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Some new solutions for the nonlinear dispersive-dissipative equation with a modified F-expansion method *

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Abstract. The modified F-expansion method is effective to solve nonlinear partial differential equations. By using the method and Mathematica, a series of solutions for the nonlinear dispersive-dissipative equation are derived. Some solutions are newly proposed and we haven't found in present works.

Keywords: nonlinear dispersive-dissipative equation, modified F-expansion method, exact solution, homogeneous balance method

1 Introduction

In recent years, it is well known that nonlinear complex physical phenomena are related to nonlinear partial differential equations (NLPDEs) which are involved in many fields from physics to biology, chemistry, mechanics, etc. As mathematical models of the phenomena, the investigation of exact solutions of NLPDEs will help one to understand these phenomenons better. Various methods for obtaining exact solutions of NLPDEs have been presented, such as inverse scattering method, Hirota's bilinear methods, Bäcklund transformation, Painlevé expansion, sine-cosine method, Adomian Pade approximation, homogeneous balance method[15], variational iteration method[5], direct algebraic method[9], tanh function method[4], linearized perturbation technique[6], multiple Riccati equations rational expansion method[11], variational method[1], the rational expansion method, and so on. Recently, F-expansion method[7, 10] was proposed to construct periodic wave solutions of NLPDEs, which can be thought of as an overall generalization of Jacobi elliptic function expansion method. F-expansion method was later further extended in different manners.

In this paper, we used a modified F-expansion method[2, 3] to construct more general exact solutions of the nonlinear dispersive-dissipative equation:

$$u_t + uu_x + \alpha u_{xxx} - (u_t + \beta uu_x)_x = 0 \tag{1}$$

where α, β are constants. The solutions of the nonlinear dispersive-dissipative equation possess their actual physical application; this is the reason why so many methods, such as bilinear method[14], modified homogeneous balance principle[12], truncated adjunct function method[13], have been applied to obtain exact solutions of the nonlinear dispersive-dissipative equation.

The rest of this paper is organized as follows: in section 2, we'll introduce the modified F-expansion method we presented in [2, 3]; in section 3, we'll apply this method to equation (1); in section 4, some conclusions are given.

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2 Description of the modified f-expansion method

For a given NLPDE with independent variables $x = (t, x_1, x_2, \cdots x_n)$ and dependent variable u:

$$P(u, u_t, u_{x_1}, u_{x_2}, \cdots, u_{x_m}, u_{x_1 t}, u_{x_2 t}, \cdots, u_{x_m t}, u_{x_1 x_1}, u_{x_2 x_2}, \cdots, u_{x_m x_m}, \cdots) = 0 \tag{2}$$

We seek the solutions in this form:

$$u = \sum_{i=-n}^{n} a_i F^i(\xi) \tag{3}$$

where $\xi = k(x - \lambda t)$ and k, λ are both constants $(k \neq 0)$. Here k denotes value of waves, λ denotes speed of waves, $a_i (i = -n, \cdots 0, \cdots, n), k, \lambda$ are constants to be determined, $F(\xi)$ and in (3) satisfy Riccati equation[8]

$$F'(\xi) = A + BF(\xi) + CF^2(\xi) \tag{4}$$

And

$$F''(\xi) = AB + (B^2 + 2AC)F(\xi) + 3BCF^2(\xi) + 2C^2F^3(\xi)$$
$$F'''(\xi) = (AB^2 + 2A^2C) + (B^3 + 8ABC)F(\xi) + (7B^2C + 8AC^2)F^2(\xi) + 12BC^2F^3(\xi) + 6C^3F^(\xi) \tag{5}$$

where A, B and C are all parameters, the prime denotes $d/d\xi$. Given different values of A, B and C, the different Riccati function solution $F(\xi)$ can be obtained from equation (4) (see Appendix A). To determine u explicitly, we take the following four steps:

Step 1. Determine the integer n by balancing the highest order nonlinear terms and the highest order partial derivative of u in equation (2).

Step 2. Substitute (3) along with (4) and (5) into equation (2) and collect coefficients of $F_i(\xi)(i = 01, \cdots, n)$, then set each coefficient to zero to derive a set of over-determined partial differential equations for $a_0, a_{-n}, \cdots, a_n$ and ξ.

