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A class of multiobjective vehicle routing optimal model under fuzzy random environment and its application*

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Abstract. This paper considers a capacitated vehicle routing problem with fuzzy random travel time and demand (FRVRP). A chance-constrained multiobjective programming is presented based on fuzzy random theory and converted to a crisp equivalent models under some assumptions. To solve such a multiobjective combinatorial optimization problem, this paper presents a hybrid multiobjective particle swarm optimization that incorporates a specific heuristics. The developed algorithm is further validated on a FRVRP instances about refined oil Secondary allocation problem.

Keywords: fuzzy random variable, vehicle routing, particle swarm optimization, multiobjective optimization

1 Introduction

The vehicle routing problem (VRP)[2] can be described as the problem of designing optimal delivery or collection routes from one or several depot(s) to a number of geographically scattered customers subject to side constraints.

This problem was first introduced by Dantzig and Ramser[2]. The classical VRP is categorized by the capacitated vehicle routing problem (CVRP), the vehicle routing problem with time windows (VRPTW), the multi-depot vehicle routing problem (MDVRP) and so on with different constraints. Because the VRP is a NP-hard problem, all kinds of exact algorithms are not efficient. Several families of heuristics have been proposed for the VRP[8]. A number of metaheuristics, such as simulated annealing (SA), genetic algorithms (GA), tabu search (TS), ant colony optimization (ACO) and particle swarm optimization (PSO) are designed for VRP.

The classical VRP statement does not capture an important aspect of real-life distribution problems, namely that several of the problem parameters (service time, customer locations, demands, etc.) are not known with certainty. These give rise to Stochastic Vehicle Routing Problems (SVRPs) and Fuzzy Vehicle Routing Problems (FVRPs). Stochastic Vehicle Routing Problems (SVRPs) arise whenever some elements of the problem are random. There are three basic classes of SVRP: stochastic customers, stochastic demands, and stochastic travel and service times[12]. Stochastic VRPs can be cast within the framework of stochastic programming. A stochastic program is usually modeled either as a chance constrained program (CCP) or as a stochastic program with recourse (SPR)[9, 12]. In some new systems, it is hard to describe the parameters of the problem as random variables because there are not enough data to analyze. Fuzzy Vehicle Routing Problems (FVRPs) arise whenever some parameters of the problem are treated as fuzzy variables[17, 19].

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In practice, fuzziness and randomness simultaneously appear in decision systems in many cases. For example, travel time of vehicle in VRP is dependent on weathers, i.e., fine, cloudy, rain, etc, and traffic, i.e., jam, open, etc, which are considered as scenarios that occur randomly. When each scenario is realized, the travel time of vehicle is estimated as a fuzzy number, such as “about 50 minutes”, “less than 30 minutes”, etc. It turns out that travel time of vehicle is expressed by a fuzzy random variable. Fuzzy random variables[7, 10] are mathematical descriptions for fuzzy stochastic phenomena. In this article, the travel time and demand of customer are treated as fuzzy random variables and the VRP is developed in fuzzy stochastic surrounding, namely fuzzy random vehicle routing problem (FRVRP).

The VRP is inherently a multiobjective optimization problem[6]. In this paper, a fuzzy random chance-constrained multiobjective programming (FRCCMOP) model is given for the capacitated vehicle routing problems (CVRP) with fuzzy random travel time and demand. In section 2 and 3, formulate a FRCCMOP model for the CVRP with fuzzy random travel time and demand. In section 4, recalls some definitions and results about fuzzy random variables. A crisp equivalent model is proposed for a special type of fuzzy random variables. In section 5, a hybrid multiobjective particle swarm optimization (PSO) is presented to solve this model. In section 6, the proposed models and algorithms is applied to Refined oil Secondary allocation problem. The results show that the hybrid PSO is effective for the fuzzy random VRP.

