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Optimal environmental charges under imperfect compliance

Dajin Liu[1] , Ya Wang[2*]

Taizhou Institute of Science and Technology, Nanjing university of Science and Technology, Taizhou 225300, P. R. China

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Abstract. This paper proposes a modeling framework for the design of optimal environmental charges, in an environmental management problem implicating firms with different characteristics decide on both the level of emissions and their reports. There is an enforcement agency whose objective is to control pollutions under desired levels at receptors. We show that optimal taxes can be achieved even if firms' reports are not the same as their own emissions, but if there are firms hard to monitor, taxes should be higher than firms' marginal revenues. Moreover, the optimal taxes can be figured out under complete information.

Keywords: environmental charging, mathematical modelling, monitor policy

1 Introduction

In recent years, environmental protection has been put more and more importance in many countries, and become a challenge for their enforcement agencies. Pollution should be controlled under desired levels at all receptors and revenue from economy of the whole society should not smaller than strictly necessary. Taxes and standards are the common policy instruments to regulate the environmental quality. And in traditional approach these policies are achieved by assuming that polluters comply with the environmental regulation [1–3]. However, this is not guaranteed.

In this paper, we take this situation into consideration, and extend traditional models by studying the optimal taxes under the situation where firms try to evade the taxes by providing false information of their pollution emission to maximize their profits, the total of which is assumed as revenue of the whole society. The hypothesis of firms' imperfect compliance was studied by Sandmo[4] and Inés and David[5]. Sandmo explored the conditions under which the efficiency property of taxes continues to hold under imperfect compliance. Based on it, Inés and David[5] extend his model. They suppose that firms' behaviors are characterized by both the productivity and evasion possibilities, and studied two optimal environmental policies in and out of government's budget. By learning firms' behaviors, we build the model to design taxes under the assumption that government's aim is to control pollution but taxing, and we do not consider its budget.

Following the model built by Baumol and Oates[1], we study the optimal emission taxes under one receptor, at which the pollution should be controlled under desired level. And on the framework of modelling by Tietenberg[2, 3], optimal charges under a pollution control system of multi-receptor are analyzed. Moreover, we also study the model under stochastic condition, in which the monitor probability for each firm is not a constant but a stochastic variable satisfying certain distribution. With complete information on pollution benefit function, monitored probability and penalty function for every firm, even if firms are trying to evade their costs of pollution, optimal environmental and emission taxes can be achieved. The more they evade, the higher taxes, which may be more than their marginal revenues, will be enforced. However, in multi-receptor system, it is possible that environmental taxes will be higher, even if all firms hold the same ability to evade from monitoring.

* Corresponding author. wangya@mail.sdu.edu.cn.

The paper is organized as follows: in section 2, we present the theory of firms' incomplete compliance and extend a new corollary for the proposition given by Inés and David[5]. Section 3 develops traditional models with one receptor and multi-receptor. Then, we extend the model with the analysis of stochastic situation. And numerical examples are present in section 4, to illustrate how the adjustment procedure functions to achieve the optimal taxes, and conclusion is in section 5.

2 Firms' behaviors

In this section, we present firms' behaviors studied by Inés and David[5], and give a new corollary of their proposition, which is necessary for the modelling of optimal taxes.

We suppose that the firm chooses the level of emission e, where $e \in [0, E]$. Hence, E is the emission level of the firm when pollution is free. If pollution at receptor is under control, even when all firms emit freely, no taxes are needed. So in the following study, we do not take this situation into consideration. To control pollution, Emissions are taxed at rate t: We suppose that t is exogenously given; it is set by the enforcement agency. The firm's benefit function from pollution emission e is represented as $g(e)$. Here $g(\cdot)$ is increasing and concave: $g'(e) > 0$ and $g''(e) < 0$ for all $e \in (0, E)$. Also, as the framework used by Sandmo[4], $g'(0) = +\infty$ and $g'(E) = 0$. This is possible in reality: a small level of emission has a big marginal impact on the firm's profits, while marginal profits at very high emission levels are very small. If the firm's emission level is not perfectly monitored, and the firm tries its best to evade emission tax, then the situations for the setting of environmental taxes may change. We denote by ρ the probability that the enforcement agency will monitor and identify firm's true emission. Different from Inés and David[5], in this paper, ρ is the only necessary probability that an evader is caught. Since the main purpose of this paper is to study setting the environmental tax, not firm' behavior, the probability that the difficulty in detecting a violation or finding strong evidence that allows the sanctioning of firms is not considered.

