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Optimization of infiltration parameters in hydrology

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Abstract. The movement of water into the soil under specific conditions is called infiltration. The study of infiltration is one of the most important necessities in irrigation and drainage engineering. Two popular infiltration models called Kostiakov and Modified Kostiakov, which are largely used in the fields, are considered for analysis. The problem is modeled as a least sum of squares of the differences in observed infiltration rates and computed infiltration rates over a specified time interval. In this paper, six optimization approaches are implemented to solve the models. They are (i) software Language for INteractive General Optimization (LINGO); (ii) Binary Genetic Algorithm (BGA); (iii) Hybrid Binary Genetic Algorithm (HBGA); (iv) Real Coded Genetic Algorithm using Laplace Crossover and Power Mutation (LX-PM); (v) its Hybrid version (H-LX-PM); and (vi) Random Search Technique (RST). The comparative performance of these methods is displayed for obtaining the infiltration parameters for the datasets (two each) of popular soils, namely Plain Field Sand (PFS) and Columbia Sandy Loam (CSL). To test model performance, the Nash-Sutcliffe efficiency is employed in this study.

Keywords: infiltration parameters, least square approach, Nash-sutcliffe efficiency, binary GA, real coded GA

1 Introduction

Infiltration is the movement of water into the soil under specific conditions. The actual rate at which water enters into the soil at any given time is termed as the infiltration rate and it has the dimensions of velocity. Infiltration plays a fundamental role in surface and sub-surface hydrology as well as in irrigation and drainage engineering. The rate of infiltration at a time describes the capacity of a soil to absorb water. Its estimation enables determination of soil moisture status, which is of interest to soil scientists, agricultural and irrigation engineers, and ground water hydrologists. After accounting for the losses, in which infiltration may significantly depend on soil characteristics, the remaining rainfall-excess leads to runoff generation, which is of interest to surface water hydrologists. It is due to these and many other reasons that the infiltration has received a great deal of attention from soil and water scientists, and large number of models for computation of infiltration have been developed. A robust infiltration model, predicting the actual infiltration correctly, can be quite effective in planning and design of water resources systems.

When water is added to a dry soil either by rain or irrigation, it is distributed around the soil particle where it is held by adhesive or cohesive forces. It gradually displaces air in the pore spaces and eventually fills the pores. When all the pores (large and small) are filled, the soil is said to be saturated and it is at its maximum retentive capacity. The time at which the capillary potential (Ψ) approaches zero, the beginning of runoff and decay of infiltration rate begins. This time to ponding (t_p) (Smith, 1972, Mishra and Singh, 2003b)[16, 24] is a function of initial moisture content of the soil. More the soil moisture content less is the time to ponding and vice versa. In addition, 't_p' shows a strong dependence on the rainfall rate (R) given as $t_p = a_1 R^{a_2}$ (Smith, 1972)[24], where 'a_1' and 'a_2' are constants, which depend on the type of soil and initial moisture

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content. Under given antecedent moisture condition, a soil exhibits the maximum rate of infiltration at time $t = 0$ (initial), which decreases as more water infiltrates into the soil with increasing time and finally achieves almost a constant rate, known as ultimate infiltration capacity (f_c). This theory influences and describes the general behavior of infiltration model under uniform rainfall or irrigation rate for all types of soils.

Infiltration models can be generally classified into three groups (Mishra et al., 2003a)[17] (i) physically based models, (ii) semi-empirical models and (iii) empirical models. Physically based models rely on the law of conservation of mass and the Darcy law. Depending on the considerations of flow dynamics, hydraulic conductivity, moisture content, initial and boundary conditions, physically based models of varying complexity have been derived. Among the physically based models, the models of Green and Ampt (1911)[7], Philip (1957, 1969)[21, 22], Mein and Larson (1971, 1973)[13, 14], Smith (1972)[24], Smith and Parlange (1978)[25] etc. are worth citing. Semi-empirical models employ simple forms of continuity equation and simple hypothesis on the rate-cumulative infiltration. These models are based on the systems approach, popularly employed in surface water hydrology. Examples of semi-empirical models are the models given by Horton (1938)[9], Overton (1964)[20], Singh and Yu (1990)[23] etc. Empirical models are derived from data observed either in field or in laboratory. The models of Kostiakov (1932)[11], Huggins and Monke (1966)[10], modified Kostiakov, (Smith, 1972)[24], etc. fall in category of empirical models.

Despite the availability of a large number of infiltration models, some of the available empirical models have been quite popular and frequently used in various water resource applications world over, owing to their simplicity and yielding reasonably satisfactory results in most applications. The wide-spread application of the Kostiakov (KT) and modified Kostiakov (MKT) models in irrigation engineering (Michael, 1982)[15] are worth citing. In this paper an attempt is made to solve the infiltration problem to get the optimum parameter values with least error using KT and MKT models, which are discussed in the next subsection.

