

Topsis-AHP simulation model and its application to supply chain management*

Min Wu[†]

School of public Administration, Sichuan University, Chengdu 610065, P.R.China

(Received October 1 2006, Accepted January 2 2007)

Abstract. SCM (supply chain management) emphasizes on the strategic cooperative relationship between core enterprise and enterprise alliance. The selection of strategic partners is an important decision-making problem in SCM and is the key to successful SCM. In this paper, aiming for the supplier selection problem, the author discusses a class of AHP (analytical hierarchy process) technique—simulation approach, which is valuable in that it examines the uncertainty in AHP and helps to reduce the uncertainty in AHP to some extent. Then the approach is illustrated by solving a simplified supplier selection problem in SCM.

Keywords: AHP, simulation, SCM, selection, uncertainty

1 Introduction

There are various decision-making methods, among which AHP (analytical hierarchy process) technique has been developed as a useful and simple method to deal with decision-making problems. Just as it was said, “AHP (Saaty^[8-10], Golden et al.^[4]) has been applied in a variety of areas as a useful and practical multi-criteria decision analysis tool.”^[6] In the area of SCM (supply chain management), we prefer to use AHP method. Recently, SCM is becoming a hot issue in international academic circle. Here goes the definition of supply chain and supply chain management given by American Supply Chain Association. The supply chain—a term now commonly used internationally—encompasses every effort involved in producing and delivering a final product or service, from the supplier’s suppliers to the customer’s customers. The core of SCM is a decision-making problem. Suppliers are very important in the supply chain because their performances not only influence the benefits of the core enterprise in the supply chain but also determine whether win-win can be achieved or not. Therefore, supplier selection is both a key problem and a tough one. We believe that by using a class of technique we may solve the key and tough problems of supplier selection in SCM. Thus we develop an AHP simulation methodology to deal with SCM problems.

Though AHP is advantageous in many aspects, the problem of uncertainty becomes the bottleneck of AHP. The core of AHP is the preference matrix consisting of pair-wise comparison, which involves some subjective and uncertain factors. Therefore the problem of uncertainty appears and has been studied by several authors for years. The concept of judgmental uncertainty was put forth by Saaty^[7] in 1978 and was later elaborated it by Vargas and Zahedi^[12] by treating the pairwise comparisons as random variables in 1982. Saaty and Vargas^[11] outlined the mechanism analytically how the uncertainty affects the ranks of the decision alternatives. Zahedi has investigated how judgmental uncertainty can be incorporated within the AHP framework. However there are some limitations in their methodologies, such as difficult algorithm, overestimates of the rank uncertainty, and no consideration of the probability distribution underlying the judgmental uncertainties. Later, in the paper entitled “Consequences of Uncertainty in the Analytic Hierarchy Process: A Simulation

* The work is supported by Foundation of Sichuan Provincial Bureau of Science and Technology in 2007. Corresponding author.

[†] Corresponding author. Tel.: +86-28-85412429; E-mail address: helenmin@263.net.

Approach”, Dan Paulson and Sajjad Zahir put forward a simulation approach of AHP to analyze the uncertainty in AHP. This new technique is different and advanced in that it overcomes the limitations of traditional AHP method and helps to reduce the uncertainty in AHP, thus it breaks down the bottleneck of AHP. In this paper, we concentrate on this class of AHP technique, and we are motivated to study the application of this technique to SCM. Although for a decision-making problem, many models are available, the simulation approach of AHP is preferred because it can be applied to handle uncertainty of decision-making process. In this paper, we propose a simulation approach of AHP model for supplier selection in SCM.

This paper is organized as follows. In Section II, aiming for the supplier selection problem in SCM, a simulation approach of AHP is proposed and the methodology is introduced briefly. In Section III, as an application of the technique, we discuss a simplified specific supplier selection problem in SCM, examining the uncertainty by using simulation approach of AHP. Some concluding remarks are finally given in Section IV.

2 Methodology

Consider a scenario where a core enterprise in a supply chain is faced with a decision-making problem, i.e. choosing one best supplier from the four suppliers. Aiming for this supplier selection problem in SCM, we prefer to use AHP simulation approach, because it not only overcomes the limitations of AHP but also seems technically feasible.