Just like Inés and David[5], let z be the reported emission level for the firm. Since it will report no more than its true emission level e, then we have $z \leq e$. And a penalty $\Theta(\cdot)$ will be imposed to those whose reports are not identified with their emissions. Here $\Theta(\cdot)$ is increasing and convex in the level of evasion: $\Theta(0) = 0, \Theta'(x) > 0$, and $\Theta''(x) > 0$ for $x > 0$. Therefor, the expected profit of the firm with monitor probability ρ, when it chooses an emission level e and report z, can be written as:

$$E\Pi(e,t) = g(e) - tz - \rho t[e - z] - \rho\Theta(e - z)$$

To maximize the expected profit, the firm will choose the optimal emission level e and report z under the proposition given by Inés and David[5]:

(a) If $\rho = 0$, then $e = E$ and $z = 0$.

(b) If $\rho \in (0, \frac{t}{\Theta'(e^*)+t})$, where e^* is the optimal emission level for the firm when it do not evade the tax, then $e \in (e^*, E)$ as defined by following equation, and $z = 0$, with: $g'(e) - \rho t - \rho\Theta'(e) = 0$.

(c) If $\rho \in [\frac{t}{\Theta'(e^*)+t}, \frac{t}{\Theta'(0)+t})$, then $e = e^*$ and $z \in [0, e^*)$ as defined by following equation: $[1 - \rho]t = \rho\Theta'(e^* - z)$.

(d) If $\rho \geq \frac{t}{\Theta'(0)+t}$, then $e = e^*$ and $z = e^*$.

Definition 1. $\hat{\rho} = \frac{t}{t+\Theta'(e^*)}$ *and* $\hat{\rho'} = \frac{t}{t+\Theta'(0)}$.

Evidently, when (a): $\rho \leq \hat{\rho}$, then the firm will report 0, and emission more than traditional optimal emission; when (b): $\hat{\rho} \leq \rho \leq \hat{\rho'}$, then firm will report less than true emission level, but emit the same as in traditional model; when (c): $\rho \geq \hat{\rho'}$, it will perfectly comply.

Based on the above hypothesis and Inés and David' study, we get the following conclusion:

Corollary 1. *$\hat{\rho}$ and $\hat{\rho'}$ are increasing in t.*

Proof. We check the first order condition:

$$\frac{\partial\,\hat{\rho}}{\partial\,t} = \frac{\Theta'(e^*) - t\Theta''(e^*)e^{*\prime}}{(t + \Theta'(e^*))^2} \qquad \frac{\partial\,\hat{\rho'}}{\partial\,t} = \frac{\Theta'(0)}{(\Theta'(0) + t)^2}$$

Evidently, $\frac{\partial \hat{\rho}'}{\partial t} > 0$. Since $e^* = \arg g'(t)$ and $g(\cdot)$ is concave, then $e^*(t)$ is decreasing with t, $\frac{\partial \hat{\rho}}{\partial t} > 0$.

So with the increasing of t, firms' compliance decrease.

3 Mathematical model and analysis

In this section, we extend traditional models by considering firms' imperfect compliance decision. Models in three kinds of situations are studied: when the pollution control system contains one receptor, multi-receptor and when the monitor probability is not a constant for certain firm but satisfying some distribution.

3.1 One receptor

In this section, when there is only one receptor, we first analyze an ideal model, of which the characteristics of all firms are the same, such as their benefit functions and the government's monitor probability for each them. And then we generalize this model, so that all the characteristics of firms are different. In both of the models, we do not take into consideration of firms' different contributions to the pollution concentration of this receptor, which is similar as the situation of multi-receptor, and we will study it in next section.

