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Numerical simulation of temperature and pressure distribution in producing gas well*

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Abstract. Temperature and pressure are two important parameters in the management, design and dynamic analysis of gas well. In this paper we establish a numerical model of temperature and pressure distribution for gas well based on fluid phase theory of gas well and mass, momentum and energy conservation. The model, which accords with the actual situation of the gas well, allows for the oblique angle, structure of well and tubing string, the radial heat transfer of the wellbore , different heat transfer medium in annular and the physical character of the stratum. We set up four order Range-kuta method to solve the numerical model. In calculation, we divide the wellbore into several intervals. Temperature and pressure are just the unknown factors in every interval of the object, and it is necessary to use iterative method to solve them. Then we compile the corresponding simulation software with computer programming language named C♯ to simulate the temperature and pressure distributions and plot the distribution curves based on the basic data of Da Well. The curves can reflect the gas flowing law intuitively and provide the technical reliance and dynamic analysis of production well.

Keywords: temperature and pressure distribution, producing gas well, numerical simulation, software development

1 Introduction

Producing gas wells generally are associated with high temperature, high flow velocity and high pressure due to friction, the change of well structure, heat transfer etc.. With the evolution of producing environments to include combined deepwater and high temperature-high pressure conditions, it became necessary to accurately predict temperature and pressure distributions to aid in the design of production facilities in petroleum engineering, optimization of hydrate prevention, and dynamic analysis of production wells. Therefore the problem has been studied by several authors for years. Early in 1959, Kirkpatrick presented a simple flowing-temperature-gradient chart that can be used to predict gas-lift valve temperatures at the injection depth[7]. Approximate methods for predicting the temperature of either a single-phase incompressible liquid or a single-phase ideal gas flowing in injection and production wells[9] was presented by Ramey and was later elaborated it by Satter considering phase changes that occur within steam-injection projects[11]. Shiu and Beggs applied Ramey's correlation to producing oil wells for which the temperature of the fluid entering the wellbore was known[12]. This work proposed an alternative method for estimating the relaxation distance parameter, A, in Ramey's equation, in order to eliminate the complex calculation of the overall heat transfer coefficient in the wellbore. Coulter and Bardon developed a method in which a rigorous analysis of the thermodynamic behavior of the fluid was performed[4]. Sagar, et al. developed a model for multi-phase flow in wells based on

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Coulter and Bardon equation and incorporated Ramey's heat transfer mechanisms in a well[10]. The correlation accounted for the kinetic energy effects and the Joule–Thomson effect due to heating/cooling caused by pressure changes within the fluid as it flows up the well. Sagar, et al. also presented a simplified version of the correlation intended for hand calculations, in which the Joule–Thomson and kinetic energy terms were replaced with a correction factor. Alves, et al. presented a model applicable to both wellbores and pipelines[1]. The method, which is applicable for the entire range of inclination angles, single phase and two–phase flow, estimated a method for calculating the Joule–Thomson coefficient for black oils but made strong assumptions for the heat transfer of horizontal regions of wells. Hasan and kabir proposed a heat transfer model to predict transient temperature behavior in the formation at all times in 1991[5]. The model incorporated the hydrodynamics of the different flow patterns, the influence of well deviation and geometry, and the heat transfer mechanisms in the tubing/casing annulus. Takacs and Guffey in 1989 derived a series of algebraic equations based on statistical analytical method to Predict flowing bottomhole pressures in gas wells[13].

However, there are some limitations in their methodologies, such as difficult algorithm and only a few models synthetically calculate the temperature and pressure distribution. Later, some researchers have studied the application of coupling component model to calculate temperature and pressure. These new methods are different and advanced in that they overcome the limitations of previous studies through synthetically considering the interrelation between temperature and pressure [16]. Motivated by this, we propose a comprehensive model to study transient temperature and pressure behavior in a producing gas well based on mass, momentum and energy conservation. The numerical model, which accords with the actual situation of production in most gas wells, allows for gas velocity and density in gas wells and consisted of four coupled nonlinear differential equations that were solved with the four order Runge-Kuta method as shown by Pacheco for simultaneous equations[8]. We divide the wellbore into several intervals for nodal analysis in production well. In every interval of the object, temperature and pressure are just the unknown factors and it is necessary to use iterative method to solve them.