Step 3. Solve the system of over-determined partial differential equations obtained in Step 2 for $a_0, a_{-n}, \cdots, a_n$ and ξ by use of Mathematica.

Step 4. Select A, B, C and $F(\xi)$ from Appendix A and substitute them along with $a_0, a_{-n}, \cdots, a_n$ and ξ. into (3) to obtain Riccati function solutions of equation (2) (see Appendix B for $F(\xi)$), from which hyperbolic function solutions and trigonometric function solutions can be obtained.

3 Exact solutions of the nonlinear dispersive-dissipative equation

By balancing uu_{xx} and u_{xxx} in equation (1), we get $n = 1$. In order to search for explicit solutions of equation (1), we set $\xi = k(x - \lambda t), k$ is a non-zero constant. Thus the ansatze solution of equation (1) can be expressed by

$$u(\xi) = a_{-1}F^(- 1)(\xi) + a_0 + a_1 F(\xi) \tag{6}$$

With the aid of Mathematica, substituting (6) along with (4) and (5) into equation (1), the left hand side of equation (1) is converted into a polynomial of $F_i(\xi)(i = 0, \pm 1, \cdots, \pm n)$, then setting each coefficient to zero, we get a set of over-determined algebraic equations for $a_i (i = -1, 0, 1), k$, and λ

$$-6A^3k^3\alpha a_{-1} - 3A^2k^2\beta a_{-1}^2 = 0$$
$$-12A^2Bk^3\alpha a_{-1} + 2A^2k^2\lambda a_{-1} - Aka_{-1}^2 - 2ABk^2\beta a_{-1}^2 - k^2\beta(3ABa_{-1}^2 + 2A^2a_{-1}a_0) = 0$$
$$Ak\lambda a_{-1} + 3ABk^2\lambda a_{-1} + k^3\alpha(-6AB^2a_{-1} + 2ACa_{-1}^2 - A(B^2a_{-1} + 2ACa_{-1}))$$
$$+k(-Ba_{-1}^2 - Aa_{-1}a_0) - k^2\beta(B^2a_{-1}^2 + 2ACa_{-1}^2 - 2A^2a_{-1}(\tfrac{2Ck\alpha}{\beta}))$$
$$-k^2\beta(a_{-1}(B^2a_{-1} + 2ACa_{-1}) + 3ABa_{-1}a_0 + 2A^2a_{-1}(\tfrac{2Ck\alpha}{\beta})) = 0$$
$$Bk\lambda a_{-1} + k^2\lambda(B^2a_{-1} + 2ACa_{-1}) + k^3\alpha(-8ABCa_{-1} - B^3a_{-1})$$
$$+k(-Ca_{-1}^2 - Ba_{-1}a_0) - k^2\beta(2BCa_{-1}^2 - 4ABa_{-1}(\tfrac{2Ck\alpha}{\beta}))$$
$$-k^2\beta(BCa_{-1}^2 + (B^2a_{-1} + 2ACa_{-1})a_0 + 4ABa_{-1}(\tfrac{2Ck\alpha}{\beta})) = 0$$
$$Ck\lambda a_{-1} + BCk^2\lambda a_{-1} - C^2k^2\beta a_{-1}^2 - Ck^3\alpha(B^2a_{-1} + 2ACa_{-1})$$
$$-Cka_{-1}a_0 - BCk^2\beta a_{-1}a_0 + AB^2k^3\alpha(\tfrac{2Ck\alpha}{\beta}) + 2A^2Ck^3\alpha(\tfrac{2Ck\alpha}{\beta})$$
$$-Ak\lambda(\tfrac{2Ck\alpha}{\beta}) + ABk^2\lambda(\tfrac{2Ck\alpha}{\beta}) + 2B^2k^2\beta a_{-1}(\tfrac{2Ck\alpha}{\beta})$$
$$+4ACk^2\beta a_{-1}(\tfrac{2Ck\alpha}{\beta}) - k^2\beta(B^2a_{-1} + 2ACa_{-1})(\tfrac{2Ck\alpha}{\beta}) + Aka_0(\tfrac{2Ck\alpha}{\beta})$$
$$-ABk^2\beta a_0(\tfrac{2Ck\alpha}{\beta}) - A^2k^2\beta(\tfrac{2Ck\alpha}{\beta})^2 - k^2\beta(B^2a_1 + 2AC(\tfrac{2Ck\alpha}{\beta})) = 0$$
$$-Bk\lambda(\tfrac{2Ck\alpha}{\beta}) + k^2\lambda(B^2 + 2AC)(\tfrac{2Ck\alpha}{\beta}) + k^3\alpha(B^3 + 8ABC)(\tfrac{2Ck\alpha}{\beta})$$
$$+k(Ba_0 + A(\tfrac{2Ck\alpha}{\beta}))(\tfrac{2Ck\alpha}{\beta}) - k^2\beta(-4BCa_{-1} + 2AB(\tfrac{2Ck\alpha}{\beta}))(\tfrac{2Ck\alpha}{\beta})$$
$$-k^2\beta(4BCa_{-1}(\tfrac{2Ck\alpha}{\beta}) + AB(\tfrac{2Ck\alpha}{\beta})^2 + a_0(B^2 + 2AC)(\tfrac{2Ck\alpha}{\beta})) = 0$$
$$-Ck\lambda(\tfrac{2Ck\alpha}{\beta}) + 3BCk^2\lambda(\tfrac{2Ck\alpha}{\beta}) + k^3\alpha(7B^2C + 8AC^2)(\tfrac{2Ck\alpha}{\beta})$$
$$+k(Ca_0 + B(\tfrac{2Ck\alpha}{\beta}))(\tfrac{2Ck\alpha}{\beta}) - k^2\beta(-2c^2a_{-1} + B^2(\tfrac{2Ck\alpha}{\beta}) + 2AC(\tfrac{2Ck\alpha}{\beta}))(\tfrac{2Ck\alpha}{\beta})$$
$$-k^2\beta(2C^2a_{-1}(\tfrac{2Ck\alpha}{\beta}) + 3BCa_0(\tfrac{2Ck\alpha}{\beta}) + (B^2 + 2AC)(\tfrac{2Ck\alpha}{\beta})^2) = 0$$
$$12BC^2k^3\alpha(\tfrac{2Ck\alpha}{\beta}) + 2C^2k^2\lambda(\tfrac{2Ck\alpha}{\beta}) + Ck(\tfrac{2Ck\alpha}{\beta})^2$$
$$-2BCk^2\beta(\tfrac{2Ck\alpha}{\beta})^2 - k^2\beta(2C^2a_0 + 3BC(\tfrac{2Ck\alpha}{\beta}))(\tfrac{2Ck\alpha}{\beta}) = 0$$