We suppose that the government's aim is to maximize the total economy revenue of the whole society, which in this paper is considered as the total revenue from all the firms, and control pollution in each receptor. Let $i = 1, ..., n$ be firms as sources of emission. Firm i chooses the level of emission e_i, where $e_i \in [0, E_i]$. Each firm's benefit function from pollution emission e are represented as $g_i(e)$.

So, to make the pollution under desired level at the receptor in an effective way, we have the following model:

$$Max \quad \sum_{i=1}^{n} g_i(e)$$
$$\text{s.t.} \quad \sum_{i=1}^{n} e_i \leq Q^*$$

Where, Q^* is the limitation of pollution in this receptor.

Since in an ideal model, all firms' characteristics are the same, then $g_1(e_1) = g_2(e_2) = \ldots = g_n(e_n) = g(e)$, $\rho_1 = \rho_2 = \ldots = \rho_n = \rho$ and $E_1 = E_2 = \ldots = E_n = E$. Make t_i the emission tax for firm i.

Following the four situations of Inés and David[5], it is evident that:

In situation (c) and (d), when $\rho > \hat{\rho}$, firms may report less than their true emissions, environmental condition is under control even using the traditional model. So, $t = t_i = g'(e) = \tau$. Here τ is Lagrangian factor, and thought as the environmental tax in multi-receptor system. Since the purpose of environmental agency is to protect environment but to tax, I suppose in the process of setting environmental tax, compliance will not be considered and the desired environmental quality is the only aim of controlling pollution for government. So, we just need to discuss the first two situations.

In situation (b), agency's problem can be written as:

$$Max \quad \sum_{i=1}^{n} g_i(e) = ng(e)$$
$$\text{s.t.} \quad \begin{cases} ne \leq Q^* \\ g'(e) - \rho t - \rho \Theta'(e) = 0 \\ 0 \leq e \leq E \end{cases}$$

Because $g(e)$ is increasing with e, when $Q^* < nQ$, by K-T condition, we can get the optimal tax as:

$$\tau = t = \frac{g'(Q^*/n) - \rho \Theta'(Q^*/n)}{\rho}$$

This is different from traditional model, of which $t = t_i = g'(Q^*/n) = \tau$.

Compared with traditional model, in this situation, the optimal environmental tax and emission tax are not only relative to firms' benefit functions but also government's monitor probability and penalty function for evasion. We draw the following conclusion:

Proposition 1. *When $0 < \rho < \hat{\rho}$, emission tax will more than firms' marginal benefit, so that the pollution at this receptor is under control.*

Proof. Suppose t_0 and τ_0 are the optimal emission tax and environmental tax respectively in traditional model, then $g'(e^*)\tau_0 = t_0$.

Because $\rho \in (0, \frac{\tau_0}{\Theta'(e^*)+\tau_0})$, then $\tau = t = [g'(e^*) - \rho\Theta'(e^*)]/\rho$, so $\tau = t > [\tau_0(\Theta'(e^*) + \tau_0)]/\tau_0 - \Theta'(e^*) = \tau_0$

That is when there is firm whose monitor probability is not large enough, to control the environment quality, the environmental tax will be larger than firm's marginal revenue (MR).

In situation (a),when there is not any monitor at all ($\rho = 0$), firms' emissions are the same as a constant E, independent of emission tax. This means no matter what the environmental tax it is, firm will emit as more as they can and report 0 to get the largest profit. In such a situation, the strategy of setting environmental tax means nothing. It needs more strategy to control the environment. So, in the following chapters, I do not take this situation, where there is such firm, into consideration.

When all firms' characteristics are different from each other, their behaviors will be different because $\hat{\rho}_i$ for each of them are not the same, and their emissions will vary.

In this situation, to design the optimal emission and environmental tax, government should first consider firms' behavior.