2 Problem statement

For vehicle routing problem under uncertain environment, travel time of each vehicle and demand of each customer are affected by randomness and fuzziness simultaneity. When we estimate the travel time of a vehicle on the way, on one side it will be affected by traffic jam or vehicle damage, etc, on the other hand, it will also affected by the fuzzy cognition which the decision-maker how to understand how great these situation affect it when these situations happened. Thus, the conclusive estimation is generally expressed as “If traffic is smooth, it will take 10mins approximately” or “If the traffic is not smooth, it will take 18mins approximately”. “If” means randomness and “approximately” means fuzziness. When we estimate demand of a customer, on one side it will be affected by marketable or unmarketable, on the other hand, it will also affected by the fuzzy cognition which the decision-maker how to understand how great these situation affect it when these situations happened. Thus, the conclusive estimation is generally expressed as “If the goods are marketable, demands will be 30units approximately” or “If the goods are unmarketable, demands will be 30units approximately”. Thus randomness and fuzziness coexist in estimate to travel time or demand. In this article, we describe them as fuzzy random variables.

Fuzzy random variable, which was introduced by Kwakernaak in 1978, is a concept to depict the phenomena in which randomness and fuzziness appear simultaneously[7]. Roughly speaking, a fuzzy random variable is a measurable function from a probability space to a collection of fuzzy variables[10].

Definition 1. [10] *Let $(\Theta, \mathcal{P}(\Theta), Pos)$ be a possibility space, and A be a set in $\mathcal{P}(\Theta)$. Then the credibility measure of A is defined by*

$$Cr\{A\} = \frac{1}{2}(Pos\{A\} + Nec\{A\})$$

Thus the credibility measure is self dual, i.e., $Cr\{A\} + Cr\{A^c\} = 1$ for any $A \in \mathcal{P}(\Theta)$.

Definition 2. [10] *A fuzzy random variable is a function ξ from a probability space $(\Omega, \mathcal{A}, Pr)$ to the set of fuzzy variables such that $Cr\{\xi(\omega) \in B\}$ is a measurable function of for any Borel set B of $\Re$.*

We assume that: (a) each vehicle has a container with a physical capacity limitation and the total loading of each vehicle cannot exceed its capacity; (b) a vehicle will be assigned for only one route on which there may be more than one customer; (c) a customer will be visited by one and only one vehicle; (d) each route begins and ends at the company site (depot); and (e) the travel times between customers and demand of each customer are fuzzy random variables.

3 Modelling

Let us first introduce the following indices and model parameters. Let vertex set $V = \{0, 1, \cdots, n\}$, customers set $V' = V\backslash\{0\}$ and vehicles set $K = \{1, 2, \cdots, m\}$.Vertex 0 represents a depot, and each of the vertices in $V' = V\backslash\{0\}$ represents a customer. Let $\tilde{\bar{t}}_{ij}, i, j \in V$, be the fuzzy random travel time from customer i to customer j and assume it is symmetric, namely $\tilde{\bar{t}}_{ij}$ and $\tilde{\bar{t}}_{ij}$ are identified, $i, j = 1, \cdots, n$. For each customer $i \in V'$,we let $\tilde{\bar{g}}_i$ denote the fuzzy random variable describing the demand at customer i. Travel times and demands are nonnegative and independent fuzzy random variables. The vehicle capacity is denoted as Q. Let x_{ijk} be a binary variable equal to 1 if vehicle k travel from customer i to customer j, or equal to 0. Also, Let y_{ik} be a binary variable equal to 1 if and only if vertex i is visited by vehicle k in the solution, or equal to 0.

For dealing with constrains with fuzzy random variable coefficient, we introduce conception chance of fuzzy random events.

Definition 3. [10] *Let $\xi = \{\xi_1, \xi_2, \cdots, \xi_n\}$be a fuzzy random vector on the probability space $(\Omega, \mathcal{A}, Pr)$ and $f : R^n \to R$ be continuous functions. Then the primitive chance of fuzzy random event characterized by $f(\xi) \le 0$ is a function from $(0, 1]$ to $[0, 1]$, defined as*

$$Ch\{f(\xi) \le 0\}(\alpha) = \sup\{\beta | Pr\{\omega \in \Omega | Cr\{f(\xi) \le 0\} \ge \beta\} \ge \alpha\}$$