Solving these over-determined algebraic equations by use of Mathematica, we get the following results:

Case 1. $\lambda = \frac{\alpha}{\beta(\beta-1)}, a_0 = \frac{\alpha - Bk\alpha + Bk\alpha\beta}{\beta(\beta-1)}, a_{-1} = 0, a_1 = \frac{2Ck\alpha}{\beta}$, where k is an arbitrary constant.

Case 2. $\lambda = \frac{\alpha}{\beta(\beta-1)}, a_0 = \frac{\alpha(1+6\beta)}{7\beta^2(\beta-1)}, a_{-1} = 0, a_1 = -\frac{2Ck\alpha}{7\beta^2 B}, k = -\frac{1}{7B\beta}$.

Case 3. $\lambda = \frac{\alpha}{\beta(\beta-1)}, a_0 = \frac{\alpha(B^2+AC+4AC\beta)}{(B^2+5AC)(\beta-1)\beta^2}, a_{-1} = 0, a_1 = -\frac{2C\alpha(B^2+AC)}{\beta^2 B(B^2+5AC)}, k = -\frac{(B_A^2 C)}{B\beta(B^2+5AC)}$.

Case 4. $\lambda = \frac{\alpha}{\beta(\beta-1)}, a_0 = \frac{\alpha(2B^2+^2\beta+4AC\beta)}{(5B^2+4AC)(\beta-1)\beta^2}, a_{-1} = 0, a_1 = -\frac{4BC\alpha}{\beta^2 B(B^2+5AC)}, k = -\frac{(2B}{\beta^2(5B^2+4AC)}$.