The above concepts illustrate the complexity of the situation under consideration. For such a complex problem, numerical simulation is usually the best approach. Therefore, in order to simulate the temperature and pressure distribution, we compile the corresponding simulation software with computer programming language named C♯. Without discussing the pros and cons of different programming languages in this place, it should be mentioned why it is reasonable to use C♯. First of all, C♯is nowadays accepted by a large part of the developer community mainly because its ease to use, especially compared to C++. There is excellent tool support and rich documentation available for free. C♯ is platform independent and suitable for both, developer and userfriendly.The software architecture is composed of four main modules: user interface module, input/output module, algorithm module and chart module. Every part is skilfully integrated with the others and has its own function.

The rest of this paper is organized as flows. In section 2, we state some basic concepts of thermal behavior and pressure drop analysis in producing gas well and roughly describe the structure of well segment. In section 3, aiming for the distribution problem of temperature and pressure, the model is set up and the solution approach is introduced briefly. In section 4, through the corresponding program, transient pressure and temperature at any point in the wellbore can be computed which are shown in curves. In section 5, we compare our simplified model with Beggs–Shiu model[12] and show that there is good agreement between field data and results computed by our simplified model. Some concluding remarks are finally given in section 6.

2 Preliminaries

When a liquid is initially produced from a zone, its temperature at the sandface may be assumed to the same as that of the formation. While this is not true of gases, inlet gas temperature may be estimated from the formation temperature if the Joule–Thompson effect is properly accounted for. Thus, the bottomhole temperature of a produced fluid may be reliably estimated. However, as the fluid rises up the well in production, its temperature soon becomes significantly higher than the surrounding earth temperature because of the general decline in earth temperature with decreasing depth.

Heat is transferred to or from the wellbore when there is a difference in temperature between the surrounding formation and the produced fluid. At any depth, the formation temperature would vary not only with radial distance from the well, but also with production time. However, when steady flow has been attained, fluid turbulence ensures a constant fluid temperature at a given depth. Hence, heat loss from the fluid decreases with time and depends on the various resistances to heat flow between the hot fluid in tubing and the surrounding earth.

The complete system consists of the fluid, the tubing, the annular space containing low– pressure air, the casing, the cement, and the formation as is shown in Fig. 1. The inside radius of the tubing is r_{ti}, and the outside radius of the tubing is r_{to}. The inside radius of the casing is r_{ci}, and the outside radius of the casing is r_{co}. Heat is conducted through the air contained in the annulus. Radiation and natural convection also occur. When a body is heated, radiant energy is emitted at a rate dependent on the temperature of the body. The amount of radiant energy transported between the tubing and casing depends on the view the surfaces have of each other and the emitting and absorbing characteristics of their surfaces. In many cases, the annulus between the casing and the hole is cemented. Because the conductivity of cement may be lower than that of the surrounding earth. The calculation starts at the bottom of the wellbore and proceeds stepwise upward. We

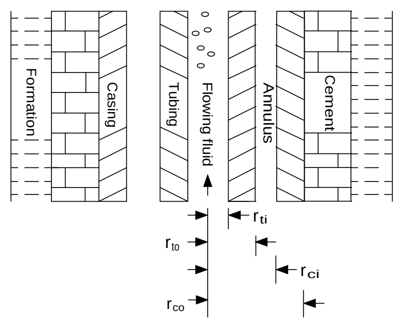

Fig. 1. Sketch map of well segment structure

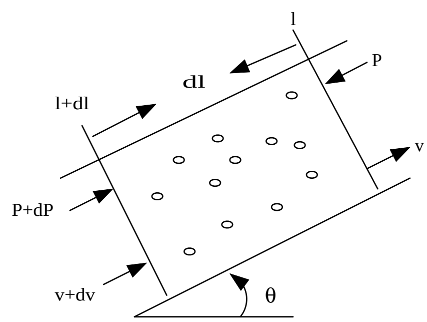

Fig. 2. Pressure drop analysis of pipe flow

establish the model using the inlet of gas well as the point of intersection of coordinate axes and the direction down along the tubing as the positive direction of coordinate axes. Fig. 2 shows pressure drop analysis on a short element of wellbore, where P represents pressure of flowing fluid; v is regarded as gas velocity; l is considered as well depth; dv means a velocity increment in interval dl; dp is deemed the pressure increment in interval dl; θ is the angle of inclination of wellbore.