Assume that each vehicle start from depot at same time, for completing the task in the shortest time at lest cost, the objectives are to minimize the maximal time spent in each rout (vehicle) and minimize the number of vehicles needed. Shorter delivery time results in higher level of customer satisfaction and fewer vehicles imply that the total operation cost is reduced. Then the problem can be formulated as follows:

$$\max\{f_1, f_2\}$$

$$s.t. \begin{cases} Ch\{\sum\limits_{i\in V}\sum\limits_{j\in V} \tilde{\bar{t}}_{ij} x_{ijk} \le f_1\}(\gamma) \ge \delta & \forall k \in K & (3.1) \\ \sum\limits_{j\in V'}\sum\limits_{k\in K} x_{0jk} \le f_2 & & (3.2) \\ Ch\{\sum\limits_{i\in V'} \tilde{\bar{g}}_i y_{ik} \le Q\}(\gamma) \ge \delta & \forall k \in K & (3.3) \\ \sum\limits_{k\in K} y_{ik} = 1 & \forall i \in V' & (3.4) \\ \sum\limits_{i\in V} x_{ijk} = y_{jk} & \forall j \in V'; \forall k \in K & (3.5) \\ \sum\limits_{j\in V} x_{ijk} = y_{ik} & \forall i \in V'; \forall k \in K & (3.6) \\ \sum\limits_{i\in R}\sum\limits_{i\in R} x_{ijk} \le |R| - 1 & \forall i \in V'; \forall k \in K & (3.7) \\ x_{ijk} \in \{0, 1\} & \forall i \in V; \forall j \in V; \forall k \in K & (3.8) \\ y_{ik} \in \{0, 1\} & \forall i \in V; \forall k \in K & (3.9) \end{cases}$$

The objective function f_1 and f_2 represents the maximal travel time of each route (vehicle) and the total number of vehicles needed , respectively, to be minimized. Constraints (3.1) express that the maximal travel time of each route (vehicle) not exceed f_1. The objective of minimization of f_1 associated with constraints (3.1) show that the objective realized is the maximal travel time of vehicle minimize. Constraints (3.2) shows the objective function f_2 represent the number of vehicles needed. Constraints (3.3) ensure that the vehicle capacity constraints is not violated at a specified level of chance. Each customer is visited by one single vehicle, thanks to constraints (3.4). (3.5) and (3.6) ensure the continuity of each route. Constraints (3.7) are classical subtour elimination constraints.

4 Crisp equivalents

One way of solving above model is to convert the chance constraints of the model into their respective crisp equivalents. As we know, this process is usually a hard work and only successful for some special cases. Consequently, we assume the fuzzy random parameters $\tilde{\bar{t}}_{ij}$ and $\tilde{\bar{g}}_i$ are L-R fuzzy random variables.

Definition 4. *[15] Fuzzy random variable $\tilde{\bar{c}}$ is called L-R type fuzzy random variable if $\tilde{\bar{c}}$ take fuzzy numbers under the occurrence of each elementary event $\omega \in \Omega$, which are characterized by the following membership functions:*

$$\mu_{\tilde{\bar{c}}(\omega)}(t) = \begin{cases} L(\dfrac{\bar{c}(\omega)-t}{\bar{a}(\omega)}) & if \quad t \le \bar{c}(\omega), \bar{a}(\omega) > 0 \\ \\ R(\dfrac{t-\bar{c}(\omega)}{\bar{b}(\omega)}) & if \quad t \ge \bar{c}(\omega), \bar{b}(\omega) > 0 \end{cases}$$

where $\tilde{\bar{c}}(\omega)$ is a realization of the fuzzy random variables $\tilde{\bar{c}}$ under the occurrence of each elementary event ω . The parameters $\bar{c}, \bar{a}$ and $\bar{b}$ are random variables. Reference functions $L, R : [0,1] \to [0,1]$ with $L(1) = R(1) = 0$ and $L(0) = R(0) = 1$ are non-increasing, continuous functions. It is denoted by $\tilde{\bar{c}} = (\bar{c}; \bar{a}, \bar{b})_{LR}$. It should be noted here that $\bar{a}(\omega)$ and $\bar{b}(\omega)$ are to be positive for ω any because they are spread parameters of L-R fuzzy numbers.

Based on possibility measure and necessary measure of fuzzy event, Dubois and Prade defined several ranks of two fuzzy numbers[3]. Sakawa and Yano have given the following results when the membership functions are assumed to be upper semicontinuous[15].