The estimation of infiltration parameters of the above models is most important from the view point of its field applicability. Various researchers applied various techniques to determine the infiltration parameters. Davis (1943)[1] used the method of averages to determine the values of parameters. Maheshwari et al. (1988)[12], used optimization method to compute parameters of empirical and theoretical infiltration equations for an irrigated border using a pattern-search optimization technique which is amenable to any form infiltration equation. Singh and Yu (1990)[23] showed that various well-known infiltration models as mentioned above and their parameters could be estimated using systems approach. At present the optimization method is widely used for the estimation of infiltration parameters of any form of infiltration equation, as the method has a potential to overcome the limitations of the other methods reported so far (Esfandiari and Maheshwari, 1997)[6].

Although various optimization techniques are used for this purpose, very few specific literature is available showing comparative performance of various optimization techniques. The objective of the present study is to solve two popular infiltration models, namely the KT model and the MKT model as nonlinear optimization models. A comparative performance of six different optimization techniques is shown to solve these two models. These optimization techniques are the software package called Language for INteractive General Optimizer (LINGO) (2003), Binary Genetic Algorithm (BGA) of (Deep and Das, 2007a)[3], Hybrid Binary GA (Deep and Kedar (2008)[4], Real Coded Genetic Algorithm, called LX-PM (Deep and Thakur (2007)[5], Hybrid Real Coded GA, called H-LX-PM of (Deep and Das (2007b)[2] and Random Search Technique (RST) (Mohan and Shanker(now Deep), 1994)[18].

This paper is organized as follows. In section 2, the KT and MKT models are stated. In section 3, the optimization problem is modeled and the techniques used to solve it are described. In section 4, results are discussed with their performance evaluations. Lastly, conclusions are drawn in section 5.

2 Kostiakov (KT) and modified kostiakov (MKT) infiltration models

The popular KT (Kostiakov, 1932)[11] and MKT (Smith, 1972)[24] infiltration models frequently used in irrigation engineering are derived using the data observed in either field or laboratory. Kostiakov (1932)[11] suggested the following best-fit equation for infiltration:

$$f = \alpha t^{-\beta} \text{ for } t \neq 0,\ \alpha > 0 \text{ and } \beta > 1 \tag{1}$$

where α is the coefficient and β is the exponent. Both the parameters rely on soil type, initial moisture content, rainfall rate, vegetative cover, etc. These can be determined empirically or by optimization techniques.

Smith (1972) modified the KT equation by including the term ultimate infiltration capacity (f_c). The 'f_c' was included for the reasons that the infiltration rate decreases as more water infiltrates into the soil to finally achieve a constant or ultimate infiltration rate. The modified KT equation is expressed as:

$$f = f_c + \alpha_1 t^{-\beta_1} \tag{2}$$

where $\alpha 1$ and $\beta 1$ are same as that of KT model and are determined empirically or by optimization techniques. According to Smith (1972)[24], the ultimate infiltration capacity 'f_c' occurs when the soil is fully saturated. Under steady state (no storage change), it will be equal to or less than the rate at which water percolates and flows into deep ground water system. Both 'f_c' and decay of infiltration capacity are functions of soil characteristics, moisture condition, vegetation, rainfall intensity and soil surface condition. Fig. 1 describes all the above components and the typical final decaying infiltration rate curve.

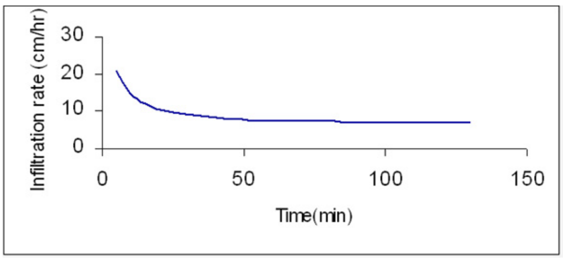

Fig. 1. A typical decaying infiltration curve

3 Optimization model and approaches

Optimal values of parameters of the above mentioned models are estimated using the method of least squares, a device for finding the equation of a specified type of curves, which best fits a set of observations. The method suggests that, for the best fit, the sum of the squares of differences between the observed and the corresponding estimated values should be minimized. Thus the optimization problem is mathematically modeled as

$$\text{Min } Z = \sum_{i=1}^{N} \{f_{obs}(i) - f_{com}(i)\}^2 \tag{3}$$

where Z = error; N = the number of observations or times; $f_{obs}(i)$ = observed infiltration rate at ith time; $f_{com}(i)$ = computed infiltration rate at i^{th} time. In order to solve this optimization problem the following approaches are used.

