between - 0.5 and 0.5. The population is updated by including the new offsprings in the population.

### 2.2 Particle swarm optimization

Particle swarm optimization technique is a population based stochastic search technique first suggested by Kennedy and Eberhart in 1995[14]. The mechanism of PSO is inspired from the complex social behavior shown by the natural species. For a $D$-dimensional search space the position of the $i$th particle is represented as $X_i = (x_{i1}, x_{i2}, \ldots, x_{iD})$. Each particle maintains a memory of its previous best position $P_i = (p_{i1}, p_{i2}, \ldots, p_{iD})$ and a velocity $V_i = (v_{i1}, v_{i2}, \ldots, v_{iD})$ along each dimension. At each iteration, the $P$ vector of the particle with best fitness in the local neighborhood, designated $g$, and the $P$ vector of the current particle are combined to adjust the velocity along each dimension and a new position of the particle is determined using that velocity. The two basic equations which govern the working of PSO are that of velocity vector and position vector are given by:

$$v_{id} = \omega v_{id} + c_1 r_1 (p_{id} - x_{id}) + c_1 r_1 (p_{gd} - x_{id}) \tag{4}$$
$$x_{id} = x_{id} + v_{id} \tag{5}$$

The first part of equation (4) represents the inertia of the previous velocity, the second part tells us about the personal thinking of the particle and the third part represents the cooperation among particles and is therefore named as the social component[13]. Acceleration constants $c_1$, $c_2$[4] and inertia weight[23] are predefined by the user and $r_1$, $r_2$ are the uniformly generated random numbers in the range of $[0,\ 1]$.

### 2.3 Differential evolution

Differential Evolution is a simple powerful evolutionary algorithm for global optimization proposed by Storn and Price[24]. It is a population based algorithm like genetic algorithms using the similar operator; crossover, mutation and selection. The main difference in constructing better solutions is that genetic algorithms rely on crossover while DE relies on mutation operator[12]. DE works as follows: First, all individuals are initialized with uniformly distributed random numbers and evaluated using the fitness function provided. Then the following will be executed until maximum number of generation has been reached or an optimum solution is found.

For a $D$-dimensional search space, each target vector $x_{i,g}$, a mutant vector is generated by

$$v_{i,g+1} = x_{r_1,g} + F \times (x_{r_2,g} - x_{r_3}, g) \tag{6}$$

where $r_1, r_2, r_3 \in \{1, 2, \ldots, NP\}$ are randomly chosen integers, must be different from each other and also different from the running index $i$. $F$ $(> 0)$ is a scaling factor which controls the amplification of the differential evolution $(x_{r_2,g} - x_{r_3}, g)$. In order to increase the diversity of the perturbed parameter vectors, crossover is introduced[25]. The parent vector is mixed with the mutated vector to produce a trial vector $u_{ji,g+1}$,

$$u_{ji,g+1} = \begin{cases} v_{ji,g+1} & \text{if} \quad rand_j \le CR \quad \text{or} \quad (j = j_{rand}) \\ x_{ji,g} & \text{if} \quad rand_j > CR \quad \text{or} \quad (j \neq j_{rand}) \end{cases} \tag{7}$$

where $j = 1, 2, \ldots, D$; $rand_j \in [0, 1]$; CR is the crossover constant takes values in the range $[0,\ 1]$ and $j_{rand} \in 1, 2, \ldots, D$ is the randomly chosen index.

Selection is the step to choose the vector between the target vector and the trial vector with the aim of creating an individual for the next generation.

$$x_{i,g+1} = \begin{cases} u_{i,g+1} & \text{if } f(u_{i,g+1}) \le f(x_{i,g}) \\ x_{i,g} & \text{else} \end{cases} \tag{8}$$

## 3 Induction motor efficiency optimization

In general, there are three different approaches to improve the induction motor efficiency especially under light load conditions[10] namely loss model controller (LMC), search controller (SC), and lookup table scheme. This paper considers the loss model controller. In order to illustrate the importance of stochastic algorithms in the industrial process, the practical textile mill load diagram applied to shaft of the IM is considered. Power factor at which maximum efficiency occur in the IM is also observed.