Lemma 1. *[15] If $\tilde{a}$ and $\tilde{b}$ are fuzzy numbers, then:*

$$(1) Pos\{\tilde{a} \ge \tilde{b}\} \ge \delta \Leftrightarrow \tilde{a}^R_\delta \ge \tilde{b}^L_\delta$$
$$(2) Nec\{\tilde{a} \ge \tilde{b}\} \ge \delta \Leftrightarrow \tilde{a}^L_{1-\delta} \ge \tilde{b}^L_\delta$$

where $\tilde{a}^L_\delta, \tilde{b}^L_\delta, \tilde{a}^R_\delta$ and $\tilde{b}^R_\delta$ are the left and right side extreme points of the δ-level sets $[\tilde{a}^L_\delta, \tilde{a}^R_\delta]$ and $[\tilde{b}^L_\delta, \tilde{b}^R_\delta]$ of $\tilde{a}$ and $\tilde{b}$, respectively.

Lemma 2. *[10] Let $(\Theta, \mathcal{P}(\Theta), Pos)$ be a possibility space, and A be a set in $\mathcal{P}(\Theta)$, if $Pos\{A\} < 1$, we have $Nec\{A\} = 0$.*

Theorem 1. *If $\tilde{a} = (m_a; \alpha_a, \beta_a)_{LR}$ and $\tilde{b} = (m_b; \alpha_b, \beta_b)_{LR}$ are L-R fuzzy numbers, then:*

$$Cr\{\tilde{a} \ge \tilde{b}\} \ge \delta \Leftrightarrow \begin{cases} m_a + \beta_a \cdot R_a^{-1}(2\delta) \ge m_b - \alpha_b \cdot L_b^{-1}(2\delta), & 0 \le \delta < \frac{1}{2} \\ m_a - \alpha_a \cdot L_a^{-1}(2-2\delta) \ge m_b - \alpha_b \cdot L_b^{-1}(2\delta - 1), & \frac{1}{2} \le \delta \le 1 \end{cases}$$

where $L_a^{-1}, L_b^{-1}, R_a^{-1}$ and R_b^{-1} are inverse of left and right reference functions for L-R fuzzy numbers $\tilde{a}$ and $\tilde{b}$, respectively.

Theorem 2. *Let $\tilde{\bar{c}} = (\bar{c}; \bar{a}, \bar{b})_{LR}$ is a L-R fuzzy random variable,where $\bar{c} = c^e + \bar{r}c^\sigma, \bar{a} = a^e + \bar{r}a^\sigma$, and $\bar{b} = b^e + \bar{r}b^\sigma$. $c^e, a^e, b^e, c^\sigma, a^\sigma$ and b^σ are constant. $\bar{r} \sim N(0,1)$. Then $\forall h \in R$*

$$Ch\{\tilde{\bar{c}} \le h\}(\gamma) \ge \delta \Leftrightarrow \begin{cases} (c^e - a^e L^{-1}(2\delta)) + \Phi^{-1}(\gamma)\sqrt{(c^\sigma)^2 + (a^\sigma L^{-1}(2\delta))^2} \le h, & 0 \le \delta < \frac{1}{2} \\ (c^e - a^e L^{-1}(2\delta - 1)) + \Phi^{-1}(\gamma)\sqrt{(c^\sigma)^2 + (a^\sigma L^{-1}(2\delta - 1))^2} \le h, & \frac{1}{2} \le \delta \le 1 \end{cases}$$

In model, assume that $\tilde{\bar{t}}_{ij} = (\bar{t}_{ij}; \bar{a}_{ij}, \bar{b}_{ij})_{LR}$, where $\bar{t}_{ij} = t^e_{ij} + \bar{r}t^\sigma_{ij}, \bar{a}_{ij} = a^e_{ij} + \bar{r}a^\sigma_{ij}$, and $\bar{b}_{ij} = b^e_{ij} + \bar{r}b^\sigma_{ij}$. It is noted that $t^e_{ij}, a^e_{ij}, b^e_{ij}, t^\sigma_{ij}, a^\sigma_{ij}$ and b^σ_{ij} are nonnegative real number, $i = 1, 2, \cdots, n; j = 1, 2, \cdots, m$, and $\bar{r} \sim N(0,1)$. For x_{ijk} is 0-1 variable, it follows from extension principle[18] that fuzzy number $\sum\limits_{i \in V} \sum\limits_{j \in V} \tilde{\bar{t}}_{ij}(\omega) x_{ijk}$ is a L-R fuzzy number.