In most situations the decision maker’s pair-wise comparisons would contain a degree of uncertainty. We adhere to the original axioms of AHP (Saaty^[8], 1980) with one exception: we assume a continuous ratio scale from 1 to 9 for preference matrices so that we can observe the probability of rank reversal over a wide range of uncertainties (between 2% and 20%). We assume that all uncertainties in the preference matrix result from doubts expressed by an individual decision maker as to the accuracy of his or her judgments.

According to Zahedi^[14] (1986), there are two sources of this judgmental uncertainty in this specific decision-making problem in SCM. External source of uncertainty refers to the procedure or environment for collecting preference data of suppliers. Internal sources refer to the ambiguity and uncertainty, which result from the limited amount of information available to the decision-maker and the level of his or her understanding of the problem. Information, as the communication carrier in the supply chain, plays a significant role both on the cooperation among enterprises and on the utilization of resources. Yet, informational risk is inevitable because of dissymmetric or insufficient information. Both sources can lead to rank reversals and thus weaken the decision-maker’s confidence in the results of the AHP.

While using this type of model, it is assumed that the credibility, quality and efficiency of the suppliers in SCM are normally distributed. After careful consideration of all the criteria of the suppliers, the decision maker has entered his or her relative preferences in a 4 by 4 preference matrix A based on pair-wise comparison among the suppliers. We assume that each element A_{ij} of the matrix has associated with it a standard deviation σ which expresses the uncertainty caused by external and internal sources. Our goal is to compute the uncertainty in the principal eigenvector w of A , which results from the uncertainties in A . We employ a simulation technique:

Step 1. A reciprocal matrix A is generated by the judgment of decision maker: $A = [A_{ij}]$, where A_{ij} is a uniform variable with values between 1 and 9 or is the reciprocal of such a variable. A_{ji} is set equal to $1/A_{ij}$.

Step 2. Then, a set of N random matrices are generated: $[A^1, A^2, \dots, A^N]$, where each A^K is obtained by assuming that each element of A greater than or equal to 1 is a normally distributed random variant with mean A_{ij} and standard deviation σ_{ij} . That is, for all $A_{ij} \geq 1$, A_{ij}^K is a normally distributed random variant with mean A_{ij} and standard deviation σ_{ij} .

Step 3. For each A^K , the principal eigenvector w^K is calculated.

Step 4. Each w^K is interpreted as measurement of the random variant w (the principal eigenvector of A). The mean w_i and standard deviation σ_i of each element w is calculated. An aggregate measure of the rank uncertainty RUrms in w is obtained using the following formula (adapted from Saaty^[8]):

$$RU_{rms} = \sqrt{\frac{1}{n} \sum_1^n (\sigma_j / w_j)}$$

We also calculate the probability of a rank reversal P_{rev} by examining each w^k and comparing it to w (the principal eigenvector of A). The reversal of the rankings of any two alternatives is considered as a rank reversal. The probability is calculated as in [11].

Step 5. Notice that the above steps derive RU_{rms} and P_{rev} for a particular A . In order to generalize the results to any A , the steps must be applied to a larger number of random matrices. $\overline{RU_{rms}}$ is the average RU_{rms} for a large sample of randomly generated A matrices. $\overline{P_{rev}}$ is the average P_{rev} for a large sample of randomly generated A matrices.

RU_{rms} and P_{rev} imply the uncertainty of the preference matrix. If the calculation consequences are large, it is shown that the matrix should be altered, the reliability of the principle eigenvector w^k is uncertain, and the confidence of the final choice is doubtful and should be reduced. Thus the uncertainty problem in supplier selection process is examined scientifically.