3 Temperature and pressure distribution model

We propose a hybrid model derived from theoretical principles such as energy, mass and momentum balances and heat-transfer mechanisms of Ramey in a wellbore[9]. Then, the solution approach is introduced briefly.

3.1 Modeling

Aiming for corresponding to the actual situation of the production wells,we assume that: the flowing state of gas is unilateralism and at steady-state; the heat transfer in the wellbore was considered at steady-state; the heat transfer of earth is astaticism based on dimensionless transient heat conduction time function; the casing and tubing is concentric. Based on the assumption, the equations of energy, mass and momentum conservation can be reduced as follows:

(1) energy-balance

$$q + \frac{d}{dl}(h + v^2/2 - gl\sin\theta) = 0 \tag{1}$$

(2) mass-balance

$$\rho\frac{dv}{dl}+v\frac{d\rho}{dl}=0 \tag{2}$$

(3) momentum-balance

$$-\frac{1}{\rho}\frac{dP}{dl}+f\frac{v^2}{2r_{ti}}+g\sin\theta=\frac{vdv}{dl} \tag{3}$$

Ramey discussed the radial transfer of heat between the fluid and the earth in detail[9]. Over the differential element dl, the radial heat transfer from the fluid to the cement/earth interface can be described by

$$q=\frac{(-2\pi)r_{ti}U_{ti}}{w}(T-T_h) \tag{4}$$

The radial heat transfer from the cement/earth interface to the surrounding earth is

$$q=-\frac{2\pi ke}{wf(t_D)}(T_h-T_e) \tag{5}$$

Combining equations (4) and (5) gives the equation for the radial heat transfer between the flowing fluid and the surrounding earth:

$$q=\frac{2\pi}{w}\left[\frac{r_{ti}U_{ti}ke}{r_{ti}U_{ti}f(t_D)+ke}\right](T-T_e) \tag{6}$$

Then combining equations (1) and (6) yield the ordinary differential equation as follows:

$$\frac{2\pi}{w}\left[\frac{r_{ti}U_{ti}ke}{r_{ti}U_{ti}f(t_D)+ke}\right](T-T_e)+\frac{d}{dl}(h+v^2/2-gl\sin\theta)=0 \tag{7}$$

When flowing fluid in wellbore, Joule–Thomson coefficient is negligible and can be ignored because of the small change of diameter of wellbore. Therefore, considering $dh=C_pdT$, the overall enthalpy change in a flowing fluid is

$$\frac{2\pi}{w}\left[\frac{r_{ti}U_{ti}ke}{r_{ti}U_{ti}f(t_D)+ke}\right](T-T_e)+C_p\frac{dT}{dl}+v\frac{dv}{dl}-g\sin\theta=0 \tag{8}$$

Combining equations (1), (2), (3) and (8), we can expand the above equations to four order ordinary differential equation group about the flow pressure, temperature, gas velocity and density in gas wells. The equation group is as follows:

$$\begin{cases} \frac{d\rho}{dl}=\dfrac{\frac{-8.314z\rho}{C_pM}[\frac{(2\pi)r_{ti}U_{ti}ke}{w[r_{ti}U_{ti}f(t_D)+ke]}(T-T_e)-g\cos\theta]-f\frac{v^2\rho}{2r_{ti}}-g\rho\cos\theta}{v^2-(\frac{8.314zv^2}{C_pM}+\frac{8.314Tz}{M})} \\ \frac{dv}{dl}=-\frac{v}{\rho}\frac{d\rho}{dz} \\ \frac{dP}{dl}=\rho g\cos\theta+f\frac{v^2\rho}{2r_{ti}}+v^2\frac{d\rho}{dl} \\ \frac{dT}{dl}=\dfrac{[\frac{v^2}{\rho}\frac{d\rho}{dl}+g\cos\theta-\frac{(2\pi)r_{ti}U_{ti}ke}{w[r_{ti}U_{ti}f(t_D)+ke]}(T-T_e)]}{C_p} \end{cases} \tag{9}$$