(i) LINGO

The software package, Language for INteractive General Optimizer (LINGO; release 8.0, March, 2003) is used to minimize the error, in the present study.

(ii) Binary Genetic Algorithm (BGA)

GAs have been one of the population based paradigm pioneered by John Holland (1975)[8]. Based on genetic processes of biological organism, genetic algorithms work surprisingly well to determine the solution very close to the global optima, even for the problems with higher complexity (Viz. with higher dimensionality, multimodality, having millions of local optima, having narrow ridge etc.).

We use the Binary coded genetic algorithm (BGA) in this paper. First the search space is needed to encode with binary coding for generating the initial population. Inspired by the conclusion drawn by Deep and Kedar (2007)[3], we consider the operators Roulette Wheel selection, Single-point crossover and Bit-wise mutation for BGA. A complete elitism is performed after the mutation in each generation, to retain survival of the fittest chromosomes.

(iii) Hybrid Binary Genetic Algorithm (HBGA)

In order to improve the efficiency and reliability of the Binary Genetic Algorithm Deep and Kedar (2008)[4] have proposed a hybrid Binary Genetic Algorithm, called HBGA. The HBGA incorporates the Quadratic Approximation (QA) operator after applying the crossover and mutation operator in each generation. It explores the search space more efficiently. In this operation, first the best fit chromosome (R_1) is chosen. Then two more chromosomes (R_2 and R_3) are chosen randomly. Then the point of minima (child) of the quadratic surface passing through R_1, R_2 and R_3 are defined as:

$$\text{Child} = 0.5\left[\frac{(R_2^2 - R_3^2)f(R_1) + (R_3^2 - R_1^2)f(R_2) + (R_1^2 - R_2^2)f(R_3)}{(R_2 - R_3)f(R_1) + (R_3 - R_1)f(R_2) + (R_1 - R_2)f(R_3)}\right]$$

where $f(R_1)$, $f(R_2)$ and $f(R_3)$ are the fitness function values at R_1, R_2 and R_3, respectively. The best chromosome will be replaced by the child if the child is found better than the best chromosome and hence the new best chromosome will be the child. In this way it enhances the convergence rate. The working principle of HBGA in the form of pseudo code is given as follows.

begin
Generation = 0
Generate initial population
Evaluate fitness of each individual in the population
While (termination criterion is not satisfied) **do**
Generation = Generation + 1
Apply roulette wheel selection operator
Apply One point Crossover operator
Apply Bitwise Mutation operator
Apply Quadratic Approximation operator (To hybridize BGA)
Apply complete elitism
end do
end begin

It has already been concluded in Deep and Kedar (2008)[4] that the HBGA outperforms BGA with respect to percentage of success, average number of function evaluations, and average computational time on a large set of benchmark test problems.

(iv) Real Coded Genetic Algorithm (LX-PM)

In Deep and Thakur (2007)[5] a new real coded Genetic Algorithm called LX-PM is designed using the Laplace Crossover Operator and Power Mutation. The LX-PM is well established in comparison with the other real coded GAs of similar category.

(v) Hybrid Real Coded Genetic Algorithm (H-LX-PM)

In Deep and Kedar (2007)[2], the LX-PM has been hybridized with the (QA) Operator which is applied after the Laplace Crossover and Power Mutation. This QA is exactly same as discussed in HBGA. The working principle of H-LX-PM in the form of pseudo code is given as follows.

begin
Generation = 0
Generate initial population
Evaluate fitness of each individual in the population
While (termination criterion is not satisfied) **do**
Generation = Generation + 1
Apply tournament selection operator (of size 3)
Apply Laplace Crossover operator
Apply Power Mutation operator
Apply Quadratic Approximation operator (To hybridize LX-PM)

Apply complete elitism
end do
end begin

It is shown in Deep and Kedar (2007)[2] that the H-LX-PM outperforms LX-PM with respect to percentage of success, average number of function evaluations, and average computational time on a large set of benchmark test problems.

(vi) Random Search Technique (RST)

This approach was given by Mohan and Shanker (now Deep) (1994)[18]. The algorithm works in two phases. In the first phase the objective function is evaluated at a number of randomly generated feasible solutions. In the second phase, these solutions are manipulated to yield possible candidates for the global minimum. The iterative process stops when all the feasible solutions converge to the global minimum or near global minimum.

4 Results and discussions

To verify the suitability of various optimization techniques used to estimate the infiltration parameters of different infiltration models, the infiltration data for Plainfield Sand (PFS) and Columbia Sandy Loam (CSL) are generated from several laboratory tests reported by Mein and Larson (1971)[13]. Two datasets each from PFS and CSL are considered for analysis.