### 3.1 Efficiency calculation

The efficiency of an electric motor represents the machine behavior during the conversion of the electric power into mechanical power[2], and is defined as

$$\eta = \frac{output}{input} = \frac{output}{output + losses}$$

The losses comprises of copper losses in stator and rotor, iron losses due to eddy current and hysteresis, stray losses arise on the copper and iron of the motor, friction losses and finally converter loses due to the resistance offered by the solid state switches. Power output is the product of shaft load and its speed. The induction motor efficiency extracted from per-unit model (which is easy for analysis) and can be minimized by selecting optimal value of slip speed especially light loaded condition. For optimization a good algorithm is needed, which is efficient as well as time effective.

IM efficiency equation presented is given by [10, 17]:

$$\eta = \frac{T_L \times \omega_r}{\left[\begin{array}{l} \left[\left(R_s I_s^2 + R_r' I_r'^2\right) + \left(k_e\left(1+s^2\right)a^2\varphi_m^2\right) + \\ \left(k_h(1+s)a\varphi_m^2\right) + \left(C_{str}\omega_r^2 I_r'^2\right) + \\ \left(C_{fw}\omega_r^2\right) + \left(k_{1conv}I_s^2 + k_{2conv}I_s\right) + \left(T_L \times \omega_r\right)\right] \end{array}\right]} \tag{9}$$

Where,

$T_L$ Load torque
$\omega_r$ Rotor speed
$R_s$ Stator resistance
$R_r'$ Rotor resistance
$s$ Slip
$a$ Supply frequency
$\varphi_m$ Air gap flux
$k_h, k_e$ Eddy current and hysteresis coefficients
$C_{str}$ Stray losses coefficient
$C_{fw}$ Mechanical losses coefficient
$I_s$ Stator current
$I_r'$ Rotor current
$k_{1conv}, k_{2conv}$ Switching coefficients

Equation (9) is treated as an objective function of both the algorithms so that efficiency maximization will be achieved by searching optimum value of slip 's'. Load torque ($T_L$) in pu and rotor speed ($\omega_r$) in pu are given values.

### 3.2 Power factor calculation

Power factor (PF) in alternating circuits is calculated by dividing real power in kilowatts by total power in KVA (PF= KW/KVA). Referring equations presented in [9, 28], PF is given by

$$PF = \frac{k_1 \times k_2 + k_3}{\sqrt{\left(k_2^2 + k_3^2\right)\left(k_1^2 + 1\right)}} \tag{10}$$

$$\text{where, } k_1 = \frac{\omega_s \times X_{rr}}{\omega_b \times R_r} \tag{11}$$

$$k_2 = R_s \times k_1 + \frac{\omega_e}{\omega_b}\left[X'' + \frac{X_m^2}{X_{rr}}\right] \tag{12}$$

$$k_3 = R_s - \frac{\omega_s}{\omega_b}\left(X'' \times k_1\right) \tag{13}$$

$$\text{where } X'' = X_{ls} + \frac{X_m \times X_{lr}}{X_m + X_{lr}} \tag{14}$$

$$\omega_e = \omega_r + \omega_s \tag{15}$$

$$X_{rr} = X_{lr} + X_m \tag{16}$$

$\omega_s$ Supply frequency
$X_{ls}$ Stator leakage reactance
$X_{lr}$ Rotor leakage reactance
$X''$ Subtransient reactance
$\omega_s$ Slip speed
$X_m$ Magnetizing reactance

### 3.3 Textile spinning machine

A ring spinning frame manufactures the cotton into yarn that winded in spindles and used to feed cone winding machine. Constant speed three-phase IM is used in this application, and its shaft load determines by the quantity of yarn in the spindles which varies from zero (when process starts) to full (when process completes). Referring the case-study presented in [21], the average load diagram of the spinning drive motor is shown in Fig. 1. 'T' is the time consumption for completion of one process.