3 Application

3.1 Backgrounds

Since 1990s, with the rapid development of IT, the forming of globalization, the aggravation of severe competition, the ceaseless changes of market demand, enterprises are faced with the challenges of reducing the cost, decreasing the storage, improving the quality and service, enhancing customer satisfaction, shortening the delivery date, raising efficiency, and heightening competitive awareness. The new economy, characterized by net economy, knowledge economy and commerce, is forging ahead and bringing about great changes of enterprise development trend day by day. If enterprises can not only plan and operate internal resources but also integrate external resources, they can ensure their competitive advantages for survival and development in the fiercely competitive environment. Nowadays, costumers begin to require that products be obtained at any time, any place, the lowest price and the fastest speed. In order to meet the demands of customers, enterprises have to adjust the logistic process driven by customers' services and implement SCM, a new management model, which adapts to social, economic and technological environments in the new era.

SCM emphasizes on the strategic cooperative relationship between core enterprise and enterprise alliance. SCM includes managing supply and demand, sourcing raw materials and parts, manufacturing and assembly, warehousing and inventory tracking, order entry and order management, distribution across all channels, and delivery to the customer. Under the environment of globalization market competition and cooperation, SCM is an effective model of enterprise operation and management. As enterprises pay more and more attention to their core competence, they are increasingly unwilling to devote the capital, time and energy to those businesses which they are not familiar with and not good at. This change is also reflected in supply system, i.e., enterprise outsourcing or seeking for proper suppliers who can provide the businesses or services that are provided by the enterprise itself in the past.

Under the integrated SCM environment, for some specific goals and benefits, enterprises incline to form this kind of strategic cooperative relationship. It is beneficial to each side and it is favorable to reduce total cost, decrease the storage, enhance information sharing, improve mutual communication, keep the consistent partnership, and create better competitive advantages. Thus at every node in the supply chain, the financial situation, quality, production, customer satisfaction and performance can be enhanced and improved. Of course, strategic cooperation requires the emphasis on cooperation and confidence. Some failures of operation and management in enterprises result from either inner instability of core enterprise or instability of suppliers. In order to reduce the cost and risk of SCM, core enterprise should make sound decisions on supplier selection and then share benefits with the suppliers. Supplier management should include supplier's credit and reputation, product price, quality, delivery date etc. Supplier, as the object of enterprise purchasing activities, directly determines the quality of the raw material and parts purchased by the core enterprise, and the supplier greatly influences the competitive competence of the products produced by the core enterprise. Therefore, a

good decision-making method of supplier selection is quite necessary. The simulation approach of AHP may serve the purpose.

3.2 Establish the hierarchy

To show the method proposed in the previous section, we discuss a simplified problem. Here is the scenario: ABC Mechanical Manufacturing Company is the core enterprise in the supply chain. It encounters the problem of supplier selection. The important selection criteria for selecting the supplier include quality, price, delivery, service, management and culture, technology, financial situation etc. There are four available suppliers: Supplier 1, Supplier 2, Supplier 3, and Supplier 4.

As we know, “In the AHP, a decision process is modeled as a hierarchy. At each level in the hierarchy, the decision maker is required to make pair-wise comparisons between decision alternatives and criteria using a ratio scale. The AHP then determines the relative ranks of the decision alternatives. The ranks of the decision alternatives are given by the elements of the normalized principal right-hand eigenvector of a preference matrix consisting of the pair-wise comparisons between alternatives.”^[6] The traditional methodology of AHP includes four steps: firstly, establish the structure of the hierarchy; secondly, construct the pair-wise judgmental matrices; thirdly, sole rank and consistency inspection.

Thus, we construct the general hierarchy investigated in this research. All preference matrices were 4 by 4 and there were four matrices at each level, shown by Fig. 1

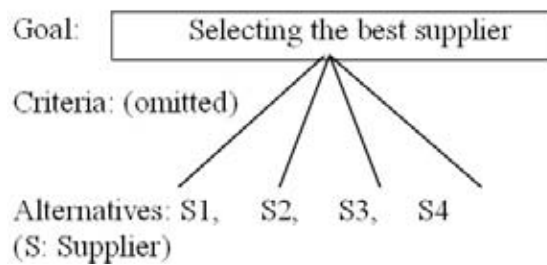


Fig. 1. the AHP hierarchy of supplier selection

3.3 Construct pair-wise comparison matrix

We assume that all comparison matrices are 4 by 4 and there are four matrices at each level as depicted in Fig. 1.