Substituting Cases 1-4 into (6) respectively, we have four kinds of formal solutions of equation (1):

$$\text{(a)} \quad u(\xi) = \frac{\alpha - Bk\alpha + Bk\alpha\beta}{\beta(\beta-1)} + \frac{2Ck\alpha}{\beta}F(\xi), \xi = k(x - \frac{\alpha}{\beta(\beta-1)}t)$$

From Appendix A, choosing $A, B, C, F(\xi)$, inserting them into (a), we obtained seven kinds of Riccati function solutions of equation (1):

Choosing $A = 0, B = 1, C = -1, F(\xi) = \frac{1}{2} + \frac{1}{2}tanh(\frac{\xi}{2})$, inserting them into (a), we obtain

$$u_1(\xi) = \frac{\alpha}{\beta(\beta-1)} - \frac{k\alpha tanh(\frac{\xi}{2})}{\beta}$$

where $\xi = k(x - \alpha t/\beta(\beta-1))$;

Choosing $A = 0, B = -1, C = 1, F(\xi) = \frac{1}{2} + \frac{1}{2}coth(\frac{\xi}{2})$, inserting them into (a), we obtain

$$u_2(\xi) = \frac{\alpha}{\beta(\beta-1)} - \frac{k\alpha coth(\frac{\xi}{2})}{\beta}$$

where $\xi = k(x - \alpha t/\beta(\beta-1))$;

Choosing $A = 1/2, B = 0, C = -1/2, F(\xi) = coth(\xi) \pm csch(\xi)$or $tanh\xi \pm isech\xi$, inserting them into (a), we obtain

$$u_3(\xi) = \frac{\alpha}{\beta(\beta-1)} - \frac{k\alpha coth(\xi) \pm csch(\xi)}{\beta},$$

$$u_4(\xi) = \frac{\alpha}{\beta(\beta-1)} - \frac{k\alpha tanh\xi \pm isech\xi}{\beta},$$

where $\xi = k(x - \alpha t/\beta(\beta-1))$;

Choosing $A = 1, B = 0, C = -1, F(\xi) = tanh\xi$ or isech ξ, inserting them into (a), we obtain

$$u_5(\xi) = \frac{\alpha}{\beta(\beta-1)} - \frac{2k\alpha tanh\xi}{\beta},\ u_6(\xi) = \frac{\alpha}{\beta(\beta-1)} - \frac{2k\alpha coth\xi}{\beta},$$

where $\xi = k(x - \alpha t/\beta(\beta-1))$;

Choosing $A = 1/2, B = 0, C = 1/2, F(\xi) = sec\xi + tan\xi, orcsc\xi - cot\xi$, inserting them into (a), we obtain

$$u_7(\xi) = \frac{\alpha}{\beta(\beta-1)} - \frac{k\alpha(sec\xi + tan\xi)}{\beta},\ u_8(\xi) = \frac{\alpha}{\beta(\beta-1)} - \frac{k\alpha(csc\xi - cot\xi)}{\beta},$$

where $\xi = k(x - \alpha t/\beta(\beta-1))$;

Choosing $A = -1/2, B = 0, C = -1/2, F(\xi) = sec\xi - tan\xi, orcsc\xi + cot\xi$, inserting them into (a), we obtain

$$u_9(\xi) = \frac{\alpha}{\beta(\beta-1)} - \frac{k\alpha(sec\xi - tan\xi)}{\beta},\ u_{10}(\xi) = \frac{\alpha}{\beta(\beta-1)} - \frac{k\alpha(csc\xi + cot\xi)}{\beta},$$

where $\xi = k(x - \alpha t/\beta(\beta-1))$;

Choosing $A = 1(-1), B = 0, C = 1(-1), F(\xi) = tan\xi, orcot\xi$, inserting them into (a), we obtain

$$u_{11}(\xi) = \frac{\alpha}{\beta(\beta-1)} + \frac{2k\alpha cot\xi}{\beta},\ u_{12}(\xi) = \frac{\alpha}{\beta(\beta-1)} - \frac{2k\alpha cot\xi}{\beta},$$

where $\xi = k(x - \alpha t/\beta(\beta-1))$;