and the four order ordinary differential equation group can be simplified using $y_i(i=1,2,3,4)$ represent ρ, v,P,T respectively and $F_i(i=1,2,3,4)$ represent right function of corresponding grads equations. The new equation group is as follows:

$$\frac{dy_i}{dl}=F_i(y_1,y_2,y_3,y_4)\quad(i=1,2,3,4) \tag{10}$$

To use equation (8), it is necessary to estimate U_{ti}, C_p and $f(t_D)$. Evaluation of the overall heat transfer coefficient is a difficult and critical step in finding an accurate solution. Willhite and Brid, et al. provided a detailed method for calculating the overall heat-transfer coefficient in terms of natural convection, conduction and radiation[3, 14]. The thermal resistance of the pipe and casing are negligible compared with the thermal resistance of the material in the tubing/casing annulus because of the high thermal conductivity of steel, and the equation developed by Willhite can be simplified as follows:

$$U_{ti}^{-1} = \frac{1}{hc + hr} + \frac{r_{ti}\ln(\frac{r_{cem}}{r_{co}})}{k_{cem}} + \frac{r_{ti}\ln(\frac{r_{ci}}{r_{to}})}{k_{ang}} \tag{11}$$

equation (11) shows that the material present in the tubing/casing annulus (gas, oil, water or any combinations) is important in determining the heat transfer in a wellbore system. The equation which is used to calculate specific heat to certain pressure C_p[17] is as follows:

$$C_p = 1243 + 3.14T + 7.931 \times 10^{-4}T^2 - 6.881 \times 10^{-7}T^3 \tag{12}$$

Ramey suggested values for both the transient-time function and the thermal conductivity of the earth[9]. The equations can be described as follows:

$$\begin{cases} f(t_D) = 1.1281\sqrt{t_D}(1 - 0.3\sqrt{t_D} & (t_D \leq 1.5) \\ f(t_D) = (1 + \frac{0.6}{t_D})[0.4063 + 0.5\ln(t_D)] & (t_D > 1.5) \end{cases} \tag{13}$$

where t_D suggested by Ramey is

$$t_D = \frac{t\alpha}{r_{wb}^2} \tag{14}$$

Jain in 1976 presented the equation to calculate friction coefficient f[6] in equation (3) which is as follows:

$$\frac{1}{\sqrt{f}} = 1.14 - 2lg(\frac{0.00001524}{r_{ti}} + \frac{21.25}{Re^{0.9}}) \tag{15}$$

3.2 Solution method

Because specific heat at constant pressure C_p and the inclination angle θ are not constant, the well must be partitioned into several length intervals using step length as h. And we assume that the well can be partitioned into n intervals where j ($j = 1, 2, 3, 4 \cdots n$) represents node and n represents the bottom of a well. According to the temperature and pressure of the bottom of well, we can calculate the corresponding gas density and fluid velocity. The boundary condition of the above differential equation group is as follows:

$$P[n] = P_0, T[n] = T_0, \rho[n] = 0.000001 \times 3484.48\gamma_g\frac{P_0}{zT_0}, v[n] = \frac{101000 \times 300000T_0}{293 \times 86400P_0A}$$

In order to calculate the other nodes, iterative equations are as follows:

$$y_i[j-1] = y_i[j] - \frac{h}{6}(a[i] + 2b[i] + 2c[i] + d[i]) \quad (i = 1, 2, 3, 4; j = 1, 2, 3, 4 \cdots n) \tag{16}$$