From Theorems 2,we have that the constraints (3.1) of the model is equivalent to the constraints (4.1)

$$\sum_i \sum_j t^e_{ij} x_{ijk} - (\sum_i \sum_j a^e_{ij} x_{ijk}) L^{-1}(2\delta - 1) + \Phi^{-1}(\gamma) \sqrt{\sum_i \sum_j (t^\sigma_{ij} x_{ijk})^2 + \sum_i \sum_j (a^\sigma_{ij} x_{ijk})^2 (L^{-1}(2\delta - 1))^2} \le f_1 \qquad (4.1)$$

Assume that $\tilde{\bar{g}}_i = (\bar{g}_i; \bar{u}_i, \bar{v}_i)_{LR}$, where $\bar{g}_i = g_i^e + \bar{r}g_i^\sigma, \bar{u}_i = u_i^e + \bar{r}u_i^\sigma$, and $\bar{v}_{ij} = v_i^e + \bar{r}v_i^\sigma$. Similarly, constraints (3.3) of the model is equivalent to the constraints (4.2)

$$\sum_i g_i^e y_{ik} - (\sum_i u_i^e y_{ik}) L^{-1}(2\delta - 1) + \Phi^{-1}(\gamma) \sqrt{\sum_i (g_i^\sigma y_{ik})^2 + \sum_i (u_i^\sigma x_{ijk})^2 (L^{-1}(2\delta - 1))^2} \leq Q \quad (4.2)$$

5 Hybrid multi-objective particle swarm optimization

Particle swarm optimization (PSO) is an evolutionary optimization algorithm proposed by Kennedy and Eberhart[4] in the mid 1990s while attempting to simulate the choreographed, graceful motion of swarms of birds as part of a socio-cognitive study investigating the notion of "collective intelligence" in biological populations. If the scale of swarm is N, then the position of the ith $(i = 1, 2, 3, \cdots, N)$ particle is expressed as X_i. The "best" position discovered by the ith particle is expressed as $pBest_i$. The index of the position of the particle of the swarm, with best solution is expressed a $gBest$. Therefore, a swarm particle i will update its own speed and position according to the following equations:

$$V_{i+1} = \omega \cdot V_i + c_p \cdot r_1 \cdot (pBest_i - X_i) + c_g \cdot r_2 \cdot (gBest - X_i) \quad (5.1)$$

$$X_{i+1} = X_i + V_{i+1} \quad (5.2)$$

where c_p is the Cognitive learning rate and c_g is the Social learning rate. The factors r1 and r2 are randomly generated within the range $[0, 1]$ and ω is the inertia factor. The equation is essentially made up of three parts. The first part is the former speed of the swarm, which shows the present state of the swarm; the second part is the cognition modal, which expresses the thought of the individual swarm particle itself; the third part is the social modal. These three parts together determine the solution space searching ability.

There are numerous methods for solving multiple objective problems using the PSO algorithm[1, 13]. For the Traveling Salesman Problem (TSP), a sub-problem of the vehicle routing problem, many versions of the PSO have been reported to generate satisfactory results[11]. A computational-efficient PSO algorithm for VRPTW has been presented[20]. In this article, a hybrid PSO is presented and based on SPEA2 [21] and MOPSO [1] for the FRMOVRP.

5.1 Particle representation

In general, a route for an n customers and k vehicles VRP problem can be presented as a $n + k - 1$ dimension vector with each dimension defining the sequence of the corresponding customers or the center node. In designing the PSO algorithm, we also define a real-number $n + k - 1$ dimension particle, which can be normalized into integer index as follows:

Node:	1	2	3	4	5	6	0	0
X :	3.2	4.1	1.5	2.5	1.2	3.6	2.3	3.1
Index:	6	8	2	4	1	7	3	5

The path representation of particle X is (5 3 0 4 0 1 6 2), which represented the following solution.