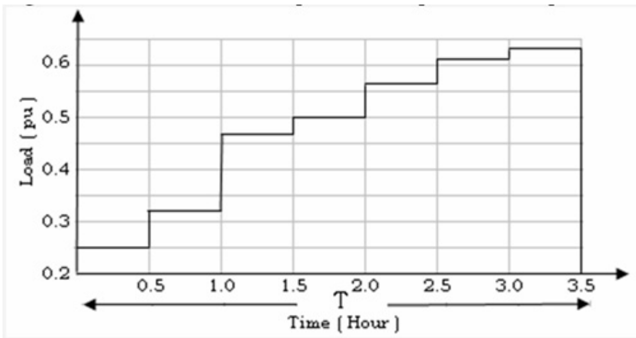


**Fig. 1.** Load diagram of a typical spinning ring frame

### 3.4 Experimental settings

For maximizing steady state efficiency of IM, a 100HP, 400V, 50 Hz motor operating with the given load diagram (Fig. 1) in textile spinning machine is considered for this study. The motor parameters are $R_s = 0.029$, $R_r' = 0.02$, $k_h = 0.006$, $k_e = 0.006$, $C_{str} = 0.025$, $C_{fw} = 0.01$, $X_{lr} = 0.067$, $X_{ls} = 0.059$, $X_m = 1.88$, $k_{1conv} = 3.1307e - 05$, and $k_{2conv} = 0.025$. Remaining parameters can be calculated by using equations given in [14,15].

In order to make good comparison of SAs we fixed the same seed for random number generation so that the initial population is same for all the four algorithms. The number of particles in the swarm (swarm size) is 30. For PSO, a linearly decreasing inertia weight is used which starts at 0.9 and end at 0.4, with the user defined parameters $c_1 = 2.0$ and $c_2 = 2.0$. In DE, the crossover constant $CR$ is set as 0.5 and the scaling parameter $F$ is 0.5. In Genetic algorithms Roulette wheel selection is used and probability of crossover and mutations are taken as 0.7 and 0.9 respectively.

### 3.5 Simulation results

For the comparison of stochastic algorithms with conventional electrical controller, constant voltage/frequency (constant flux) controller is considered in this paper. Tab. 1 show the results of both the algorithms in terms of efficiency and powerfactor for the given load diagram. Tab. 1 shows the generation at which the maximum efficiency is occurred for all the algorithms. Efficiency improvement (%) of stochastic algorithms in comparison with Constant V/f controller is shown in Tab. 3. Fig. 1 shows the generation comparison of SAs. Fig. 2, 3 shows the variations of efficiency and power factor with respect to the wide range of load torque. Table 4 and 5 gives the results for different load torque and rotor speed.

## 4 Discussions and conclusions

For the present study, all the three algorithms were implemented using Turbo C++ on a PC compatible with Pentium IV, a 3.2 GHz processor and 2 GB of RAM. Since all the three algorithms are stochastic in nature, more than one execution is needed to reach to a solution. A maximum of 50 iterations was fixed for all the three algorithms; In terms of number of generations DE gave a better performance in comparison to GA and PSO.