From Inés and David[5], we know when government's monitor probability for firm i satisfies: $\rho_i < \hat{\rho}_i$, firms' emission will larger than e_i^*, that is:

$$e_i = \begin{cases} e^* & \rho_i \geq \hat{\rho} \\ e_i^{**} & \rho_i < \hat{\rho} \end{cases}$$

Here, e_i^{**} satisfies the function $g'(e_i^{**}(\rho_i)) - \rho_i t - \rho_i \Theta(\rho_i) = 0$, $\rho_i \in (0, \hat{\rho}_i]$, and $e_i^{**}(0) = E$. e_i^* subject to $g_i'(e^*) = t$, and$\hat{\rho}_i$ satisfies $\hat{\rho}_i = \frac{t}{t+\Theta'(e_i^*)}$.

When, the emission tax is designed as t, the total concentrate of pollution at the receptor is:

$$e^R = (\sum_{\rho_i<\hat{\rho}_i} e_i^{**} + \sum_{\rho_i>\hat{\rho}_i} e_i^*)$$

The first term illustrates the total pollution of those who will pollute more than in traditional model. And the second represents those who will not pollute more but may also evade tax. So the government's problem can be described as:

$$\begin{aligned} Max \quad & \sum_{i=1}^{n} g(e_i) \\ \text{s.t.} \quad & e^R \leq Q^* \end{aligned}$$

Because t is increasing with firms' emission e_i, to solve this problem, we can also learn from K-T condition that the optimal tax satisfies the following functions:

$$\begin{cases} \sum_{\rho_i<\hat{\rho}} e_i^{**} + \sum_{\rho_i>\hat{\rho}} e^* = Q^* \\ g'(e_i^{**}(\rho_i)) - \rho_i t - \rho_i \Theta'(e_i^{**}(\rho_i)) = 0 \\ g'(e^*) = t = \tau \\ \hat{\rho} = t/[\Theta'(e^*) + t] \\ e_i^{**}(0) = E_i \end{cases}$$

This model improves the first ideal model and we achieve the extension of proposition 1:

Proposition 2. *When there exists firms whose monitor probabilities are not large enough, to control the environment quality, the optimal tax is larger than that in traditional model.*

Proof. Since in the first term of total emission, firms' true emissions satisfy the function $g'(e_i^{**}(\rho_i)) - \rho_i t - \rho_i \Theta'(e_i^{**}(\rho_i)) = 0$, $e_i^{**} > e_i^*$, and $e^R > \sum e^*$ under the same tax rate t. Because $\partial\, e/\partial\, \tau < 0$, then we get the proposition.

3.2 Multi-receptor

In this section, we study a system of multi-receptor. Generally, for better monitor the environmental quality, environmental enforcement always set more than one receptor to monitor the environmental quality. So that government's problem is to find a series of environmental taxes at these receptors and make the balance of revenue and environment.

Let m be the number of receptors$j = 1, ..., m$ represent each receptor. Then the problem for environmental enforcement is:

$$\begin{aligned} & Max \quad \sum_{i=1}^{n} g_i(e_i) \\ & \text{s.t.} \quad \sum_{i=1}^{n} \alpha_{i,j} e_i \le Q_j^* \ (j = 1, ..., m) \end{aligned}$$

Here, Q_j^* is the upper limit of pollution level at receptor j, $\alpha_{i,j}$ is firm i's emission transfer coefficient to receptor j. In traditional model, $g_i'(e_i) = t_i = \sum_{j=1}^m \lambda_j \alpha_{i,j}$. Here, λ_j is the shadow price at receptor j. Let $\tau_j = \lambda_j$, then λ_j is the environmental tax at receptor j, which is relative to the emission taxes for firms.

When all firms are trying to evade tax, their total emission at receptor j will be:

$$e_j^R = \sum_{\rho < \hat{\rho}_{\tau,i}} \alpha_{i,j} e_i^{**} + \sum_{\rho \ge \hat{\rho}_{\tau,i}} \alpha_{i,j} e_i^*$$

Here, $\hat{\rho}_{\tau,i} = [\sum_j \tau_j \alpha_{i,j}]/[\Theta'(e_i^*) + \sum_j \tau_j \alpha_{i,j}]$, and e_i^*, e_i^{**} are similar as discussed above, subject to the following functions respectively: $g_i'(e_i^*) = t_i = \sum_{j=1}^m \alpha_{i,j}\tau_j$ and $g_i'(e_i^{**}) - \sum_{j=1}^m \alpha_{i,j}\tau_j\rho_i - \rho_i\Theta'(e_i^{**}) = 0$.