Choosing $A = 0, B = 0, C \neq 0, F(\xi) = -1/(C\xi + m)$ (m is an arbitrary constant), inserting them into (a), we obtain

$$u_{13}(\xi) = \frac{\alpha}{\beta(\beta-1)} - \frac{2Ck\alpha}{\beta(C\xi + m)},$$

where $\xi = k(x - \alpha t/\beta(\beta-1))$;

$$(b) \quad u(\xi) = \frac{\alpha(1+6\beta)}{7(\beta-1)\beta^2} - \frac{2C\alpha}{7B\beta^2}F(\xi), \xi = -\frac{1}{7B\beta}(x - \frac{\alpha}{\beta(\beta-1)t})$$

Choosing $A = 0, B = 1, C = -1, F(\xi) = \frac{1}{2} + \frac{1}{2}tanh(\frac{\xi}{2})$, inserting them into (b), we obtain

$$u_{14}(\xi) = \frac{\alpha(1+6\beta)}{7(\beta-1)\beta^2} - \frac{2\alpha}{7\beta}(\frac{1}{2} + \frac{1}{2}tanh(\frac{\xi}{2})),$$

where $\xi = \frac{1}{7\beta}(x - \frac{\alpha}{\beta(\beta-1)}t)$;

Choosing $A = 0, B = -1, C = 1, F(\xi) = \frac{1}{2} + \frac{1}{2}coth(\frac{\xi}{2})$, inserting them into (b), we obtain

$$u_{15}(\xi) = \frac{\alpha(1+6\beta)}{7(\beta-1)\beta^2} - \frac{2\alpha}{7\beta}(\frac{1}{2} + \frac{1}{2}coth(\frac{\xi}{2})),$$

where $\xi = \frac{1}{7\beta}(x - \frac{\alpha}{\beta(\beta-1)}t)$;

Choosing A as an arbitrary constant, B as a non-zero arbitrary constant, $C = 0, F(\xi) = (exp(B\xi) - A)/B$, inserting them into (b), we obtain

$$u_{16}(\xi) = \frac{\alpha(1+6\beta)}{7\beta^2(\beta-1)}$$

where $\xi = -\frac{1}{7B\beta}(x - \frac{\alpha}{\beta(\beta-1)}t)$;

$$(c)\quad u(\xi)=\frac{\alpha(B^2+AC+4AC\beta)}{(B^2+5AC)(\beta-1)\beta}-\frac{2C\alpha(B^2+AC)}{B\beta^2(B^2+5AC)}F(\xi)$$

$$\xi=-\frac{(B^2+AC)}{B\beta(B^2+5AC)}(x-\frac{\alpha}{\beta(\beta-1)}t)$$

Choosing $A=0,B=1,C=-1,F(\xi)=\frac{1}{2}+\frac{1}{2}tanh(\frac{\xi}{2})$, inserting them into (c), we obtain

$$u_{17}(\xi)=\frac{\alpha}{\beta^2(\beta-1)}-\frac{2\alpha}{\beta^2}(\frac{1}{2}+\frac{1}{2}tanh(\frac{\xi}{2})),$$

where $\xi=-(x-\frac{\alpha}{\beta(\beta-1)}t)/\beta$;

Choosing $A=0,B=-1,C=1,F(\xi)=\frac{1}{2}-\frac{1}{2}coth(\frac{\xi}{2})$, inserting them into (c), we obtain

$$u_{18}(\xi)=\frac{\alpha}{\beta^2(\beta-1)}-\frac{2\alpha}{\beta^2}(\frac{1}{2}+\frac{1}{2}coth(\frac{\xi}{2})),$$

where $\xi=(x-\frac{\alpha}{\beta(\beta-1)}t)/\beta$;

Choosing A as an arbitrary constant, B as a non-zero arbitrary constant, $C=0,F(\xi)=(exp(B\xi)-A)/B$, inserting them into (c), we obtain

$$u_{19}(\xi)=\frac{\alpha}{\beta^2(\beta_1)}$$

where $\xi=-(x-\frac{\alpha}{\beta(\beta-1)}t)/B\beta$;

$$(d)\quad u(\xi)=\frac{\alpha(2B^2+3B^2\beta+4AC\beta)}{(5B^2+4AC)(\beta_1)\beta^2}-\frac{4BC\alpha}{(5B^2+4AC)\beta^2}F(\xi),$$

$$\xi=-\frac{2B}{(5B^2+4AC)\beta^2}(x-\frac{\alpha}{\beta(\beta-1)}t).$$

Choosing $A=0,B=1,C=-1,F(\xi)=\frac{1}{2}+\frac{1}{2}tanh(\frac{\xi}{2})$, inserting them into (d), we obtain

$$u_{20}(\xi)=\frac{\alpha(2+3\beta)}{5\beta^2(\beta-1)}-\frac{4\alpha}{5\beta^2}(\frac{1}{2}+\frac{1}{2}tanh(\frac{\xi}{2})),$$

where $\xi=-\frac{2(x-\frac{\alpha}{\beta(\beta-1)}t)}{5\beta}$;