From the numerical results none of the algorithms can be claimed superior to the other as all of them gave more or less similar results, which are better than the results obtained by the traditional method, particularly

**Table 1.** Comparison of Efficiency and Power factor for the given load diagram

| Te (pu) | Constant V/f | | Stochastic Algorithms | |
|---|---|---|---|---|
| | Efficiency | PF | Efficiency | PF |
| 0.25 | 0.831018 | 0.423261 | 0.866565 | 0.646295 |
| 0.32 | 0.853864 | 0.507883 | 0.875106 | 0.673638 |
| 0.46 | 0.875685 | 0.635822 | 0.882818 | 0.71969 |
| 0.5 | 0.878815 | 0.663791 | 0.883809 | 0.730742 |
| 0.56 | 0.881984 | 0.699962 | 0.884698 | 0.745777 |
| 0.61 | 0.883563 | 0.725534 | 0.885023 | 0.757034 |
| 0.63 | 0.88398 | 0.734747 | 0.885068 | 0.761243 |

**Table 2.** Number of iterations for finding the maximum efficiency of all algorithms

| Te | PSO | DE | GA |
|---|---|---|---|
| 0.25 | 26 | 14 | 28 |
| 0.32 | 31 | 11 | 32 |
| 0.46 | 33 | 10 | 37 |
| 0.5 | 20 | 11 | 30 |
| 0.56 | 22 | 9 | 28 |
| 0.61 | 33 | 11 | 35 |
| 0.63 | 31 | 8 | 33 |

**Table 3.** Efficiency improvement (%) of stochastic algorithms in comparison with V/f controller

| Te | Improvement (%) | Te | Improvement (%) |
|---|---|---|---|
| 0.25 | 4.277525 | 0.56 | 0.307715 |
| 0.32 | 2.487745 | 0.61 | 0.165240 |
| 0.46 | 0.814562 | 0.63 | 0.123079 |
| 0.5 | 0.568265 | | |

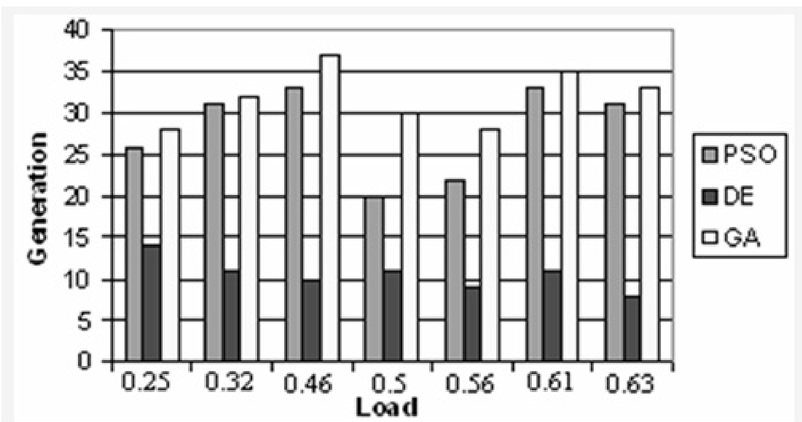


**Fig. 2.** Generation comparison of SAs for efficiency optimization

for light loads which are used more in industrial drives like spinning drives, mine hoist, etc. Thus it can be concluded that in comparison to traditional techniques stochastic algorithms can be used for optimizing the efficiency of electric motors and will help in the conservation of electrical energy.

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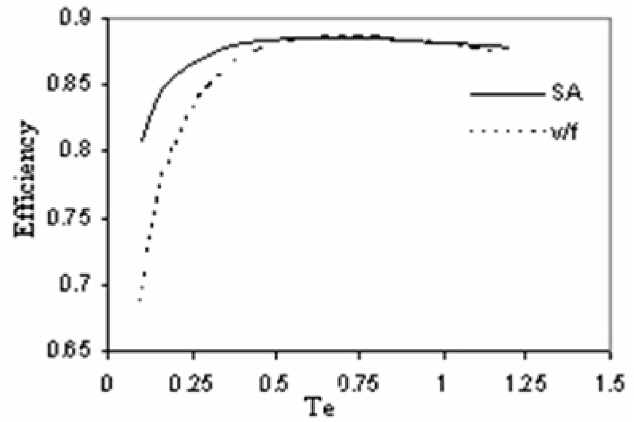