Then the problem for the government is:

$$\begin{aligned} & Max \quad \sum_{i=1}^{n} g_i(e) \\ & \text{s.t.} \quad e_j^R \le Q_j^* \quad j = 1, \ldots, m \end{aligned}$$

This problem can be solved by iteration problem studied by Y. Ermoliev, G. Klaassen and A. Nentjes[7, 8].

Let $\tau^0 = (\tau_1^0, \ldots, \tau_m^0)$ be the initial vector of environmental taxes at m receptors, and $\tau^k = (\tau_1^k, \ldots, \tau_m^k)$ be the vector of environmental taxes at m receptors in step k iteration. Firm i will change its emission e_i^k according to environmental τ^k and emission tax t^k, to achieve its expected profit:

$$E\Pi_i(e,t) = g_i(e_i) - t_i z_i - \rho_i t_i [e_i - z_i] - \rho_i \Theta(e_i - z_i)$$

Because the government know polluters' information completely, they can adjust the tax according to the following iteration, by calculating their true emissions.

$$\tau_j^{k+1} = max\{0,\ \tau_j^k + \gamma_k (\sum_{i=1}^n e_i^k \alpha_{i,j} - Q_j^*)\}$$

In which,

$$e_i^k = \begin{cases} e_i^{k*}, & \rho_i \ge \hat{\rho}_i^k \\ e_i^{k**}, & \rho_i < \hat{\rho}_i^k \end{cases}$$

Here, $\hat{\rho}_i^k = [\sum_j \tau_j^k \alpha_{i,j}]/[\Theta'(e_i^{k*}) + \sum_j \tau_j^k \alpha_{i,j}]$, and e^{k*} and e^{k**} satisfy respectively the function: $g_i'(e_i^{k*}) = t_i^k = \sum_{j=1}^m \alpha_{i,j}\tau_j^k$ and $g_i'(e_i^{k**}) - \sum_{j=1}^m \alpha_{i,j}\tau_j^k \rho_i - \rho_i \Theta'(e_i^{k**}) = 0$. γ_k is the step size of iteration factor, satisfying $\gamma_k > 0$. Evidently, when $\tau_j^k + \gamma_k(\sum_{i=1}^n e_i^k \alpha_{i,j} - Q_j^*) < 0$, there is $\tau_j^{k+1} = 0$.

The the convergence of vector $\tau^k = (\tau_1^k, \ldots, \tau_m^k)k \to \infty$ is relative to γ_k. Learning from [7], when series $\{\gamma_k\}$ satisfying $\gamma_k > 0$, $\sum_{k=0}^\infty \gamma_k = \infty$ and $\sum_{k=0}^\infty \gamma_k^2 < \infty$, this method is effective.

3.3 Improvement for monitor probabilities

The above model is built on the assumption that the monitor probability for each firm is different from each other but constants. In this section, we extend the above model by considering that the probabilities are not constant, but satisfying the same stochastic distribution.

Suppose the government's monitor probabilities for each firm is stochastic and independent. They subject to the same distribution. Because these firms are under the same government, the assumption of the distribution is reasonable. Other conditions are the same as studied before.

Let x_i be the stochastic variable of monitor probability of firm i, and $f(x_i)$ be the density function. It is evident that $f(x_i) = \rho_i$, and let $F(x_i)$ be the distribution function.

Let $x_i \in [x_i', x_i''] = \sigma_i$ be the domain of variable x_i, when $\rho_i \leq \hat{\rho}_i$. Therefore, when firm i, $i = 1, \ldots, n$ trying to evade tax, their expectation emission is:

$$Ee_i = \int_{x_i'}^{x_i''} e_i^{**}(x) f(x) \mathrm{d}x + [1 - F(\sigma_i)] e_i^*$$

That is:

$$Ee_i = \int_0^{\hat{\rho}_i} e_i^{**}(\rho_i) \rho_i \mathrm{d}\rho_i + [1 - F(\sigma_i)] e_i^*$$