To use the above equations, it is necessary to estimate $a[i]$, $b[i]$, $c[i]$ and $d[i]$. The equations are as follows:

$$\begin{cases} a[i] = F_i(y_1[n], y_2[n], y_3[n], y_4[n]) \\ b[i] = F_i(y_1[n] + \frac{h}{2}a[1], y_2[n] + \frac{h}{2}a[2], y_3[n] + \frac{h}{2}a[3], y_4[n] + \frac{h}{2}a[4]) \\ c[i] = F_i(y_1[n] + \frac{h}{2}b[1], y_2[n] + \frac{h}{2}b[2], y_3[n] + \frac{h}{2}b[3], y_4[n] + \frac{h}{2}b[4]) \\ d[i] = F_i(y_1[n] + hc[1], y_2[n] + hc[2], y_3[n] + hc[3], y_4[n] + hc[4]) \end{cases} \tag{17}$$

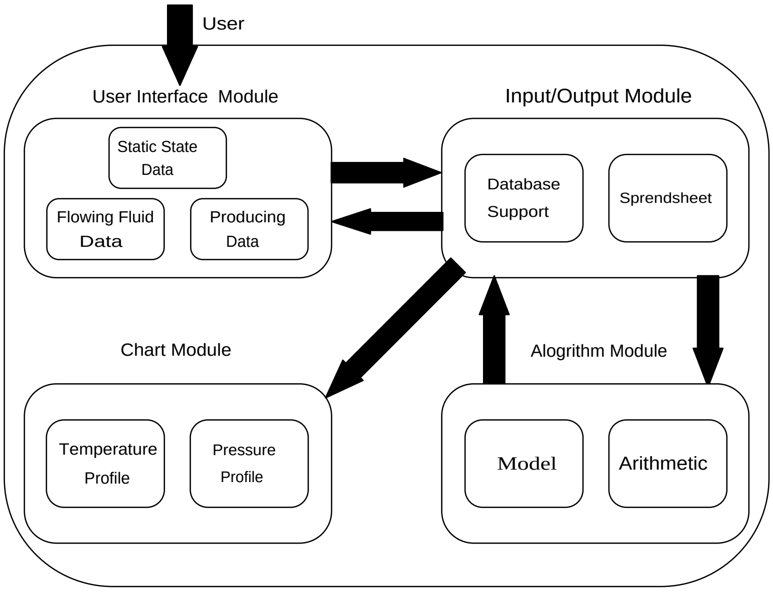

Fig. 3. Software architecture

4 Simulation and analysis

In order to simulate the temperature and pressure distribution, we compile the corresponding simulation software with computer programming language named C♯. The basic data of Da Well, 5160 meters of depth, are used for case calculation.

4.1 Software architecture

The software architecture (Fig. 3) is composed of four main modules:

(1) User interface module: This interface is concerned with the initial information of gas well which can be divided into static state data, flowing fluid data and producing data. We give a concise introduction in the following:

(i) Static state data: The data describe the characteristic of gas well in static state. S in short.

(ii) Flowing fluid data: The data are used to show the state of fluid such as water, oil and gas. F in short.

(iii) Producing data: The data represent the given parameters in producing stage. P in short. main modules:

All the data are compiled in Tab. 1 (2) Input/Output module: This module is responsible for read-

Table 1. Input data

name	value	unit	type
length of well	5160	m	P
critical temperature	189	K	P
critical pressure	4.57	Mpa	P
flowing fluid temperature at bottom	396.38	K	P
flowing fluid pressure at bottom	70	Mpa	P
production time	30	d	P
thermal conductivity of cement	0.52	W/m.C	S
thermal conductivity of gas in annulus	0.03	W/m.C	F
thermal conductivity of earth	2.06	W/m.C	S
thermal diffusivity of earth	1.03×10^{-6}	m^2/s	S
mixed gas specific gravity	0.56	dimensionless	F

ing/saving the data from/to a permanent storage. Both databases are built about wellbore and result using software named Microsoft SQL Server2000. Firstly, read the basic data from wellbore database. Then, save the result to the other database after calculating. Finally, read the result from database to plot curves.

(3) The algorithm module: The algorithm module is used for all mathematical computations. And we can compute temperature and pressure of any node from bottom to inlet.

(4) Chart module: The module is responsible for the graphical presentation of data. Through CrystalReports, we can plot curves of gas temperature and pressure and export the result.