**Fig. 3.** Efficiency curves for SA and V/f controller

**Table 4.** Comparison of Efficiency and Power factor under different load ($\omega_r = 0.2\ pu$)

| Te | V/f | | SA | |
|---|---|---|---|---|
| | Efficiency | PF | Efficiency | PF |
| 0.2 | 0.593171 | 0.406376 | 0.703554 | 0.625513 |
| 0.4 | 0.687997 | 0.631149 | 0.716491 | 0.730246 |
| 0.6 | 0.704505 | 0.757731 | 0.709497 | 0.790725 |
| 0.8 | 0.698104 | 0.827095 | 0.698115 | 0.828284 |
| 1.0 | 0.683411 | 0.86645 | 0.685618 | 0.853201 |

**Table 5.** Comparison of Efficiency and Power factor under different load ($\omega_r = 1\ pu$)

| Te | V/f | | SA | |
|---|---|---|---|---|
| | Efficiency | PF | Efficiency | PF |
| 0.2 | 0.804529 | 0.353682 | 0.856396 | 0.62577 |
| 0.4 | 0.868887 | 0.587172 | 0.88051 | 0.701413 |
| 0.6 | 0.883312 | 0.720719 | 0.884983 | 0.754869 |
| 0.8 | 0.884065 | 0.794713 | 0.884093 | 0.791324 |
| 1.0 | 0.879421 | 0.836861 | 0.88108 | 0.816878 |

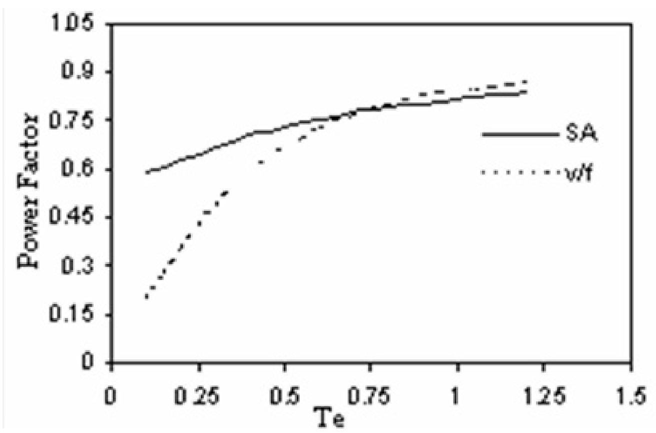


**Fig. 4.** Power Factor curves for SA and V/f controller

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**Appendix A:** Comparison of GA, PSO and DE on Some Selected Benchmark Problems for Global optimization

Tabl. 6, 7 shows the numerical bench mark problems and their results respectively.

**Table 6.** Numerical benchmark problems

| Function | Range | Optimum |
|---|---|---|
| $f_1(x) = x_1^2 + x_2^2$ | $[-5.12,\ 5.12]$ | 0 |
| $f_2(x) = 20 + x_1^2 + x_2^2 - 10(\cos(2\pi x_1) + \cos(a\pi x_2))$ | $[-5.12,\ 5.12]$ | 0 |
| $f_3(x) = \frac{1}{4000}\sum_{i=1}^{2} x_i^2 + \prod_{i=1}^{2}\cos(\frac{x_i}{\sqrt{i}}) + 1$ | $[-600,\ 600]$ | 0 |
| $f_4(x) = 100(x_2 - x_1^2)^2 + (x_1 - 1)^2$ | $[-30,\ 30]$ | 0 |

**Table 7.** Numerical results

| Function | PSO | DE | GA |
|---|---|---|---|
| F1 | 1.667377e-06 | 1.387779e-17 | 1.269822e-06 |
| F2 | 1.181916e-10 | 1.309548e-23 | 1.154321e-12 |
| F3 | 3.007688e-05 | 3.359894e-12 | 2.887793e-05 |
| F4 | 5.634183e-05 | 2.108665e-09 | 4.876983e-06 |