Choosing $A=0,B=-1,C=1,F(\xi)=\frac{1}{2}-\frac{1}{2}coth(\frac{\xi}{2})$, inserting them into (d), we obtain

$$u_{21}(\xi)=\frac{\alpha(2+3\beta)}{5\beta^2(\beta-1)}-\frac{4\alpha}{5\beta^2}(\frac{1}{2}-\frac{1}{2}tanh(\frac{\xi}{2})),$$

where $\xi=-\frac{2(x-\frac{\alpha}{\beta(\beta-1)}t)}{5\beta}$;

Choosing A as an arbitrary constant, B as a non-zero arbitrary constant, $C=0,F(\xi)=(exp(B\xi)-A)/B$, inserting them into (d), we obtain

$$u_21(\xi)=\frac{\alpha(2+3\beta)}{5\beta^2(\beta-1)}$$

where $\xi=-\frac{2(x-\frac{\alpha}{\beta(\beta-1)}t)}{5\beta}$.

Let $\alpha=k=1,\beta=2,C=1$, we get some numerical simulation images of some solutions as following:

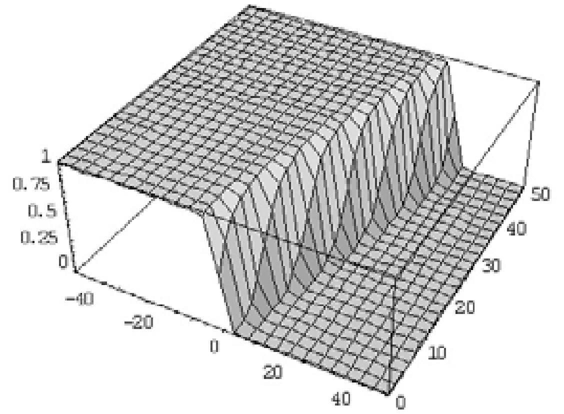

Fig. 1. $u_1(\xi) = \alpha/\beta(\beta-1) - k\alpha tanh(\frac{\xi}{2})/\beta$

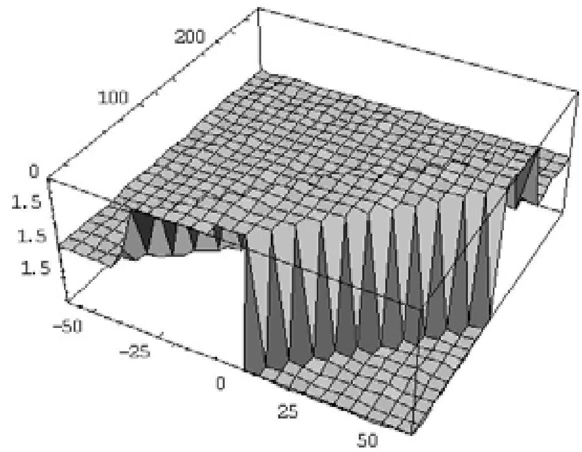

Fig. 2. $u_1(\xi) = \alpha/\beta(\beta-1) - \frac{2k\alpha coth\xi}{\beta}$

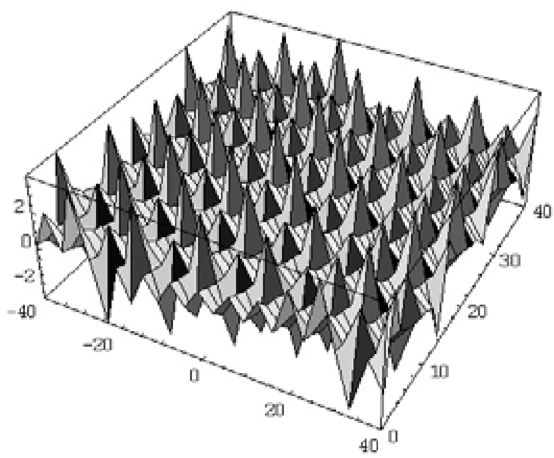

Fig. 3. $u_7(\xi) = \alpha/\beta(\beta-1) + k\alpha(sec\xi + tan\xi)/\beta,$

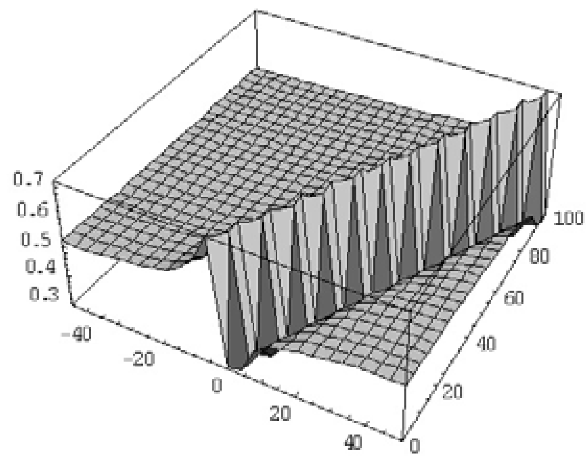

Fig. 4. $u_8(\xi) = \alpha/\beta(\beta-1) - 2Ck\alpha/\beta(C\xi + m)$