To evaluate the ability of above infiltration models to simulate infiltration process, four infiltration datasets of two different soils (PFS and CSL) are considered. The parameters of each model for different soils considered are estimated using all 6 different optimization techniques discussed in section 3.

All 5 algorithms (except LINGO) are programmed in C++ and implemented on a P-IV machine. The best parameter values in the sense to minimize the error function among 20 runs with all 6 optimization approaches, for each soil and for each model, are investigated and reported in Tab. 1 and 3 for KT and MKT models respectively. The corresponding optimal values are reported in Tab. 2 and 4 for KT and MKT models respectively. While execution, we consider Population size = $10 \times (nv)$, where nv is the number of decision variables present in the objective function. The rest of the parameters in BGA, LX-PM are kept fixed as per the corresponing referred papers. Tab. 1 and 2 represent the estimated parameter values of KT and MKT infiltration models respectively, on different soils using Lingo, BGA, HBGA, LX-PM, H-LX-PM and RST.

Table 1. Parameter values using KT model

Optimization Techniques	Soil Type							
	PFS-1		PFS-2		CSL-1		CSL-2	
	α	β	α	β	α	β	α	β
Lingo	57.4900	0.5350	50.4900	0.5029	55.5889	0.4702	44.1450	0.4867
BGA	57.5000	0.5350	50.4314	0.5035	56.6885	0.4765	44.1460	0.4867
HBGA	57.5004	0.5351	50.4314	0.5035	56.6889	0.4765	44.1460	0.4867
LX-PM	57.5009	0.5350	50.4257	0.5037	56.6836	0.4765	44.1649	0.4869
H-LX-PM	57.5016	0.5350	50.4314	0.5035	56.6843	0.4765	44.1461	0.4867
RST	57.5028	0.5350	50.42923	0.5035	56.6397	0.4762	43.7812	0.4830

From the Tab. 1, 2, 3 and 4 it is observed that the values of the estimated parameters using different optimization techniques give slightly varying results. BGA, HBGA, LX-PM and H-LX-PM (say 4 GAs) give the same and better optimum values than LINGO and RST, for both the soils and for both the models. LINGO only gives the better optimum and is same as 4 GAs for CSL-2 by KT and for PFS-1, CSL-1 and CSL-2 by MKT. Further RST gives better optimum and is same as 4 GAs only at PFS-1 by KT.

In order to get a clear picture in the comparative performance among these techniques, we run each 100 times and the percentage of success (Tab. 5 and 6 for KT and MKT respectively), number of function evaluations (Tab. 7 and 8 for KT and MKT respectively) are reported. We here consider a run to be success if

Table 2. Parameter values using MKT model

Optimization Techniques	Soil Type					
	PFS-1			PFS-2		
	α_1	β_2	f_c	α_1	β_2	f_c
Lingo	39.3227	0.7993	15.4401	34.2510	0.7298	13.0770
BGA	39.3225	0.7993	15.4403	33.9952	0.7347	13.2187
HBGA	39.3231	0.7993	15.4398	33.9955	0.7347	13.2185
LX-PM	39.3366	0.7990	15.4281	34.0027	0.7346	13.2112
H-LX-PM	39.3323	0.7990	15.4327	33.9955	0.7347	13.2185
RST	39.4334	0.7971	15.3535	33.9298	0.7359	13.2674
Optimization Techniques	Soil Type					
	CSL-1			CSL-2		
	α_1	β_2	f_c	α_1	β_2	f_c
Lingo	61.9339	0.5813	2.7030	56.3043	0.8282	5.7408
BGA	61.9262	0.5812	2.7011	56.3114	0.8283	5.7420
HBGA	61.9111	0.5811	2.7000	56.3043	0.8282	5.7411
LX-PM	61.8784	0.5802	2.6764	56.4174	0.8291	5.7419
H-LX-PM	61.8375	0.5798	2.6723	56.3629	0.8291	5.7506
RST	61.5614	0.5744	2.5393	56.3528	0.8217	5.6057

Table 3. Comparative optimum values using KT model

Optimization Techniques	Soil Type			
	PFS-1	PFS-2	CSL-1	CSL-2
Lingo	183.3072	112.9494	0.6997	0.3474
BGA	183.3050	112.9205	0.6649	0.3474
HBGA	183.3050	112.9205	0.6649	0.3474
LX-PM	183.3050	112.9205	0.6649	0.3474
H-LX-PM	183.3050	112.9205	0.6649	0.3474
RST	183.3050	112.9206	0.6650	0.3507