In this situation, the problem for government is:

$$\begin{aligned} &Max \quad \sum_{i=1}^n g_i(E(e_i)) \\ &\text{s.t.} \quad \sum_{i=1}^n \alpha_{i,j} E(e_i) \leq Q_j^* \quad j = 1, \ldots, m \end{aligned}$$

To solve this problem, we need to know the relationship between firms' expected emission Ee_i, and government's tax t. We have the following conclusion:

Proposition 3. *Under model's presupposition, the expectation of firm's true emission is decreasing with emission tax, although he is trying to evade tax.*

Proof. Assume there are two standard of emission tax: t_1 t_2, and $t_1 < t_2$.

Then $Ee_1 = \int_0^{\hat{\rho}_1} e_1^{**}(\rho)\rho \mathrm{d}\rho + [1 - F(\sigma_1)]e_1^*$, and $Ee_2 = \int_0^{\hat{\rho}_2} e_2^{**}(\rho)\rho \mathrm{d}\rho + [1 - F(\sigma_2)]e_2^*$.

Because $\hat{\rho} = \frac{t}{\Theta'(e^*)+t}$ is increasing with t, $\hat{\rho}_1 < \hat{\rho}_2$. So $\sigma_1 < \sigma_2$, then Ee_1 and Ee_2 can be written in the following forms respectively.

$Ee_1 = \int_0^{\hat{\rho}_1} e_1^{**}(\rho)\rho \mathrm{d}\rho + \int_{\hat{\rho}_1}^{\hat{\rho}_2} e_1^*(\rho)\rho \mathrm{d}\rho + [1 - F(\sigma_2)]e_1^*$.

$Ee_2 = \int_0^{\hat{\rho}_1} e_2^{**}(\rho)\rho \mathrm{d}\rho + \int_{\hat{\rho}_1}^{\hat{\rho}_2} e_2^{**}(\rho)\rho \mathrm{d}\rho + [1 - F(\sigma_2)]e_2^*$.

So $Ee_1 - Ee_2 = \int_0^{\hat{\rho}_1} (e_1^{**} - e_2^{**})\rho \mathrm{d}\rho + \int_{\hat{\rho}_1}^{\hat{\rho}_2} (e_1^* - e_2^{**})\rho \mathrm{d}\rho + [1 - F(\sigma_2)](e_1^* - e_2^*)$.

Let the three terms of the above function be $A1$, $A2$, $A3$. Evidently $A1 > 0$ and $A3 > 0$. Because firms' emission is decreasing with tax, when the monitor probability is certain, we know when ρ is fixed, $(e_1^* - e_2^{**}) > 0$, then $A2 > 0$, hence $Ee_1 > Ee_2$.

According to this proposition, to design the optimal tax, iteration method can also be used. The initial vector of environmental taxes at receptors is $\tau^0 = (\tau_1^0, \ldots, \tau_m^0)$, and the adjustment procedure follows:

$$\tau_j^{k+1} = max\{0,\ \tau_j^k + \gamma_k(\sum_{i=1}^{n} E(e_i^k)\alpha_{i,j} - Q_j^*)\}$$

Here, $\{\gamma_k\}$ still satisfies the conditions: $\gamma_k > 0$, $\sum_{k=0}^{\infty}\gamma_k = \infty$ and $\sum_{k=0}^{\infty}\gamma_k^2 < \infty$, to guarantee the convergency of iteration.

4 Numerical analysis

In this section, we give two numerical examples to illustrate models given above, and to compare our models and traditional model. There are two parts of this section. The first part gives optimal tax under traditional model, of which there is no evasion by firms, and we get the critical value of government monitor probability $\hat{\rho}_i$ for each firm. In second part, we make a numerical analysis of optimal tax, when firms are trying to evade tax.