Fig. 2 shows the ratio scale used in this research: We assumed a normal distribution for each element

1--equally important	1--equally important
3--a little important	1/3--a little unimportant
5--important	1/5---unimportant
7--equally important	1/7--very unimportant
9- extremely important	1/9--extremely unimportant

Fig. 2. ratio scale used in constructing the judgmental matrix

of the preference matrices. A reciprocal matrix A is generated by the judgments of the experts by using the Delphi method to consider all the criteria of the four supplier candidates comprehensively:

$$A = [A_{ij}], \text{ as shown in the following,}$$

$$\begin{bmatrix} 1 & 6.414 & 6.464 & 1.118 \\ 0.156 & 1 & 3.990 & 1.116 \\ 0.155 & 0.251 & 1 & 3.763 \\ 0.894 & 0.896 & 0.266 & 1 \end{bmatrix}$$

$A = [A_{ij}]$ represents the reciprocal matrix of pair-wise comparison between each two suppliers. After calculating the weight of each supplier, we found that $w^1 = 0.541, w^2 = 0.189, w^3 = 0.129, w^4 = 0.141$. The results show that supplier 1 is the best and should be selected. However, there might be some uncertainty in the experts’s judgment.

Then, in order to examine the uncertainty of the judgmental matrix, a set of N random matrices: $[A^1, A^2, \dots, A^N]$, by assuming that each element of A greater than or equal to 1 is a normally distributed random variant with mean A_{ij} and standard deviation σ_{ij} . According to the previous experiences, we assume that the σ which represents the judgmental uncertainty of experts varies at the range from 2% to 20%.

For each A^K , the principal eigenvector w^k is calculated.

$\overline{RU_{rms}}$ and $\overline{P_{rev}}$ are calculated for the hierarchy. The consequences are shown in Tab. 1 and Fig. 3 and 4. The consequences of the simulation approach show that judgmental uncertainty and probability of rank

Table 1. simulation results of varying the level of judgmental uncertainty σ in a 4 by 4 pair-wise

Judgmental uncertainty σ	Average $\overline{P_{rev}}$	Average $\overline{RU_{rms}}$
0.02	0.0027	0.0034
0.04	0.01067	0.0066
0.06	0.018	0.0098
0.08	0.021	0.0102
0.1	0.0279	0.0163
0.18	0.041	0.028
0.2	0.043	0.0318

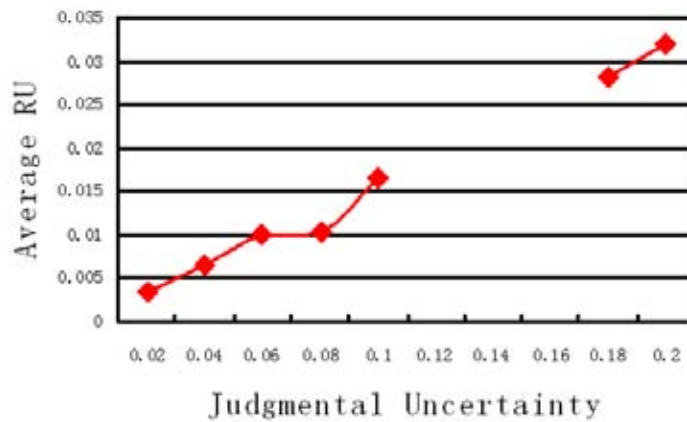


Fig. 3. Behavior of average $\overline{RU_{rms}}$ as judgmental uncertainty σ is increased in a 4 by 4 preference matrix.

reversal increase as σ increases. In this specific case, generally speaking, uncertainty is in a low degree since all the numerical values are small. In a sense, we may assume the judgmental matrix is correct and unnecessary to be altered, and the confidence in selecting Supplier 1 can be reinforced.