Table 4. Comparative optimum values using MKT model

Optimization Techniques	Soil Type			
	PFS-1	PFS-2	CSL-1	CSL-2
Lingo	82.6026	14.8092	0.6286	0.0106
BGA	82.6026	14.7522	0.6286	0.0106
HBGA	82.6026	14.7522	0.6286 0.0106	
LX-PM	82.6026	14.7522	0.6286	0.0106
H-LX-PM	82.6026	14.7522	0.6286	0.0106
RST	82.6090	14.7548	0.6290	0.0144

Table 5. Comparative Percentage of Success for KT model

Optimization Techniques	Soil Type			
	PFS-1	PFS-2	CSL-1	CSL-2
Lingo	—	—	—	—
BGA	89	100	56 736	
HBGA	100	100	100	100
LX-PM	83	84	71	47
H-LX-PM	100	100	98	88
RST	100	98	99	58

Table 6. Comparative Percentage of Success for MKT model

Optimization Techniques	Soil Type			
	PFS-1	PFS-2	CSL-1	CSL-2
Lingo	——	——	——	——
BGA	95	67	78	15
HBGA	100	100	100	89
LX-PM	96	48	24	17
H-LX-PM	100	100	99	67
RST	100	93	96	23

Table 7. Comparative average number of function evaluations for KT model

Optimization Techniques	Soil Type			
	PFS-1	PFS-2	CSL-1	CSL-2
Lingo	——	——	——	——
BGA	237	231	392	2186
HBGA	60	58	250	288
LX-PM	83	263	135	148
H-LX-PM	54	58	98	102
RST	46	47	90	89

Table 8. Comparative average number of function evaluations for MKT model

Optimization Techniques	Soil Type			
	PFS-1	PFS-2	CSL-1	CSL-2
Lingo	——	——	——	——
BGA	286	372	4144	3094
HBGA	96	213	1020	1971
LX-PM	145	261	225	390
H-LX-PM	90	171	156	611
RST	86	137	169	110

Table 9. Comparative average number of function evaluations for MKT model

Optimization Techniques	Soil Type			
	PFS-1	PFS-2	CSL-1	CSL-2
Lingo	98.2897	98.4855	99.3553	99.4280
BGA	98.2897	98.4859	99.3975	99.4280
HBGA	98.2897	98.4859	99.3975	99.4280
LX-PM	98.2897	98.4859	99.3975	99.4280
H-LX-PM	98.2897	98.4859	99.3975	99.4280
RST	98.2897	98.4859	99.3975	99.4226

Table 10. Optimization technique efficiencies using MKT model

Optimization Techniques	Soil Type			
	PFS-1	PFS-2	CSL-1	CSL-2
Lingo	99.2293	99.8014	99.4304	99.9826
BGA	99.2293	99.8022	99.4304	99.9826
HBGA	99.2293	99.8022	99.4304	99.9826
LX-PM	99.2293	99.8022	99.4304	99.9824
H-LX-PM	99.2293	99.8022	99.4304	99.9825
RST	99.2292	99.8022	99.4302	99.9762

the obtained value is in the radius of 1% accuracy of the known minima. To obtain the percentage of success the stopping criteria is a maximum of 200 generations and to obtain the number of function evaluations the stopping criteria is either to attain 1% accuracy or a maximum of 1000 generations is reached.

It is observed from Tab. 5, 6, 7 and 8 that the function evaluation is very less in RST but the success rate is not up to the mark. It is clear that the hybrid versions are dominating (marked italics) their corresponding non-hybrids in both the success rate and the number of function evaluations. Moreover, HBGA outperforms all others in success rate everywhere, RST outperforms all others in function evaluations everywhere except for the soil type CSL-1 by MKT model. We couldn't find the results in LINGO (marked dotted lines) because it is a software black box, which can't be modified to get the required results.

The generation number versus the minimum function value obtained from each generation, graph is plotted (Fig. 2 ∼ 5). Fig. 2 and 3 represent the convergence graphs for KT model taking BGA versus HBGA and LX-PM versus H-LX-PM, respectively. Similar arguments hold in Fig. 4 and 5 for MKT model. From these convergence graph it is clear that the corresponding hybrid versions works better to converge faster towards the optima in all the cases.