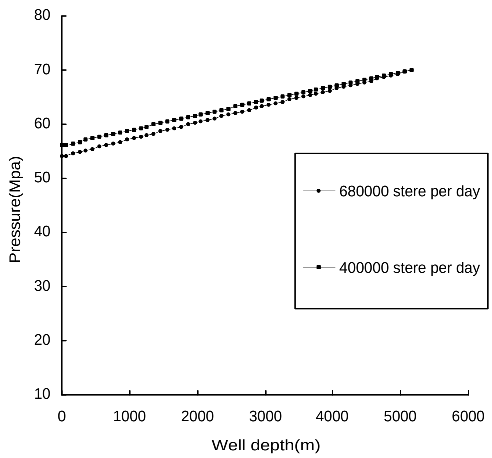

Fig. 4. Pressure distribution

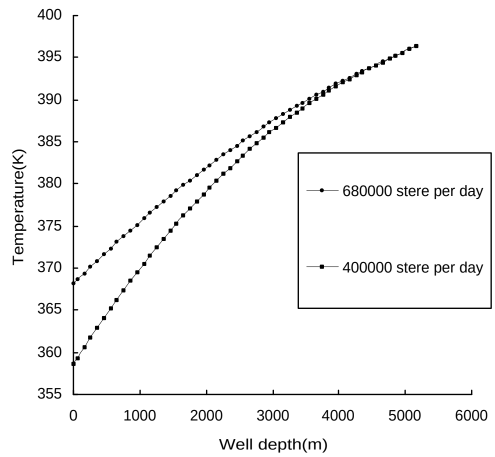

Fig. 5. Temperature distribution

4.2 Program flow

Program flow of temperature and pressure distribution is shown in Fig. 6

4.3 Results analysis

Through the distribution model and software simulation, we get a series of results. As is shown from Fig. 4 to Fig. 5, the temperature and pressure of flowing gas in producing gas well get lower from bottom to inlet due to gravitation, friction resistance and heat transferred from wellbore to earth. Temperature in gas well decreases a lot at the hight from o to 3000 meters, but it tends to be constant when the hight of gas well increase more; while pressure increases linearly when the hight increases. In addition, Fig. 4 shows that the pressure of gas become lower when gas production get bigger at the same well depth due to the accretion of frictional resistance; while temperature become higher because of the accretion of velocity of flow and the reduction of heat losing shown by Fig. 5.

5 Simplified–model evaluation

The overall performance of the simplified model was evaluated by measuring its ability to predict Measured flowing temperatures in a well accurately. We wish to compare the accuracy of the simplified model with that of Shiu and Beggs' correlation. Note that the simplified model is based primarily on fundamental principles, while the Shiu and Beggs procedure can be viewed as a correlation. Note also that Shiu and Beggs included wellhead temperatures. This temperature were excluded from our data base for reasons as these: Firstly, wellhead fluid temperatures are often unreliable because they can be influenced by errors in measurement procedure and by daily and seasonal temperature variations. Secondly, in particular, steel is a very good conductor of heat, and variations in temperature of the surface equipment can greatly influence the wellhead temperature.