4 Conclusion

There are various decision-making methods, among which AHP (analytical hierarchy process) technique has been developed as a useful and simple method to deal with decision-making problems. Just as it was

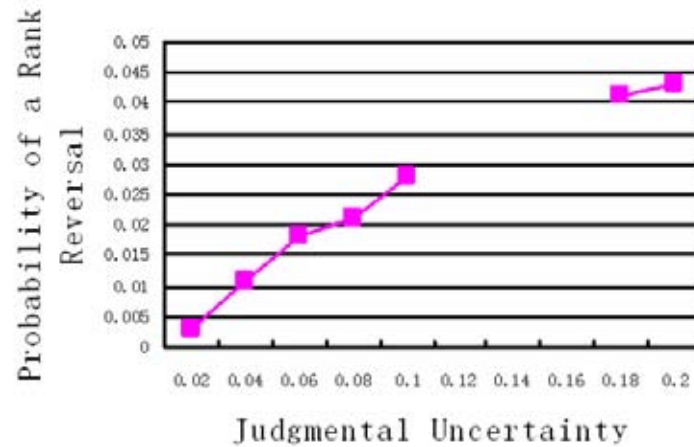


Fig. 4. The probability of a rank reversal P_{rev} as judgmental uncertainty σ is increased in a 4 by 4 preference matrix.

said, “AHP has been applied in a variety of areas as a useful and practical multi-criteria decision analysis tool.”^[6] In the area of SCM (supply chain management), we prefer to use AHP method. Recently, SCM is becoming a hot issue in international academic circle. Here goes the definition of supply chain and supply chain management given by American Supply Chain Association. The supply chain—a term now commonly used internationally—encompasses every effort involved in producing and delivering a final product or service, from the supplier’s suppliers to the customer’s customers. The core of SCM is a decision-making problem. Suppliers are very important in the supply chain because their performances not only influence the benefits of the core enterprise in the supply chain but also determine whether win-win can be achieved or not. Therefore, supplier selection is both a key problem and a tough one. We believe that by using a class of technique we may solve the key and tough problems of supplier selection in SCM. Thus we develop an AHP simulation methodology to deal with SCM problems.

References

- [1] M. Abramowitz, I. A. Stegun. Handbook of mathematical functions. *U.S. department of commerce*, 1972, 941.
- [2] A. Arbel, L. G. Varga. Preference simulation and preference programming: Robustness issues in priority derivation. *European Journal of Operational Research*, 1993, **69**: 200–209.
- [3] G. Box, M. E. Muller. A note on the generation of random normal deviates. *Annals of Mathematical Statistics*, 1958, **29**: 610–611.
- [4] B. L. Golden, E. A. Wasil, P. T. Harker. *The Analytic Hierarchy process: Applications and studies*. springer-verlag, New York.
- [5] J. Lu, C. Chen. Partner selection in supply chain based on AHP. *Hangzhou Electronic Industrial College Journal*, **21**(3): 94–98.
- [6] D. Paulson, S. Zahir. Consequences of uncertainty in the analytic hierarchy process. *European Journal of Operational Research*, 1993, 45–56.
- [7] T. L. Saaty. Modeling unstructured decision problems: The theory of analytical hierarchies. *Mathematics and Computers in Simulation*, 1978, **20**: 147–158.
- [8] T. L. Saaty. *The Analytic Hierarchy Process*. McGraw-Hill, New York, 1980.
- [9] T. L. Saaty. Axiomatic foundation of the analytic hierarchy process. *Management Science*, 1986, **32**: 841–855.
- [10] T. L. Saaty. How to make a decision: The analytic hierarchy process. *European Journal of Operational Research*, 1990, **48**: 9–26.
- [11] T. L. Saaty, L. G. Vargas. Uncertainty and rank order in the analytic hierarchy process. *European Journal of Operational Research*, 1987, **32**: 107–117.
- [12] L. G. Vargas. Reciprocal matrices with random coefficients. *Mathematical Modelling*, 1982, **3**: 69–81.
- [13] H. Yu, H. Ye. Partial selection and construction of SCM based on AHP. *Logistics Technology*, 2000, (1): 17–22.
- [14] F. Zahedi. Group consensus function estimation when preferences are uncertain. *Operations Research*, 1986, **34**(6): 883–894.
- [15] M. A. Zahir. Incorporating the uncertainty of decision judgments in the analytic hierarchy process. *European Journal of Operational Research*, 1991, **53**: 206–216.