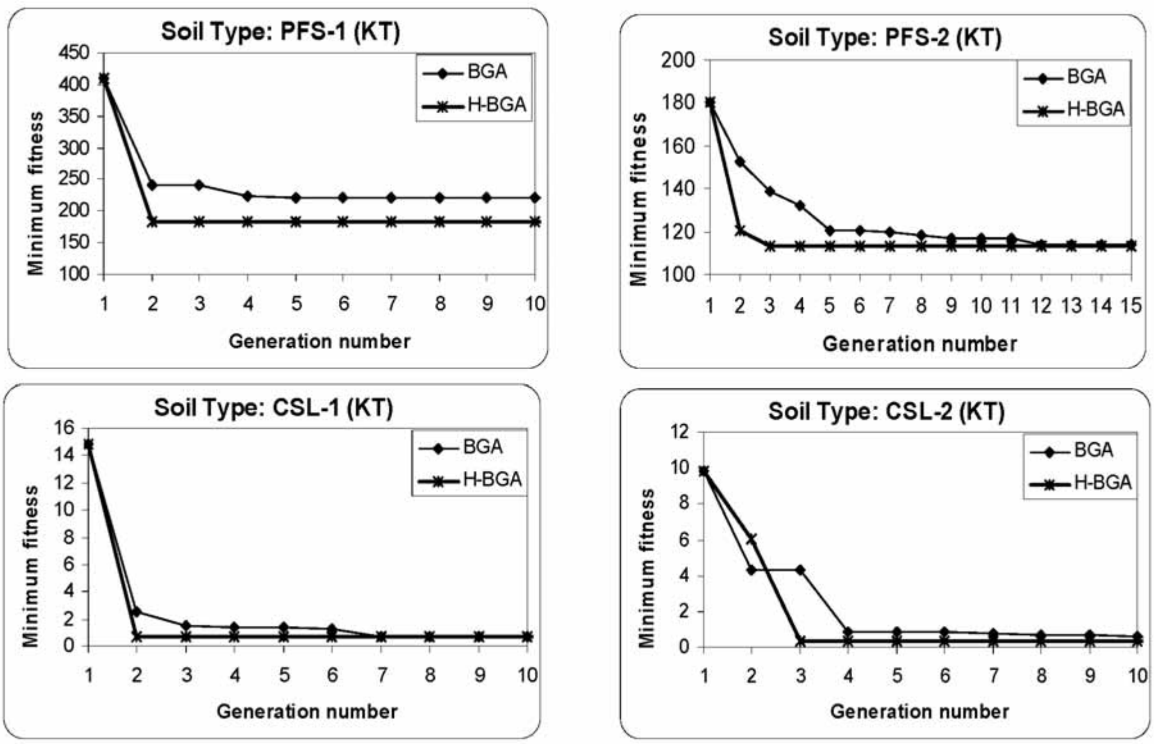

Fig. 2. Convergence graph of BGA and HBGA for KT model

Performance Evaluation Among the several statistical measures available for evaluating the performance of a model, such as correlation coefficient, relative error, standard error etc., the Nash and Sutcliffe (1970)[19] efficiency is the most frequently used (Mishra and Singh, 2003)[16] and it has been employed in this study. It is expressed in percent form as:

$$Efficiency = (1 - D_1/D_0) \times 100 \quad (4)$$

where D_1 is the sum of squares of deviations between computed and observed data, expressed as:

$$D_1 = \sum (Y_0 - Y_1)^2 \quad (5)$$

and D_0 is the initial variance, which is the sum of the squares of deviations of the observed data about the observed mean, expressed as:

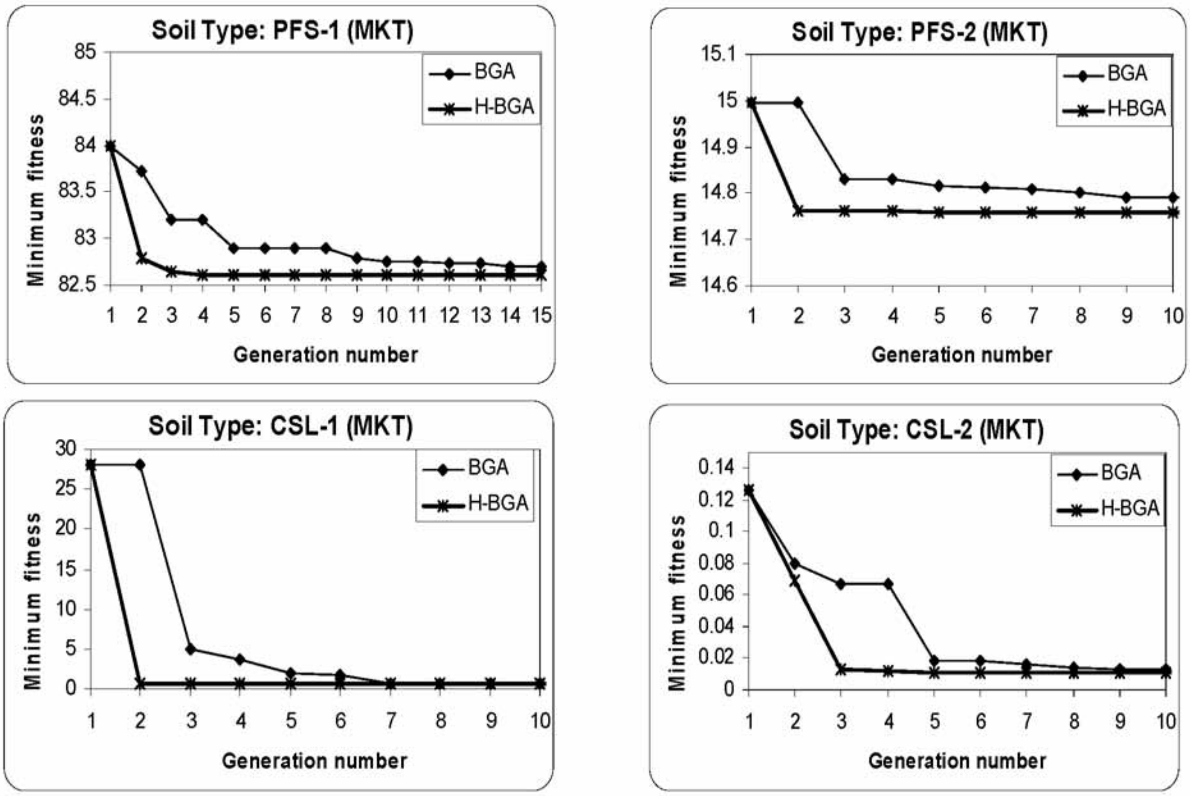

Fig. 3. Convergence graph of BGA and HBGA for MKT model

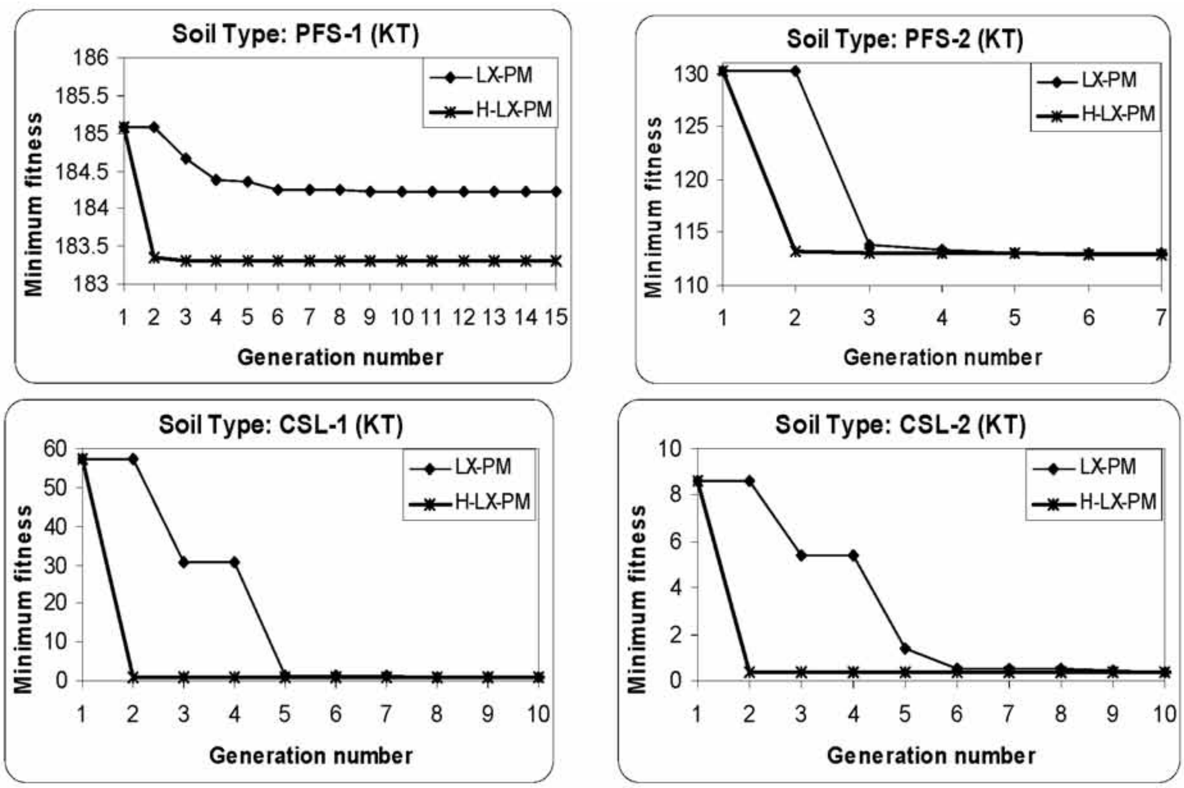

Fig. 4. Convergence graph of LX-PM and H-LX-PM for KT model

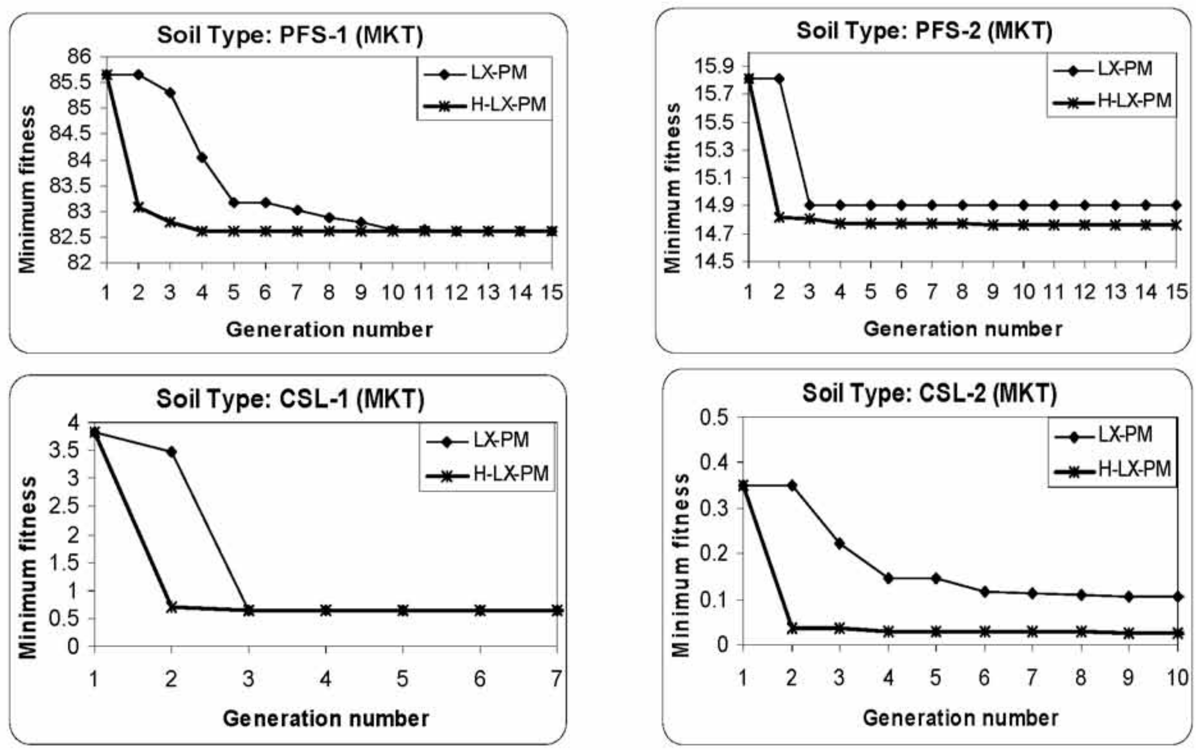

Fig. 5. Convergence graph of LX-PM and H-LX-PM for MKT model

$$D_0 = \sum (Y_0 - Y_m)^2 \tag{6}$$

where Y_0=observed data; Y_1=computed data; Y_m=mean of observed data.

The efficiencies resulting from the employment of KT and MKT models on four infiltration datasets, based on the fitting of the computed parameters derived using different optimization techniques, are shown in Tab. 9 and 10 respectively.

It is clear from the Tab. 9 that for KT model, all the optimization techniques are equally efficient for PFS-1. All 4 GAs and RST yield the same best efficiency for PFS-2 and CSL-1, where LINGO fails. But for CSL-2, LINGO along with 4 GAs are most and equal efficient whereas RST fails to be the most efficient.

From Tab. 10, clearly for MKT model, for the soil type PFS-1 and CSL-1, except RST all rest techniques give best efficiency. For PFS-2, except LINGO, all rest techniques are efficient. But for CSL-2, only LINGO, BGA and HBGA are most efficient among all 6.

5 Conclusion

In this paper an attempt is made to solve the hydro-optimization problem in finding infiltration parameters using Kostiakov (KT) and Modified Kostiakov (MKT) models. Experiments are implemented for two types of soils PFS and CSL, two from each. The results are obtained using six optimization techniques viz. LINGO, BGA, HBGA, LX-PM, H-LX-PM and RST. Computational results are obtained and discussed. To find a best optimization technique to find the optimum value of the infiltration parameters, we consider 3 factors in this paper. They are success rate, number of function evaluations and efficiency. Keeping in mind the success rate and the obtained optimum values, we recommend the HBGA is the best. If one needs to solve this problem with a less number of function evaluations, then RST is recommended. But it gives less successes rate and near optimal solution. Overall HBGA works better over all other techniques. Further, from numerical and graphical representation it can also be concluded that the hybrid versions HBGA and H-LX-PM are always better than BGA and LX-PM, respectively.

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