Vehicle 1 : 0 - 5 - 3 - 0
Vehicle 2 : 0 - 4 - 0
Vehicle 3 : 0 - 1 - 6 - 0

In the particle definition and the solution expression, the constraints (3.4) to (3.7) are automatically satisfied.

Each dimension of the particle position vector X take real number between 1 and $n + k - 1$ and each dimension of the velocity vector V take real number between $-(n + k - 2) \sim (n + k - 2)$.

5.2 Fitness assignment

SPEA2 proposed a fitness assignment method incorporation density information and dominance relation[21].Each individual i in the population pop is assigned a fitness $F(i) = R(i)+D(i)$, where $R(i)$ is the raw fitness determined by the strengths dominators in both archive and population, $D(i)$ is the density value determined by the distances (in objective space) to all individuals. Please note that fitness is to be minimized here.

5.3 Algorithm flow

Incorporating archive mechanism in MOPSO[1] and fitness assignment method in SPEA2[21], a hybrid PSO algorithm is proposed as follow:

Step 1. Initialization: Generate an initial population pop_1, temporary population pop_2 and mixed population pop_3. The size of the three populations are M, M and $2M$, respectively. Set $t = 0$. (a) For population pop_1, each dimension of the particle position vector X_i take real number between 1 and $n + k - 1$ and each dimension of the velocity vector V_i take real number between $-(n + k - 2) \sim (n + k - 2)$. (b) Improve the particles corresponding unfeasible solutions to feasible ones in pop_1. (c) For population pop_2 and pop_3, the particle present position, velocity and the best position in its history of each individual set empty.

Step 2. Evaluation: Evaluate each of the particles in population pop_1. (a) Calculate the values of objectives (f_1, f_2) and get fitness $F(i)$ of each particle as section 5.2. (b) Sorting the fitness in increasing order. (c) Update the best position of each particle in its history $pBest$.

Step 3. Determine temporary population pop_2. (a) Copy all nondominated individuals in pop_1 to the archive nonDomList. (b) In nonDomListapply roulettewheel selection to update the global best position of each particle $gBest$. (c)Compute the position and velocity of each individual in pop_1 as Eqa.(5.1) and Eqa.(5.2), respectively, and update the position and velocity of corresponding individual in pop_2. (d) Improve the particles corresponding unfeasible solutions to feasible ones in pop_2. (e) According to the best history position of each particle in pop_1 and the present position of corresponding one in pop_2, update the best position of each particle the best history position of each particle in pop_2.

Step 4. Determine mixed population pop_3. (a) Copy all individuals in pop_1 and pop_2 to pop_3, clear population pop_1, pop_2 and nonDomList. (b) Evaluate each of the particles in population pop_3 as section 4.2 and sorting the fitness in increasing order.

Step 5. According to order of fitness, copy the first M individuals in pop_3 to pop_1 and clear pop_3.

Step 6. Set $t = t + 1$. If the stopping criterion is not satisfied, and go to setp 3. Otherwise, set A to the set of decision vectors represented by the nondominated individuals in pop_1. Stop.

6 Refined oil secondary allocation problem

There are various applications of vehicle routing problem. We take account of refined oil Secondary allocation problem. The first allocation of refined oil is transportation from petroleum refinery to region center depots and the second allocation is transportation from region center depots to gas station. Assume that only a refined oil depot locate in a region and serve for 10 gas stations, i.e., the number of customers $n = 10$. The capacity of vehicle $Q = 80$, the number of available vehicles $m = 5$. The predetermined confidence levels $\gamma = \delta = 0.9$. Assume that $L(x) = R(x) = max\{0, 1 - |x|\}, a^{\sigma}_{ij} = b^{\sigma}_{ij} = 0, u^{\sigma}_{i} = v^{\sigma}_{i} = 0$. Other parameters are listed in Tab. 1 $\sim$ Tab. 3.