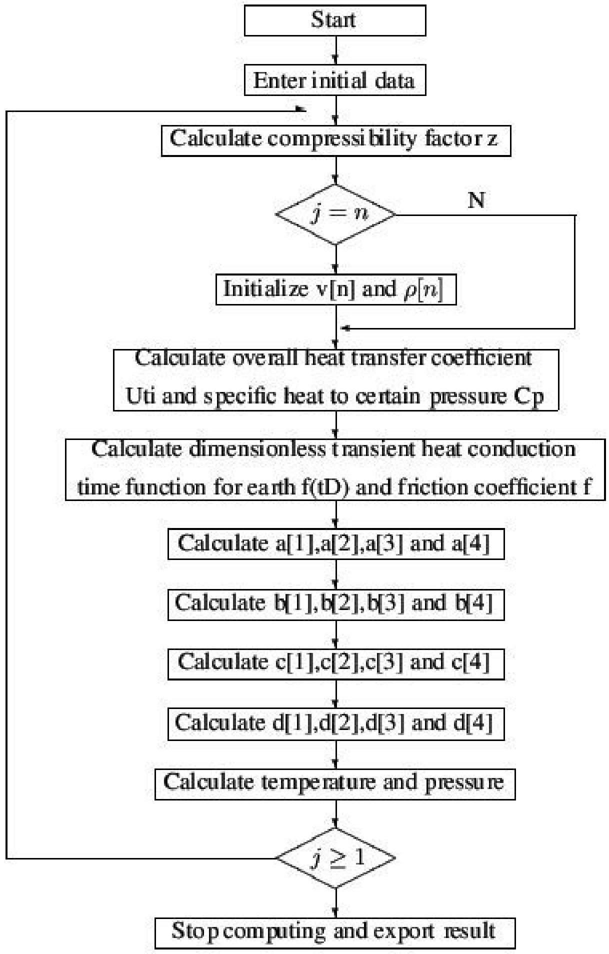

Fig. 6. Program flow figure

Fig. 7 show comparison between the computed fluid temperature profile to that predicted by each of two models(simplified model and Beggs–Shiu correlation). Note that the depths at which the temperatures were predicted were the depths at which the actual fluid temperature was measured. The overall conclusion from this comparison is that the simplified model represents an improvement over the Shiu and Beggs correlation and is accurate within its range of application.

6 Conclusion

The prediction of flowing fluid temperature and pressure has become necessary in several design problems that arise in gas production. In this paper, we establish a model based on mass, momentum and energy conservation and the interrelation of flow pressure, temperature, gas velocity and density in gas wells has been well considered to overcome the limitations of previous studies. For such a complex problem, numerical simulation is usually the best method. Thus we develop a numerical simulation software with programming language named C♯ to solve the model. Field data supports the modeling approach presented in this study. The results can provide the technical reliance for the process design of well test in high-temperature-high-pressure gas wells and dynamic analysis, such as hydrate formation prediction and tubing string design of production well.

Nomenclature

A=inside tubing acreage, m^2

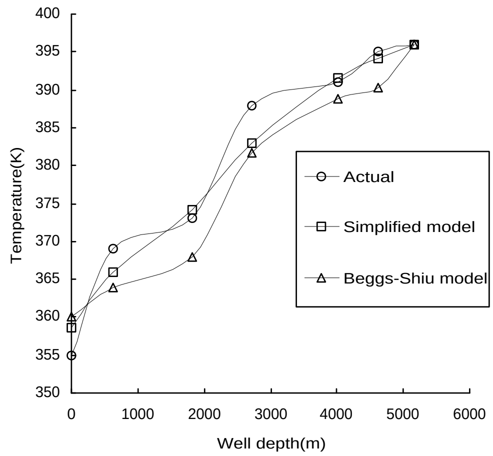

Fig. 7. Comparison of computed fluid temperature profiles to measured fluid temperature profile

C_p=specific heat at constant pressure, J/kg.K
f=friction coefficient, dimensionless
$f(t_D)$=dimensionless transient heat conduction time function for earth
g=acceleration of gravity
h=specific enthalpy
k_{ang}=thermal conductivity of gas in annulus,J/m.K
k_{cem}=thermal conductivity of cement,J/m.K
ke=thermal conductivity of earth,J/m.K
l=length of well,m
M=average molecular weight of gas,g/mol
P=flowing fluid pressure,pa
P_{pc}:critical pressure,pa
r_{cem}=ouside cement radius,m
r_{ci}=inside casing radius,m
r_{co}=ouside casing radius,m
r_{ti}=inside tubing radius,m
r_{to}=ouside tubing radius,m
r_{wb}=wellbore radius,m
Re=Renault data
t=production time,s
t_D=dimensionless time,dimensionless
T=flowing fluid temperature,K
T_e=surrounding earth temperature,k
T_h=cement/earth interface temperature,k
T_{pc}=critical temperature,K
U_{ti}=overall heat transfer coefficient,W/m.K
v=gas velocity,m/s
w=total mass flow rate,kg/s
z=compressibility factor
α=thermal diffusivity of earth,m^2/s
γ_g=mixed gas specific gravity,dimensionless
θ=angle of inclination,d
ρ=gas density,kg/m^3

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