4.1 A numerical example of traditional model

We consider such situation that there are 5 firms who emit in a district. Their upper limits of emission are as follows:

E_1	E_2	E_3	E_4	E_5
6.5	6.0	5.4	5.3	7.0

The benefit functions of these firm are $g(e_i) = a_i[\frac{1}{e_i} - \frac{1}{Q_i}] \quad i = 1, \ldots, 5$
Here a_i stand for

a_1	a_2	a_3	a_4	a_5
4.0	5.0	4.5	6.0	5.2

Four receptors are set in this district to control pollution. Here we just discuss point pollution. The upper limits of pollution at each receptor are:

Q_1^*	Q_2^*	Q_3^*	Q_4^*
5.0	5.0	5.6	5.1

And the transfer coefficient $\alpha_{i,j}$ satisfy:

$\alpha_{i,j}$	$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$
$j = 1$	0.7948	0.7568	0.5226	0.7801	0.1730
$j = 2$	0.8797	0.2714	0.2523	0.8757	0.7373
$j = 3$	0.1365	0.2318	0.8939	0.1991	0.5987
$j = 4$	0.6614	0.2844	0.4692	0.3648	0.8883

Lack of real data, all the data above are chosen randomly by computer in a reasonable range. We achieve emission taxes for firm i ($i = 1, \ldots, 5$):

t_1	t_2	t_3	t_4	t_5
2.9022	1.7650	1.3332	2.8701	1.5941

Then the optimal emission for each firm are:

e_1^*	e_2^*	e_3^*	e_4^*	e_5^*
1.1371	1.9243	2.0770	1.4992	2.2251

Suppose the differential function of government's penalty function for firms' evasion is $\Theta'(e - z) = 2[e - z]$.

Then we have the critical value of $\hat{\rho}_i(e_i^*)$:

$\hat{\rho}_1$	$\hat{\rho}_2$	$\hat{\rho}_3$	$\hat{\rho}_4$	$\hat{\rho}_5$
0.5605	0.3144	0.2429	0.4889	0.2637

When $\rho_i \geq \hat{\rho}_i$, firms may be still not compliance with taxing policy, but their emissions are under control. In the following, we illustrate our model by the result above.

4.2 Numerical analysis of our model

In this section, we use data to illustrate the optimal tax when firms are trying to evade taxes.

First, we suppose the monitor probabilities for each firm are:

ρ_1	ρ_2	ρ_3	ρ_4	ρ_5
0.3	0.4	0.45	0.42	0.5

That is $\rho_i < \hat{\rho}_i,\ i = 1, 4$, and $\rho_i > \hat{\rho}_i,\ i = 2, 3, 5$.

We achieve emission taxes for firm i $(i = 1, \dots, 5)$:

t_1	t_2	t_3	t_4	t_5
16.1374	12.6359	9.0384	15.8981	6.1441

Then the marginal revenue of each firm are:

$e_1^{*'}$	$e_2^{*'}$	$e_3^{*'}$	$e_4^{*'}$	$e_5^{*'}$
0.2388	0.3712	0.4558	0.3523	0.7550

And firms' true emissions e_i^* are as follows:

e_1^*	e_2^*	e_3^*	e_4^*	e_5^*
1.5148	1.6486	1.7400	1.6968	1.7685

Compared with traditional model, when monitor probabilities for some firms are not large enough, to control the pollution, emission tax need to be much larger than that in traditional model, and in this situation, more firms choose to evade tax. In this example, all firms' true emissions satisfy $e_i < e_i^{*'}$, that is $z_i = 0\ i = 1, \dots, 5$, and their reports are all 0. This proves the theories studied above. Compared with the firms' emission, we find that the emissions of firm 1 and 4 increase, and others' decrease. That is, when monitor probabilities are not large enough, government's policy tend to reduce the firms' pollution, who is easy to monitor. This is similar as the result in [5].

5 Summary and conclusions

This paper is studied on the basis of firms' behaviors to tax and monitor policy, and extend traditional model with one receptor and multi-receptor. Our models are built on the assumption that firms are trying to evade taxes and make false reports if it is possible. This is more reasonable than traditional models. Mathematical optimal models are built, and adjustment procedure method is used to solve these optimal program. New conclusions on the relationship of firms' emission and emission tax are drawn. In section 4, numerical examples are given to illustrate our theories.

The assumption of this paper is on the condition of complete information, however, some time, government can not know perfectly the benefit functions of firms. In such situation, how to design the optimal tax still needs to be further studied.

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