HPSO parameters setting: The maximum number of iterations G was set to 100. The population size M was set to 60. The value of inertial weight ω_t is allowed to decrease linearly with iteration from ω_1 to ω_2 [14]. The value of inertia weight at iteration t, ω_t is obtained as

$$\omega_t = \omega_2 + (\omega_1 - \omega_2)\frac{G - t}{G}$$

Table 1. Parameter $(t^e_{ij}, t^\sigma_{ij})$ of fuzzy random travel time

Gas station	0	1	2	3	4	5	6	7	8	9
1	(50,25)									
2	(10,5)	(40,20)								
3	(50,25)	(10,5)	(40,20)							
4	(15,7)	(50,25)	(15,7)	(45,22)						
5	(50,25)	(35,17)	(35,17)	(30,15)	(35,17)					
6	(50,25)	(15,7)	(40,20)	(5,2)	(45,22)	(30,15)				
7	(25,12)	(40,20)	(30,15)	(35,17)	(15,7)	(25,12)	(35,17)			
8	(15,7)	(40,20)	(10,5)	(45,22)	(20,10)	(35,17)	(40,20)	(35,17)		
9	(50,25)	(15,7)	(45,22)	(10,5)	(45,22)	(30,15)	(10,5)	(40,20)	(40,20)	
10	(20,10)	(45,22)	(25,12)	(45,22)	(15,7)	(30,15)	(10,20)	(10,5)	(25,12)	(45,22)

Table 2. Parameter (a^e_{ij}, b^e_{ij}) of fuzzy random travel time

Gas station	0	1	2	3	4	5	6	7	8	9
1	(10,15)									
2	(5,8)	(15,20)								
3	(15,15)	(6,5)	(20,12)							
4	(8,9)	(22,20)	(5,6)	(13,18)						
5	(25,10)	(10,10)	(12,10)	(20,18)	(14,15)					
6	(30,20)	(10,5)	(20,15)	(2,2)	(10,10)	(5,5)				
7	(5,10)	(10,10)	(10,15)	(15,10)	(5,6)	(5,10)	(5,15)			
8	(10,3)	(10,15)	(5,5)	(10,10)	(10,10)	(12,12)	(15,10)	(12,13)		
9	(20,15)	(5,4)	(15,20)	(5,5)	(15,25)	(10,12)	(3,5)	(10,20)	(10,20)	
10	(5,10)	(15,20)	(5,10)	(15,25)	(5,8)	(10,15)	(5,10)	(5,5)	(5,10)	(15,20)

Table 3. Parameter (g^e_i, g^σ_i) and (u^e_i, v^e_i) of fuzzy random demands

Gas station	1	2	3	4	5	6	7	8	9	10
(g^e_i, g^σ_i)	(20,10)	(10,5)	(14,7)	(16,8)	(20,10)	(6,3)	(20,10)	(13,6)	(16,8)	(17,8)
(u^e_i, v^e_i)	(3,2)	(6,3)	(1,2)	(3,5)	(4,2)	(1,3)	(4,5)	(5,5)	(6,7)	(4,8)

Where $\omega_1 = 0.7, \omega_2 = 0.4$ (as suggested in[14]). The higher values of ω help in the global search for the optimal solution, while lower values help in the local search around the current search area.The learnng rates were 1.49445 (as suggested in[16]).

The program was coded in Matlab 6.5 and performed on a personal computer with a 1.83GHz AMD Athlon XP 2500+ processor and 1GB of RAM.

The run of the HPSO by 20 times shows that the best operational plan is Tab. 4. The average runtime is 1.03 minutes.

Table 4. Pareto optimal solutions and objectives of FRCVRP by HMOPSO

Pareto optimal solutions	Objectives (f_1, f_2)
5-7-4-0-8-1-2-0-3-9-6-10	(115.3652, 3)
7-8-4-0-1-0-5-2-0-9-3-6-10	(112.7132, 4)
3-0-7-4-8-2-0-1-0-5-0-9-6-10	(108.2171, 5)

7 Conclusion

In this paper, we have proposed a capacitated vehicle routing problem with fuzzy random travel time and demand (FRVRP). We have developed a chance-constrained multiobjective programming model for FRVRP and converted to a crisp equivalent models under assumptions of LR fuzzy random coefficients. We designed a hybrid multiobjective particle swarm optimization to solve it and the developed algorithm is further validated on a FRVRP instances about refined oil Secondary allocation problem. Although the fuzzy random MOCCP model constructed in this paper should be helpful for uncertain vehicle routing problems, a CCP solution does not take into account the cost of corrective actions in case of failure. The next research will be fuzzy random VRP model with recourse.

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