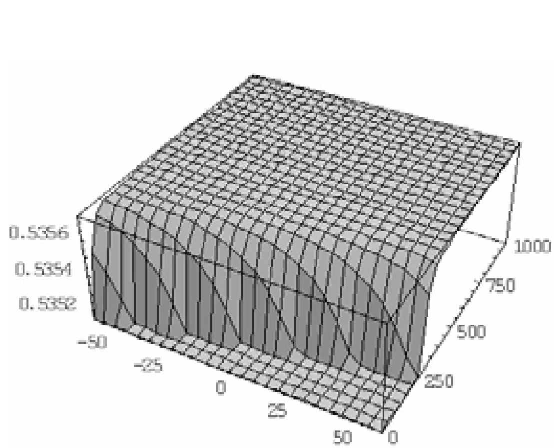

Fig. 5. $u_9(\xi) = \alpha/\beta(\beta-1) - ka(sec\xi - tan\xi)/\beta$

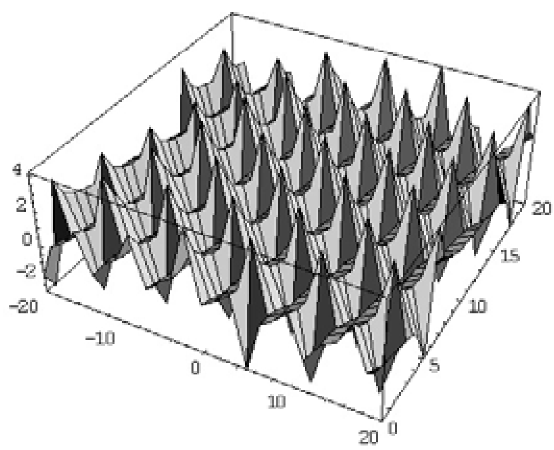

Fig. 6. $u_14(\xi) = \alpha/(1+6\beta)/7\beta^2(\beta-1) + \frac{2\alpha}{7\beta}(\frac{1}{2} + \frac{1}{2}tanh(\frac{\xi}{2}))$

4 Conclusion

In this paper, we have used a modified F-expansion method[2, 3] to construct more exact solutions of the nonlinear dispersive-dissipative equation. With the aid of Mathematica, the method provides a powerful mathematical tool to obtain more general exact solutions of a great many NLPDEs in mathematical physics.

What is remarkable is that this method is more usually used in the case where the odd and even partial differential terms don't exist simultaneously. Applying this method to the nonlinear dispersive-dissipative equation, we have successfully obtained many new and more general soliton-like solutions, trigonometric function solutions and rational solutions.

Appendix

Relations between values of (A, B, C) and corresponding $F(\xi)$ in Riccati equation

$$F'(\xi) + A + BF(\xi) + CF^2(\xi)$$

are listed in Tab. 1.

Table 1. The relations between values of (A, B, C) and corresponding $F(\xi)$ in Riccati equation

A	B	C	F
0	1	-1	$\frac{1}{2}+\frac{1}{2}tanh(\frac{1}{2}\xi)$
0	-1	1	$\frac{1}{2}-\frac{1}{2}coth(\frac{1}{2}\xi)$
$\frac{1}{2}$	0	$-\frac{1}{2}$	$coth\xi \pm csch\xi, tanh\xi \pm isech\xi$
1	0	-1	$tanh\xi, coth\xi$
$\frac{1}{2}$	0	$\frac{1}{2}$	$sec\xi + tan\xi, csc\xi - cot\xi$
$-\frac{1}{2}$	0	$-\frac{1}{2}$	$sec\xi - tan\xi, csc\xi + cot\xi$
1(-1)	0	1(-1)	$tan\xi(cot\xi)$
0	0	$\neq 0$	$-\frac{1}{C\xi+\lambda}$ (λ is an arbitrary constant)
arbitrary constants	0	0	$A\xi$
arbitrary constants	$\neq 0$	0	$\frac{exp(B\xi)-A}{B}$

References

[1] G. A. Afrouzi, S. Heidarkhani. Multiplicity results for the dirichlet boundary value problem involving the p-laplacian in N-dimensional case. *World Journal of Modelling and Simulation*, 2006, **4**(2): 222–226.

[2] G.-L. Cai, Q.-C. Wang. A modified F-expansion method for solving nonlinear PDEs. *Journal of Information and computing Science*, 2007, **1**(2): 3–16.

[3] G.-L. Cai, Q.-C. Wang, J.-J. Huang. A modified F-expansion method for solving breaking soliton equation. *International Journal of Nonlinear Science*, 2006, **2**(2): 122–128.

[4] S. A. El-Wakil, M. A. Abdou. Modified extended tanh-function method for solving nonlinear partial differential equations. *Chaos, Solitons and Fractals*, 2007, (31): 1256–1264.

[5] J.-H. He. Variational iteration method for delay differential equations. *Communications in Nonlinear Science & Numerical Simulation*, 1997, **4**(2): 235–236.

[6] J.-H. He. Analytical solution of a nonlinear oscillator by the linearized perturbation technique. *Communications in Nonlinear Science & Numerical Simulation*, 1999, **2**(4): 109–113.

[7] X.-Z. Li, Y.-M. Wang, X.-Y. Li, B.-A. Li, M.-L. Wang. A solving method for compound kdv-burgers equation. *Journal of Henan University of Science and Technology(Natural Science)*, 2003, **4**(24): 104–107.

[8] Y.-M. Li, S.-S. Hu. T-S fuzzy fault-tolerant control via riccati equation. *World Journal of Modelling and Simulation*, 2005, **1**(1): 27–34.

[9] Y.-D. Shang. Explicit and exact solutions to a nonlinear dispersive-dissipative equation. *Appl. Math. -JCU*, 1998, **3**(14): 280–284.

[10] M.-L. Wang, Y.-B. Zhou. The periodic wave solutions for the klein-gordon-schrodinger equations. *Phys. Lett. A*, 2003, (318): 84–92.

[11] Q. Wang, Y. Chen. A multiple riccati equations rational expansion method and novel solutions of the broer-kaup-kupershmidt system. *Chaos, Solitons and Fractals*, 2006, (30): 197–203.

[12] X. Wang, Y.-P. Hu. New exact solutions to a nonlinear dispersive- dissipative equation. *Journal of Zhengzhou Institute Of Light Industry(Natural Science)*, 2004, **2**(19): 57–58.

[13] X. Wang, S.-B. Wang. New solitary wave solutions and periodic wave solutions to a nonlinear dispersive-dissipative equation. *Journal of Zhengzhou University (Engineering Science)*, 2002, **3**(23): 41–43.

[14] S.-Q. Wu. Soliton solutions of a nonlinear dispersive-dissipative equation. *Journal of Sichuan Normal University (Natural Science)*, 1998, **2**(21): 174–175.

[15] J.-L. Zhang, Y.-M. Wang, D.-W. Yang, M.-L. Wang, C.-J. Qin. Exact solutions to nonlinear dispersive long wave equations with the variable coefficients. *Journal of Luoyang Institute of Technology*, 2002, **2**(23): 102–105.